Designing the learning trajectory for the topic of circles through a tambourine context

Anita Juniarti 1, Zingiswa Jojo 2, Rully Charitas Indra Prahmana 1*

1 Universitas Ahmad Dahlan, Yogyakarta, Indonesia
2 University of South Africa, Pretoria, South Africa
*Correspondence: rully.indra@mpmat.uad.ac.id

Received: 20 January 2022 | Revised: 20 February 2022 | Accepted: 10 March 2022 | Published 11 March 2022 © The Author(s) 2022

Abstract

A tambourine is one of the musical instruments commonly used in Islam. It is also used in an extracurricular activity in various schools so that many students are familiar with the instrument, making it relevant for learning mathematics. Furthermore, there is an approach to learning mathematics called Indonesian Realistic Mathematics Education (IRME), where students start the learning with contexts close to their lives. This study aims to design a learning trajectory using the IRME approach with a tambourine context to support students' understanding of circles. This learning progresses from the informal stage to the formal through the IRME approach. The study applied design research with three stages: preliminary design, design experiments, and retrospective analysis. The research subjects were 19 six-grade students in one of the elementary schools in Balangan, South Kalimantan, Indonesia. The instruments used were videos to observe the learning process and how students work on the questions given, photos to reference students’ work, and a test in a student worksheet to obtain data on students’ work. The results reveal the learning trajectory practised using the tambourine as the context seen in the student's daily activities. The learning trajectory consists of four events: assembling the tambourine, drawing an illustration of the tambourine, listing the parts of a circle, and solving a problem related to the parts of the circle. In addition, this study also shows that learning trajectory activities have essential roles in supporting students’ understanding of the concept of a circle.

Keywords: Circle, Design Research, Indonesian Realistic Mathematics Education Approach, Tambourine

Introduction

Mathematics is a science highly beneficial for human life. It plays a role in all aspects, including the development of science and technology (Jayanthi, 2019; Godino, Burgos, & Gea, 2021).
Given the importance of mathematics for human life, students study this subject from elementary school to university. However, most students think mathematics is a challenging subject (Putra & Yasin, 2021; Ng, 2021), and this culminates to lower enthusiasm for learning mathematics compared to other subjects.

Students’ lack of enthusiasm for mathematics is always associated with numbers and formulas (López-Díaz & Peña, 2021; Østbø & Zachrisson, 2021). Students mainly consider mathematics as a subject related to memorizing formulas without knowing how the formulas are obtained and what the concepts mean (Birgin & Uzun Yazıcı, 2021; Khanal, Panthi, Kshetree, Acharya, & Belbase, 2021). Without understanding the concepts, students usually find it hard to learn mathematics towards a higher learning process. But according to Zulkardi (2002) learning mathematics should not start from memorizing countless formulas, but the essence of learning mathematics is understanding the concept. Moreover, when students understand mathematics concepts, it becomes easy for them to recall and apply the associated formulae.

Gravemeijer, Stephan, Julie, Lin, and Ohtani (2017) assert that in the process of learning mathematics, understanding concepts is a critical foundation for thinking in solving mathematical and everyday problems. In addition, Zulkardi (2002) notes that mathematics lessons emphasize understanding concepts, meaning that in studying mathematics, students must understand mathematical concepts first to be able to solve problems and to apply the learning in the real world. Hence, understanding mathematical concepts is a basic and an essential ability to master other mathematical abilities.

Furthermore, researchers (Ahdhianto, Marsigit, Haryanto, & Santi, 2020; Zhang & Cai, 2021) caution that if basic skills are not well understood, students will have difficulty designing and solving problems in mathematics. Thus, a learning approach to help teachers design a lesson that can involve students in learning and support them in understanding the concept of learning easily is necessary. One approach that can be used is the Indonesian Realistic Mathematics Education (IRME) approach, adapted from Realistic Mathematics Education (RME) and has been developed following the context, cultural values, and local wisdom of Indonesia (Sembiring, Hadi, & Dolk, 2008).

The IRME is one of the learning approaches emphasizing the real-world context that students can find in everyday life (Zulkardi, 2002). It also presents realistic problems as the first step to understanding mathematical concepts. The use of realistic problems is useful for increasing students' motivation and interest in mathematics (Yuanita, Zulnaidi, & Zakaria, 2018; Julià & Antolí, 2019; Angraini, Prahmana, & Shahrill, 2021). The use of realistic problems will also encourage students to think that mathematics is part of their daily activities so that the knowledge becomes more meaningful.

Several studies have been conducted to support students' understanding by designing learning trajectories using the IRME approach. This approach uses context as a starting point to help students understand the concept of a circle. For example, Fitri and Prahma (2020) used the IRME approach through a Ferris wheel to help students learn circles. In line with those findings, the glass context in the IRME approach can also help students learn the concept of the
circumference of a circle (Juandi, Kusumah, & Tamur, 2022). Therefore, IRME is considered capable of supporting students in understanding the concept of a circle.

