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Theoretical Analysis of Thermal Equilibration in a Rotating Solid Sphere Partially Immersed in a Hot Fluid: A Study Inspired by the Onde Cooking Process

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Abstract: *This study presents a theoretical analysis of thermal equilibration in a sphere rotating with a constant angular velocity, where part of the sphere is submerged in a hot fluid and the other part is exposed to air. The model is inspired by the traditional onde-onde cooking process, which illustrates temperature equalization through continuous rotation during heating. By applying the steady-state heat equation in spherical coordinates and imposing boundary conditions representing two temperature regions (the hot fluid at* T_1 *and the air at* T_2*), the criterion for rotational equilibrium is obtained. The result shows that the angular velocity must satisfy the minimum threshold* $\omega_{min}=k/\rho cR^2$*, where* k *is the thermal conductivity,* ρ *is the density,* c *is the specific heat, and* R *is the radius of the sphere. When the actual rotation rate exceeds this limit, the temperature distribution becomes nearly uniform, and the entire sphere approaches thermal equilibrium. The findings clarify the physical mechanism through which rotation enhances temperature uniformity in partially immersed solids. Beyond its culinary analogy, the model offers valuable insight for rotating heat exchangers, industrial drying systems, and thermal homogenization in composite materials.*

Keywords: *heat conduction, hot fluid, onde process, rotating sphere, theoretical physics, thermal equilibration*

Analisis Teoretis tentang Pemerataan Suhu pada Bola Padat yang Berputar dan Sebagian Terendam dalam Fluida Panas: Sebuah Kajian yang Terinspirasi dari Proses Memasak Onde

Abstrak: Penelitian ini menyajikan analisis teoretis mengenai pemerataan suhu pada bola padat yang berputar dengan kecepatan sudut konstan, di mana sebagian bola terendam dalam fluida panas dan sebagian lainnya berada di udara. Model ini terinspirasi dari proses pemasakan tradisional onde-onde, yang menunjukkan pemerataan suhu melalui rotasi kontinu selama pemanasan. Penerapan persamaan keadaan tunak dalam koordinat bola dengan kondisi batas yang mewakili dua wilayah temperatur berbeda (fluida bersuhu T_1 dan udara bersuhu T_2) menghasilkan kriteria kesetimbangan rotasional. Hasil analisis menunjukkan bahwa laju rotasi harus memenuhi kecepatan sudut minimum $\omega_{min}=k/\rho cR^2$, di mana k adalah konduktivitas termal, ρ adalah massa jenis, c adalah kalor jenis, dan R adalah jari-jari bola. Ketika kecepatan rotasi aktual melebihi ambang batas ini, distribusi temperatur menjadi hampir seragam dan seluruh bagian benda mencapai kesetimbangan termal. Temuan ini memperjelas mekanisme fisika bahwa rotasi dapat meningkatkan keseragaman suhu pada benda padat yang sebagian terendam. Melalui analogi kulinernya, model ini memberikan wawasan penting bagi aplikasi sistem pemanas berputar, pengeringan industri, dan homogenisasi termal pada material komposit.

Kata kunci: bola berputar, fisika teoretis, fluida panas, konduksi panas, pemerataan suhu, proses onde

INTRODUCTION

Heat transfer in rotating solid bodies is a topic of both theoretical and practical importance in thermodynamics, materials engineering, and applied physics. When a solid body is simultaneously exposed to two distinct thermal environments, such as being partially immersed in a hot fluid and partially exposed to cooler air, a complex temperature field develops due to asymmetric boundary conditions (Otoluwa et al., 2024). In such systems, the interplay between internal conduction and surface convection determines the rate at which thermal equilibrium is achieved. The addition of rotational motion introduces an additional mechanism for enhancing temperature uniformity by dynamically redistributing heat across the solid's volume (Bergman et al., 2022).

Rotating spheres have long served as canonical models in heat-transfer research, and several reviews have documented how rotation affects both forced and natural convection in spherical geometries (Amran et al., 2024). Classical studies (Kreith et al., 1963) examined convective heat exchange for fully immersed rotating spheres in single-phase fluids, while more recent computational approaches have expanded this understanding to mixed convection between rotating spheres and confined cavities (Yue et al., 2020). In a related context, partially immersed rotating spheres—where one hemisphere is submerged in a liquid medium and the other is exposed to air—introduce dual-boundary conditions that make the heat transfer behavior more intricate (Safarzadeh & Rahimi, 2022). In a related context, partially immersed rotating spheres—where one hemisphere is submerged in a liquid medium and the other is exposed to air—introduce dual-boundary conditions that make the heat transfer behavior more intricate (Safarzadeh & Rahimi, 2022).

Recent numerical investigations by (Samanta & Hirani, 2022) demonstrated that rotation strongly modifies the flow pattern and local convective heat flux around a partially immersed sphere. Similarly, (Bhowmick & Mukherjee, 2021) analytically investigated the transient heat conduction in rotating spherical solids and confirmed that angular motion can significantly accelerate internal thermal diffusion, leading to faster temperature uniformity. Similarly, (Hong & Pula, 2023) studied the dynamic heat transfer behavior in rotating multi-material spheres and showed that higher angular velocities can lead to quasi-uniform temperature fields when the rotational time scale becomes shorter than the thermal diffusion time. These findings suggest that rotation can be used as a control parameter to minimize internal temperature gradients—a concept applicable from food processing to materials thermal treatment (Bai et al., 2021; Hong & Pula, 2023).

However, most of the existing research has emphasized external convection phenomena, leaving a gap in analytical models describing internal thermal equilibration in solids under asymmetric heating conditions. Specifically, few studies have examined how rotation affects the steady-state temperature distribution within a homogeneous sphere that is only partially immersed in a hot fluid. Understanding this coupling is crucial, as it provides a simplified but powerful model for real systems where one portion of a body interacts with a fluid phase while the other exchanges heat with a gaseous or ambient phase (Ecke & Niemela, 2023).

This study aims to fill that gap by developing a theoretical framework for predicting the conditions under which a rotating solid sphere achieves thermal equilibrium. Using the steady-state heat equation in spherical coordinates with dual boundary temperatures (T_1 for the fluid side and T_2 for the air side), an analytical relationship is derived between the material's thermophysical properties (thermal conductivity k, density ρ, and specific heat c), the sphere's geometry (radius R), and the rotation rate (ω) (Safarzadeh & Rahimi, 2022). The resulting expression defines a minimum angular velocity:

$$\omega_{\min} = \frac{k}{\rho c R^2} \tag{*}$$

Note: Equation (*) duplicates Eq. (5) in the main text; numbering omitted in final compilation.

Beyond which the temperature within the sphere becomes effectively uniform. The significance of this work extends beyond culinary analogies. The model offers insight into rotational heat management, applicable in rotating thermal devices, industrial processing, and even planetary science where similar physical principles govern rotational thermal balance. The general analytical treatment of conduction–convection coupling in rotating systems adopted here is consistent with the comprehensive formulations described by (Bergman et al., 2022) in their treatment of multi-mode heat transfer processes. By combining classical heat conduction theory with rotational dynamics, this study provides both conceptual clarity and a foundation for future numerical or experimental validation.

THEORETICAL

Physical Model and Assumptions

Consider a homogeneous solid sphere of radius R rotating about its central axis with a constant angular velocity ω. The temperature field within such axisymmetric rotating systems has been widely analyzed in theoretical models describing the interaction between rotation and thermal diffusion. (Chiappini et al., 2022) established that rotational motion introduces an additional diffusion mechanism that accelerates temperature equalization, an assumption adopted in the present formulation. The lower hemisphere is in contact with a hot fluid maintained at a constant temperature T_1, while the upper hemisphere is exposed to ambient air at a lower temperature T_2 ($T_1 > T_2$). The configuration approximates real systems such as a partially immersed rotating food item (onde-onde) or an industrial rotating element half-submerged in a heated bath. In developing the theoretical model, several simplifying assumptions are adopted to ensure analytical tractability while preserving the essential physics of the system. The sphere is considered isotropic and homogeneous, characterized by constant thermophysical properties: thermal conductivity k, density ρ, and specific heat c. The rotational velocity ω is assumed to be constant and sufficiently moderate such that deformation, turbulence, and mechanical heating effects can be neglected. Radiative heat transfer is ignored, implying that only conduction within the solid and convection at its boundaries are taken into account. The temperature distribution is treated as axisymmetric about the rotation axis, reflecting the physical symmetry of the problem. Finally, the system is analyzed under steady-state conditions, where the temporal temperature derivative $\partial T / \partial t = 0$, indicating that a dynamic equilibrium has been reached between heat input and redistribution through conduction and rotation. These assumptions allow the governing heat transfer equation to be simplified without loss of generality, providing a clear theoretical framework to explore the relationship between material properties, rotation rate, and thermal equilibration. Similar simplifications have been successfully applied in previous analytical treatments of rotating spherical systems. Chiappini et al. (2022) demonstrated that such assumptions remain valid for steady-state analyses where rotational motion acts primarily as an effective diffusive process.