IRME starts with the real-world context. One context close to students is the cultural context. Cultural diversity around students can be integrated into learning mathematics (Chahine, 2011; 2021; Ogunkunle & George, 2015). In addition, there is a mathematical concept in the cultural element that can be a starting point in learning mathematics and a solution to introduce culture (Maryati & Prahmana, 2018; Risdiyanti & Prahmana, 2018). In this study, researchers used the context of a tambourine to learn about circles.

Indonesia is a country where most of the population is Muslim. In Islamic culture, events are often held using traditional musical instruments, one of which is the tambourine (Jamil, 2020; Zamhari & Mustofa, 2020). Furthermore, tambourine can also be found anywhere; even in various schools, it is used in extracurricular activities. Research conducted by Agustina, Gazali, Chairani, and Nareki (2021) presented the results of the exploration of ethnomathematical forms in the traditional musical instrument of tambourine. Mathematical concepts found were the physical form of the tools used, like curved circles, tubes, and cones. On the other hand, the game technique used the mathematical concept of counting beats to hear the music released from the tambourine game in harmony. In addition, Hartini's research (2018) found the promotion of religious values, self-confidence, social care, responsibility, honesty, and discipline in the tambourine. The use of cultural contexts close and familiar to students’ daily lives makes it easier for students to understand mathematical concepts. It is easy to understand the usefulness of mathematical concepts in dealing with problems that exist around students. It is in line with D'Ambrosio's (2007) statement, who said that mathematics education needs to contextualize mathematics with the environment and culture of students. This is because, in essence, science appears as a culture in its efforts to respond to phenomena and problems in the reality of life (Rosa & Orey, 2016). So, this study discusses the context of the tambourine, especially how students learn the parts of a circle while studying culture, using a tambourine as a starting point in the learning process.

As seen in some previous studies, cultural context can be used as a starting point in mathematics learning. Risdiyanti and Prahmana (2018) explored Javanese culture in Yogyakarta containing geometry transform concepts, namely the motif of Batik. In addition, Fitri and Prahmana (2020) also made a learning design for the subject of a circle using the Ferris wheel context and supported students’ understanding. Furthermore, there is another cultural context namely Indonesian shadow puppets and Mahabharata stories that can be use as starting point in learning sets theory (Risdiyanti & Prahmana, 2021; Prahmana & Istiandaru, 2021). Therefore, a cultural context can be used as a starting point in learning mathematics.

This study designs a mathematics learning trajectory applying the IRME theory and using a cultural context, namely the tambourine as a starting point, in learning mathematics on the topic of a circle and applied it to the primary students who participated in this study. The contribution of this research is in providing an alternative framework for learning circles using daily activities close to students. This research is expected to be a reference for further researchers in developing learning media with similar approaches and topics.
Methods

The research method used is design research. Design research includes systematic learning starting from designing, developing, and evaluating all interventions related to education, such as learning process programs, learning environments, teaching materials, learning products, and learning systems (Bakker, 2018). It is one of the solutions to address research questions and examine the students' learning process. In addition, design research helps develop a Hypothetical Learning Trajectory (HLT) to support students' understanding of the circle concept. This research took place in one of the public elementary schools in South Kalimantan, Indonesia. The participants were sixth graders consisting of 7 male and 12 female students. There are three stages in this design research: preliminary design, experimental design, and retrospective analysis (Plomp & Nieveen, 2013).

Preliminary design

The preliminary design phase is to formulate a learning trajectory outlined and refined in the experimental design phase (Bakker, 2018). There were three activities conducted at this stage. The first was to conduct observations and interviews with teachers. The second was obtaining information about students' difficulties in learning circles and activities to support students’ understanding of the circle concept. The third was to prepare learning activities through library research on the circle concept using a tambourine and the IRME approach. This information was used to design the Hypothetical Learning Trajectory (HLT), consisting of three components: learning objectives, learning activities, and the hypothesized learning process (Akker, Gravemeijer, Mckenney, & Nieveen, 2006). The hypothesized learning process or the conjecture became a guideline that will develop in every learning activity. It should also be flexible and subject to revision during the design trial phase. Table 1 presents the overview of the students’ activities and conjectures.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Main Goal</th>
<th>Conjecture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Making tambourine</td>
<td>Finding the parts of the tambourine</td>
<td>Students collect information about tambourines and make the model.</td>
</tr>
<tr>
<td>model</td>
<td></td>
<td>Students are confused about arranging tambourine jingles.</td>
</tr>
<tr>
<td>Illustrating the</td>
<td>Determining the centre of the circle</td>
<td>Students draw the centre of the circle directly.</td>
</tr>
<tr>
<td>tambourine</td>
<td></td>
<td>Students measure with a ruler and find the centre.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Students draw two intersecting straight lines and mark the point of intersection.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Students fold the paper into equal parts and mark the point of intersection.</td>
</tr>
</tbody>
</table>
### Table 1. (continued)

<table>
<thead>
<tr>
<th>Listing the parts of the circle</th>
<th>Completing the table by drawing and defining the part of the circle</th>
<th>Students fill all tables correctly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solving problems related to the parts of the circle</td>
<td>Determining the relationship between the radius and the diameter of a circle</td>
<td>Students fill in several tables correctly</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Students cannot fill all tables correctly</td>
</tr>
<tr>
<td></td>
<td>Determining the difference between the diameter and the chord of a circle</td>
<td>Students cannot determine the relationship between the two.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Students can determine the difference in diameter and chord</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Students cannot determine the difference in diameter and chord, so they cannot determine the difference between the two.</td>
</tr>
</tbody>
</table>