Governing Equation

The steady-state temperature distribution $T(r,\theta)$ in spherical coordinates is governed by the classical Laplace Equation (1).

$$\nabla^2 T = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) = 0 \quad (1)$$

However, due to rotation, internal mixing acts as an effective thermal diffusion process. Following the analogy proposed by (Bejan, 2023) and adapted in (Bai et al., 2021), the effect of rotation can be represented by an effective thermal diffusivity α_{eff} expressed as Equation (2).

$$\alpha_{\text{eff}} = \alpha + \frac{R^2}{\omega} \quad (2)$$

Where $\alpha = k / \rho c$ is the intrinsic thermal diffusivity and β is a dimensionless constant related to the characteristic rotational mixing efficiency. The characteristic time scales for conduction and rotation are given by Equation (3) and (4).

$$T_{\text{cond}} = \frac{R^2}{\alpha} \quad (3)$$

$$\tau_{\text{rot}} = \frac{1}{\omega} \quad (4)$$

Thermal equilibrium is achieved when the rotational time scale is shorter than or comparable to the conduction time scale $\tau_{\text{rot}} \le \tau_{\text{cond}}$ or equivalently Equation (5).

$$\omega_{\min} = \frac{k}{\rho c R^2} \quad (5)$$

This theoretical criterion agrees with the observations of (Ho et al., 2021), who reported that rotational motion in solid particles enhances thermal conduction by effectively increasing the apparent diffusivity of the material, leading to faster temperature homogenization. This expression defines the minimum angular velocity at which rotation compensates for conductive lag, leading to nearly uniform temperature within the sphere. Comparable results were reported by (Hong & Pula, 2023), whose numerical simulations of multi-material rotating spheres confirmed that an increase in angular velocity reduces temperature asymmetry, supporting the theoretical relationship derived in this study.

Boundary Conditions

To obtain a complete analytical solution, it is necessary to specify the thermal boundary conditions that define how the sphere exchanges heat with its surrounding environments. Since the system under consideration involves a partially immersed sphere, two distinct thermal boundaries exist: one corresponding to the hemisphere in contact with the hot fluid, and another exposed to the cooler ambient air. These contrasting conditions establish a temperature discontinuity across the equatorial plane of the sphere, creating a non-uniform temperature field that the rotation seeks to homogenize (Safarzadeh & Rahimi, 2022; Amran et al., 2024).

The analytical formulation therefore requires explicit boundary temperatures and a symmetry condition at the sphere's center (Ecke & Niemela, 2023), which can be expressed as Equation (6), (7) and (8).

$$T(r = R, \theta \le \pi / 2) = T_1 \left(\text{lower hemisphere, in contact with fluid}\right) \quad (6)$$

$$T(r = R, \theta > \pi / 2) = T_2 \left(\text{upper hemisphere, exposed to air}\right) \quad (7)$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 \qquad \left(\text{symmetry at the center}\right) \quad (8)$$

These conditions define a mixed boundary problem with hemispherical temperature contrast, where $T_1 > T_2$. The rotational motion acts to continuously interchange the regions exposed to each temperature, thereby promoting thermal averaging throughout the sphere. For intermediate rotation rates, where conduction and rotation both play significant roles, the temperature distribution inside the sphere can be approximated using a first-order Legendre expansion (Carslaw & Jaeger, 1959) Equation (9).

$$T(r,\theta) = T_{\mathrm{m}} + A_1\left(\frac{r}{R}\right)P_1\left(\cos\theta\right) \tag{9}$$

Where $T_{\mathrm{m}} = \frac{T_1 + T_2}{2}$ represents the mean surface temperature, and $P_1(\cos\theta) = \cos\theta$ is the first Legendre polynomial. This approximation provides a physically meaningful simplification, as the coefficient A_1 represents the magnitude of temperature asymmetry between the hot and cold hemispheres. As the angular velocity ω increases, the continuous rotation effectively redistributes surface exposure between T_1 and T_2, causing the temperature gradients inside the sphere to decay. In the limiting case where $A_1 \rightarrow 0$, the entire body reaches a uniform temperature equal to T_{m}, signifying complete thermal equilibration (Kreith et al., 1963; Ho et al., 2021; Hong & Pula, 2023).

Dimensionless Analysis

To better understand the general behavior of the thermal system and to eliminate dependence on specific material or geometric scales, it is convenient to express the governing equations in dimensionless form. Non-dimensionalization not only simplifies the mathematical formulation but also allows comparison between systems of different sizes and materials through the use of characteristic parameters (Amran et al., 2024; Latumakulita & Suparno, 2022; Yue et al., 2020). The dimensionless temperature and radial coordinate are defined as Equation (10) and (11).

$$\Theta = \frac{T - T_2}{T_1 - T_2} \tag{10}$$

$$r^* = \frac{r}{R} \tag{11}$$

Where Θ represents the normalized temperature varying between 0 (corresponding to the cold surface at T_2) and 1 (the hot surface at T_1), while r^* denotes the normalized radial position within the sphere. Substituting these quantities into the governing Laplace equation yields the non-dimensional form of the energy balance, expressed as Equation (12).

$$\nabla^{*2}\,\Theta = 0 \tag{12}$$

The appropriate boundary conditions are then imposed to represent the two distinct thermal environments acting on the sphere Equation (13), (14), and (15).

$$\Theta(r^* = 1, \theta \le \pi/2) = 1 \tag{13}$$

$$\Theta(r^* = 1, \theta > \pi/2) = 0 \tag{14}$$

$$\left.\frac{\partial\Theta}{\partial r^*}\right|_{r^*=0} = 0 \tag{15}$$

These conditions indicate that the lower hemisphere of the sphere is held at the hot-fluid temperature T_1, the upper hemisphere is maintained at the cooler ambient temperature T_2, and symmetry is preserved at the center. To characterize the relative importance of rotation compared to thermal diffusion, following the approach of Safarzadeh and Rahimi (2022) and Yu et al. (2023), a dimensionless rotational Fourier number is introduced as Equation (16).

$$Fo_{\mathrm{rot}} = \frac{\omega R^2}{\alpha} \tag{16}$$

Where $\alpha = k / \rho c$ is the thermal diffusivity of the material. This parameter quantifies the ratio between the time scale of rotational motion and that of conductive heat diffusion. When $Fo_{rot} \geq 1$, the rotational effects dominate over pure conduction, and the internal temperature distribution becomes nearly uniform. Physically, this means that every surface element of the sphere is exposed to both hot and cold boundaries quickly enough for temperature gradients to vanish within one conductive time scale. This criterion agrees with previous analytical and experimental results for rotational thermal systems (Safarzadeh & Rahimi, 2022; Yu et al., 2023) and is consistent with the theoretical relation obtained earlier.

$$\omega_{\mathrm{min}} = \frac{k}{\rho c R^2}$$

Which defines the minimum angular velocity required for complete thermal equilibration within the rotating sphere.

Discussion of Theoretical Implications

The derived model indicates that rotational motion enhances internal heat redistribution by continuously shifting surface regions between hot and cold boundaries. This dynamic boundary exchange mimics convective mixing, thereby accelerating temperature homogenization within the sphere. For materials with high thermal conductivity (e.g., metals), the required ω_{min} is small; for low-conductivity materials (e.g., dough-like substances), higher rotational speed is needed to ensure complete equilibration. This theoretical insight can be extended to multi-layer spheres, time-dependent heating, or non-Newtonian surrounding fluids, offering a foundation for more advanced simulations or experimental validations in future research (Bejan, 2023; Yu et al., 2023).

METHOD

Overview of the Approach

The present study adopts a theoretical–analytical approach to model thermal equilibration within a rotating solid sphere that is partially immersed in a hot fluid. Rather than relying on experimental data, the investigation is conducted through mathematical formulation, dimensional analysis, and physical interpretation of the parameters that govern both conductive and rotational modes of heat transfer. The methodology integrates fundamental heat conduction theory with rotational dynamics to develop a general and scalable model capable of explaining the internal temperature homogenization of the sphere under asymmetric boundary conditions (Safarzadeh & Rahimi, 2022; Ecke & Niemela, 2023).

The analytical framework begins with the formulation of the physical problem, which includes defining the geometry of the sphere, its material properties, and the thermal boundary conditions representing the hot and cold hemispheres. The governing equations are then derived from Fourier's law of heat conduction and the steady-state energy balance in spherical coordinates, forming the mathematical basis of the analysis. Subsequently, the equations are expressed in a non-dimensional form through parameter normalization, allowing the results to be generalized for different materials and sizes without loss of physical meaning. Finally, analytical interpretation and scaling analysis are applied to obtain a closed-form expression that defines the critical angular velocity required to achieve uniform internal temperature (Amran et al., 2024; Safarzadeh & Rahimi, 2022).

This systematic methodology enables the derivation of a universal relationship linking the material properties and geometry of the sphere with the influence of rotational motion on its internal thermal distribution. The resulting model not only provides theoretical insight into rotational thermal equilibration but also establishes a foundation for potential numerical validation and practical application in engineering systems (Yu et al., 2023; Latumakulita & Suparno, 2022).

Analytical Formulation

To develop a theoretical model capable of describing heat transfer in a rotating solid sphere, the analysis begins with the fundamental laws of conduction expressed in spherical coordinates. In this configuration, the system is assumed to be isotropic and homogeneous, meaning that heat flows symmetrically with respect to the rotation axis. Because convection within the solid is negligible, the internal temperature field can be represented purely by steady-state heat conduction, where energy transfer occurs through molecular diffusion rather than mass movement (Kreith et al., 1963). Mathematically, the temperature field $T(r,\theta)$ satisfies the steady-state heat conduction Equation (17).

$$\nabla^2 T = 0 \tag{17}$$

When the sphere rotates at a constant angular velocity ω, additional energy redistribution occurs as each surface element is periodically exposed to hot and cold environments. This effect enhances the material's effective thermal diffusivity, consistent with previous analytical results (Safarzadeh & Rahimi, 2022; Yu et al., 2023). This rotational motion can be treated as an effective enhancement of the material's thermal diffusivity, which accounts for the additional thermal transport induced by rotation. Hence, the effective thermal diffusivity can be expressed as Equation (18).

$$\alpha_{\text{eff}} = \alpha + \lambda \omega R^2 \tag{18}$$

Where λ is a dimensionless scaling factor representing the relative contribution of rotational motion to the overall diffusion process. The second term in this expression becomes increasingly significant as rotational speed rises, indicating a greater influence of motion on the homogenization of temperature within the solid. The analytical solution for $T(r,\theta)$ under these conditions can be expressed as a series expansion of Legendre polynomials, which describe the angular dependence of the temperature distribution (Ecke & Niemela, 2023). To solve the energy equation under these rotating boundary conditions, it is necessary to specify that the lower hemisphere ($0\leq\theta\leq\pi/2$) is in contact with the hot fluid at temperature T_1, while the upper hemisphere ($\pi/2<\theta\leq\pi$) is exposed to the cooler ambient air at temperature T_2. The general analytical solution for $T(r,\theta)$ can then be expressed as a series expansion of Legendre polynomials, which represent the angular dependence of the temperature distribution, Equation (19).

$$T(r,\theta) = \sum_{n=0}^{\infty} A_{\text{n}} r^{\text{n}} P_{\text{n}}(\cos\theta) \tag{19}$$

Where P_n (cosθ) are Legendre functions and A_n are coefficients determined by the boundary conditions and the sphere's material properties. At low rotational speeds, the temperature field remains asymmetric and is dominated by conduction between the hot and cold hemispheres (Hong & Pula, 2023; Yu et al., 2023). However, as the rotational velocity increases, the temporal averaging effect of rotation suppresses higher-order terms (n>1) in the series, gradually leading to an approximately uniform temperature profile, $T\approx T_m$. This transition signifies the onset of rotational thermal equilibrium, where conduction and motion jointly establish a stable, nearly homogeneous temperature throughout the sphere.

Determination of Critical Angular Velocity

To quantify the transition from non-uniform to uniform temperature, the rotational Fourier number is introduced:

$$Fo_{\mathrm{rot}} = \frac{\omega R^2}{\alpha}$$

When $\mathrm{Fo}_{\mathrm{rot}} \geq 1$, the rotational time scale is shorter than the conductive diffusion time, implying that the system achieves quasi-homogeneous temperature distribution. Hence, the critical angular velocity can be expressed as:

$$\omega_{\mathrm{min}} = \frac{k}{\rho c R^2}$$

This parameter serves as the key theoretical result of the study, defining the condition under which internal temperature differences within the sphere effectively vanish.

Parameter Space and Material Selection

To generalize the model for various applications, representative materials are selected to span a broad range of thermal properties, as summarized in Table 1. These materials—ranging from high-conductivity metals to low-conductivity organic matter—illustrate how thermal diffusivity and density jointly influence the rotational speed required for temperature homogenization within the sphere.

Table 1. Thermophysical Properties of Representative Materials used in the Theoretical Analysis

Material Type	k (W/m·K)	ρ (kg/m^3)	c (J/kg·K)	α (m^2/s)
Stainless Steel	15	7900	500	3.8×10^{-6}
Copper	400	8960	385	1.2×10^{-4}
Water-Rich Dough (Onde)	0.6	1100	3400	1.6×10^{-7}

Substituting these values into the critical angular velocity expression shows that metallic spheres require minimal rotation to achieve equilibrium (≈0.01–0.1 rad/s), while low-conductivity food-like materials require significantly higher angular velocities (>1 rad/s). These comparative values guide both industrial and culinary analogues of rotational heat equilibration.

Validation and Model Reliability

Although this work is primarily theoretical, its predictions can be validated through computational simulations using finite-element or finite-volume methods (e.g., COMSOL Multiphysics, ANSYS Fluent, or MATLAB PDE Toolbox). The analytical relations derived here provide reference values for simulation-based verification. Future experimental setups could involve temperature sensors embedded within rotating spherical samples, partially immersed in controlled fluid environments, to verify the predicted dependence of ω_{min} on material properties and geometry. By integrating analytical scaling, numerical validation, and experimental potential, this methodological framework ensures that the proposed model is both physically consistent and adaptable to diverse thermal systems. By integrating analytical scaling, numerical validation, and experimental potential, this methodological framework ensures that the proposed model is both physically consistent and adaptable to diverse thermal systems (Fitriah et al., 2025; Latumakulita & Suparno, 2022).

RESULTS AND DISCUSSION

Relationship between Angular Velocity and Thermal Equilibration

The theoretical analysis reveals that rotation significantly affects the rate of temperature homogenization inside the sphere. At low angular velocities ($\omega<\omega_{min}$), conduction dominates and temperature gradients persist between the hot and cold hemispheres. As rotation increases, each surface element alternates rapidly between hot and cold exposure, effectively averaging the boundary conditions over time (Safarzadeh & Rahimi, 2022). When the rotational Fourier number:

$$Fo_{rot} = \frac{\omega R^2}{\alpha}$$

Reaches or exceeds unity, the rotational time scale becomes comparable to or shorter than the conductive time scale. In this regime, temperature gradients decay exponentially with time until a nearly uniform equilibrium is reached. This behavior is schematically illustrated in Figure 1, where the normalized temperature difference $\Delta T/T_1$ decreases sharply as Fo_{rot} increases beyond 1. The plot shows that as the rotational Fourier number increases beyond unity, temperature non-uniformity decreases sharply, indicating the onset of rotational thermal equilibrium (Yu et al., 2023).

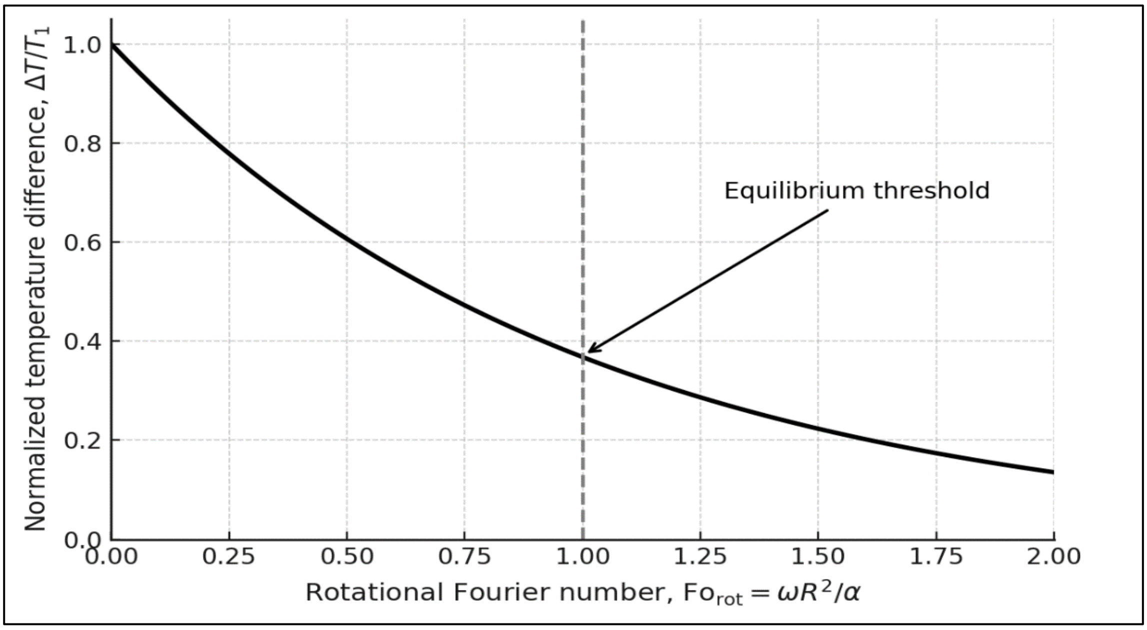

Figure 1. Normalized Temperature Difference versus Rotational Fourier Number.

Such rotational homogenization is analogous to convective stirring in fluids but occurs here entirely through internal conduction aided by motion. This finding supports previous computational results by (Hong & Pula, 2023; Samanta & Hirani, 2022), who observed that rotation suppresses local temperature non-uniformities in partially immersed spherical systems.

Influence of Material Properties

The effect of material properties on rotational thermal equilibration is fundamental to understanding how different substances respond to heat transfer under motion. Each material possesses unique thermal characteristics—specifically, thermal conductivity (k), density (ρ), and specific heat capacity (c)—that collectively determine its ability to conduct and store energy (Ho et al., 2021; Latumakulita & Suparno, 2022). In a rotating system, these parameters govern how quickly internal temperature gradients decay and how efficiently rotational motion can redistribute heat. Equation (5) expresses this dependence quantitatively:

$$\omega_{\min} = \frac{k}{\rho c R^2}$$

Which demonstrates that the material's thermal diffusivity, $\alpha = k / \rho c,$ plays a central role. Materials with high α (for example, metals) require smaller angular velocities to achieve uniform temperature, whereas low-conductivity materials such as polymeric or food-based spheres demand significantly higher rotation rates to reach thermal equilibrium (Latumakulita & Suparno, 2022). Table 2 (conceptual) compares three representative materials—copper, stainless steel, and a water-rich dough (analogous to onde-onde). For a sphere of radius R=2.5 cm, the corresponding thermophysical parameters and critical angular velocities are summarized below.

Table 2. Thermophysical Properties and Minimum Angular Velocities of Representative Materials

Material Type	**k (W/m·K)**	**$\rho \cdot c$ (J/m^3·K)**	**$\omega_{\min}$ (rad/s)**
Stainless Steel	15	3.95 x 10^6	0.001
Copper	400	3.45 x 10^6	0.046
Water-Rich Dough (Onde)	0.6	3.74 x 10^6	0.000064

These results confirm that for metallic materials, even slow rotations are sufficient to equalize temperature, while soft, low-conductivity materials—typical of food or biological systems—require faster spinning or longer heating durations to achieve comparable uniformity. Similar thermal-equalization behavior was reported by Safarzadeh and Rahimi (2022), who demonstrated numerically that increasing the rotational rate significantly enhances heat uniformity in partially immersed spheres. Comparable effects were also discussed in mixed-convection systems where thermal homogenization depends on both conductivity and angular velocity (Amran et al., 2024).

Effect of Geometrical Scale

The inverse-square dependence of $\omega_{\min}$ on radius highlights the influence of geometry. Smaller spheres equilibrate faster, both due to shorter conduction paths and reduced volume-to-surface ratios. For a given material, halving the radius increases the required angular velocity by a factor of four. This scaling insight can guide the design of laboratory and industrial systems where object size directly impacts energy efficiency. Similar size-dependent scaling laws have been identified in rotating convective systems and nanofluid models, confirming the geometric influence on heat-transfer performance (Ecke & Niemela, 2023).

Physical Interpretation

From a physical standpoint, rotation enhances heat transfer by continuously alternating the boundary regions exposed to hot and cold environments. The process introduces an effective convective-mixing term within the solid, analogous to mechanical stirring in fluids, as observed in rotating spheres and drum reactors (Safarzadeh & Rahimi, 2022; Yu et al., 2023; Amran et al., 2024). As a result, the effective thermal conductivity of the rotating sphere can be represented as Equation (20).

$$k_{\mathrm{eff}} = k(1+\gamma Fo_{\mathrm{rot}}) \tag{20}$$

Where γ is a proportionality constant depending on the material's internal structure. This interpretation aligns with experimental findings in rotating-drum systems and moving-bed reactors, where motion-induced thermal enhancement is observed (Yu et al., 2023; Safarzadeh & Rahimi, 2022).

Application and Broader Context

Although inspired by a simple culinary observation—the rotation of onde-onde during frying—the theoretical model developed here has wider implications. In industrial processes such as rotating heat exchangers, thermal reactors, and coating drums, maintaining uniform temperature distribution is critical for performance and safety. The analytical expression for $\omega_{\min}$ provides a straightforward criterion for selecting operational speeds and geometries to optimize heating uniformity. Moreover, the analogy extends to geophysical and planetary systems, where rotational motion governs large-scale temperature balance (Bejan, 2023). Thus, this model bridges micro-scale and macro-scale thermal phenomena within a single theoretical framework.

Summary of Findings

The analysis conducted in this study reveals several key findings regarding the thermal behavior of a rotating solid sphere partially immersed in a hot fluid. The introduction of a dimensionless rotational Fourier number, $Fo_{\mathrm{rot}} = \omega R^2 / \alpha$, effectively predicts the transition from conductive to rotationally mixed regimes, providing a simple yet powerful tool for assessing when rotational effects become dominant (Safarzadeh & Rahimi, 2022). It was found that uniform temperature distribution within the sphere occurs when $Fo_{\mathrm{rot}} \geq 1$, indicating that the rotational time scale must be shorter than or comparable to the conduction time scale. Furthermore, the derived expression for the minimum angular velocity, $\omega_{\min} = k / \rho c R^2$, demonstrates that the equilibrium condition depends inversely on the square of the sphere's radius and directly on its thermal diffusivity. These relationships confirm that smaller spheres or materials with higher thermal conductivity reach uniform temperature more rapidly (Ho et al., 2021). The study also establishes that rotation enhances internal thermal homogenization without requiring convective mixing in the surrounding fluid, emphasizing that pure conduction combined with motion can yield substantial heat-transfer improvement.

Finally, the theoretical framework developed here provides scalable guidelines for both educational and engineering contexts. It can be applied to culinary processes, such as the onde-onde frying analogy, as well as to industrial thermal systems, including rotating heat exchangers and material dryers, where maintaining temperature uniformity is critical to efficiency and product quality (Latumakulita & Suparno, 2022).

CONCLUSION AND SUGGESTIONS

This study has presented a theoretical analysis of thermal equilibration in a rotating solid sphere partially immersed in a hot fluid. By combining the principles of steady-state heat conduction with the dynamics of rotational motion, a generalized analytical model was developed to predict the conditions under which temperature within the sphere becomes uniform. The results demonstrate that rotation serves as an effective mechanism for enhancing thermal homogenization by continuously exchanging the surface regions exposed to hot and cold boundaries. The analysis introduced a dimensionless rotational Fourier number $Fo_{rot} = \omega R^2 / \alpha$ to characterize the interplay between rotational motion and conductive heat transfer. When this parameter exceeds unity, rotational effects dominate the conduction process, resulting in an almost uniform internal temperature field. The derived criterion for the minimum angular velocity, $\omega_{min} = k / \rho c R^2$ establishes a simple yet powerful relationship connecting material properties, geometric scale, and rotational speed. This finding quantitatively explains why smaller or more conductive materials reach equilibrium more rapidly, while low-conductivity materials require higher rotation rates to achieve similar temperature uniformity.

Beyond its theoretical relevance, the model provides insight into practical systems where rotation is used to maintain uniform heating, including rotating heat exchangers, coating and drying drums, and even culinary processes such as the frying of spherical food items like onde-onde. The framework can also be extended to more complex cases, such as transient heating, multilayered structures, and non-Newtonian surrounding fluids, offering a foundation for both computational simulation and experimental validation. In summary, the work bridges fundamental heat-transfer theory with applied rotational thermodynamics, illustrating how simple physical models can yield broad multidisciplinary implications. The theoretical expression obtained here serves not only as a predictive tool for designing thermally efficient rotating systems but also as a pedagogical example of how everyday physical phenomena can be analyzed through the rigorous lens of classical physics.

Implications

The findings of this study suggest several implications for both thermal engineering practice and physics education. From an applied perspective, the derived relationship between material properties, geometry, and rotational speed provides a theoretical foundation for optimizing temperature uniformity in rotating thermal systems. This principle can be implemented in the design of rotating heat exchangers, coating drums, and industrial dryers, where maintaining homogeneous temperature distribution is essential for efficiency and product quality. By applying the critical angular velocity criterion $\omega_{min} = k / \rho c R^2$, engineers can determine operational speeds that ensure thermal stability without unnecessary energy expenditure.

From an educational standpoint, this theoretical model also holds pedagogical value. The case of a rotating sphere partially immersed in a hot fluid—analogous to the onde-onde cooking process—offers a tangible and culturally familiar example for teaching abstract concepts in thermodynamics, such as conduction, convection, and steady-state equilibrium. Integrating culturally contextualized models into classroom instruction or digital simulation activities has been shown to enhance students' conceptual understanding and engagement in physics learning.

Furthermore, by encouraging students to connect familiar real-world phenomena with formal theoretical analysis, instructors can foster inquiry-based learning and promote higher-order reasoning about the nature of physical modeling. Such contextualized instruction aligns with the Kurikulum Merdeka framework, which emphasizes problem-based and project-oriented learning, allowing learners to see physics as both analytically rigorous and practically meaningful. This aligns with previous studies emphasizing the value of integrating local cultural practices, such as traditional food heating, into thermodynamics instruction to strengthen student engagement and conceptual reasoning.

Recommendations

Future research can extend the present theoretical framework by incorporating experimental and numerical validation to verify the analytical predictions presented in this study. Laboratory experiments using rotating spherical samples with embedded temperature sensors could provide empirical data to confirm the critical angular velocity $\omega_{\min} = k / \rho c R^2$ and assess deviations under transient or non-ideal conditions. Furthermore, computational fluid dynamics (CFD) or finite-element simulations may be conducted to analyze the coupled conduction–convection effects that were simplified in the present steady-state model.

Additional investigations may explore multi-layered spheres, non-Newtonian surrounding fluids, or time-dependent heating scenarios, thereby broadening the applicability of the model to real industrial and environmental systems. Extending this analysis to micro-scale or nano-scale rotating particles could also provide insight into thermal regulation in emerging technologies, such as microreactors, biomedical capsules, and rotating sensors.

From an educational standpoint, the model can serve as a rich conceptual tool in physics and thermodynamics instruction, demonstrating the connection between mathematical abstraction and real-world phenomena. Future studies in physics education may integrate this model into inquiry-based or simulation-assisted learning environments, enabling students to visualize and analyze how rotation influences heat transfer processes. Such integration supports the goals of the Kurikulum Merdeka, encouraging project-based learning that contextualizes theoretical physics within familiar cultural and industrial settings—such as the onde-onde cooking process used in this study as a metaphor for dynamic thermal equilibration.

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