### Design experiment

This phase was divided into two cycles: teaching experiments and pilot experiments (Bakker, 2018). In the teaching experiment, the HLT designed in the previous phase was implemented in small groups of nine students. The purpose of this stage was to explore and observe students' strategies and understanding during the learning process. Furthermore, the HLT was revised and improved based on the evaluation suggestions in the first stage. The revised HLT in the first cycle was implemented in this second cycle. The second stage, the pilot experiment, was carried out in a large class of 19 students. Data were collected through classroom observations with video recordings and student worksheets. Finally, the group discussion documentation was also recorded to illustrate students' understanding during the learning process.

### Retrospective analysis

After conducting the design experiment, all the data collected were analyzed at this stage by comparing the conjectures in the HLT designed in the first stage with the results of the implementation of the learning trajectory. Furthermore, the learning trajectory was the guide in analyzing the data, so it was possible to investigate and explain how students understand the concept of a circle. The analysis results resulted in a description of the learning trajectory of the circle using the context of a tambourine.
Results and Discussion

This research developed a learning trajectory for the parts of the circle through several learning activities for six graders. The learning activities consisted of four activities, making a tambourine model, illustrating a tambourine, listing the parts of a circle, and solving problems related to parts of a circle. The teacher started the lesson by asking the students about the Maulid Habsyi event. The teacher asked questions to clarify students' knowledge of the tambourine as the context used in the learning process. Students could mention the various tools used, as shown in Dialogue 1.

Dialogue 1

Teacher : Have you ever watched the Maulid Habsyi event?
Student : Yes, I have
Teacher : Which tools do you use or play?
Student : Tambourine, drum, and bass
Teacher : What kind of tambourine is it?
Student : Round, circular, when it is tapped, it makes a buzzing sound

Dialogue 1 indicates that students know about tambourines, so the teacher introduced tambourines as a context and starting point in the learning process. Next, the teacher showed the tambourine musical instrument. The teacher hoped that students had the same perception about the tambourine. Furthermore, the teacher outlines the learning objectives that students must achieve. Students had to make a tambourine model and determine the centre of the circle. The teacher also informed students about learning activities, such as group discussions, presentations, and class discussions. The teacher asked students to sit in groups of four, distributed the student worksheet, and invited students to familiarize themselves with it.

Activity 1: Making a tambourine model

In this informal stage, students were introduced to the circle through the tambourine. They tried to illustrate a tambourine based on the instructions on the student worksheets. Furthermore, there were four activities that students had to do to make a tambourine model: assembling two paper plates using staples, punching eight holes in the edge of the paper plate using a paper punch in a clockwise direction, preparing the jingles, and tucking the thread in the holes, and tying the jingles clockwise (in the order of red, gray, yellow, green, light blue, dark blue, purple, and pink) in the holes on the edge of the paper plate. The use of something tangible for students, such as a tambourine, in the learning process, is one of the characteristics of the IRME approach, namely using a contextual problem. Figure 1 shows students' activity in the process of making a tambourine model.
Activity 2: Drawing an illustration of the tambourine

In this activity, the teacher asked students to look at the problems in the student worksheet (known as LAS). LAS explained that four jingles would be attached to the tambourine in the following positions: A is on the “merah” (red) jingle, B is on the “abu-abu” (gray) jingle, C is on the “hijau” (green) jingle and D is on the “biru tua” (dark blue) jingle.

Next, students were asked to draw a circle on the worksheet that describes the position of the four jingles, with the condition that the red jingle is in the top position. The way students draw circles differed.
Groups 1 and 2 drew circles using coins (Figure 2), and Groups 3 and 4 used a slightly different strategy from the other groups (Figure 3); they drew circles using a compass (Figure 4). The second step was determining the center point of the circle with a folding strategy as they had done in the previous meeting. This is another characteristic of the IRME approach, using student contributions and is related to other learning topics. Finally, students drew a jingle showing the four-jingle positions as illustrated in Figure 4.

**Activity 3: Listing the parts of the circle**

In this activity, the teacher asks students to look at the problems in the student worksheet. Students were asked to list the parts of a circle by completing the table on the worksheet. In this activity, students discussed with their group members the strategy of drawing parts of a circle according to the instructions given (model of) and defining it (model for). Groups 1 and 2 could fill all tables correctly, in line with the conjecture on the HLT designed by the researchers.

Figure 5 shows parts of a circle prepared by Group 1. They understand the instructions given in the table so they can draw the parts of a circle correctly. In addition, they can define those parts based on the images they have created. Group 1 explained that a radius is a line
connecting the center point to the point on the circle. While the diameter is a line connecting two points on the circle and through the center of the circle.

![Table of parts of a circle listed by Group 1](image)

**Figure 5.** Table of parts of a circle listed by Group 1

As seen in **Figure 6**, Group 2 initially drew a chord by drawing a straight line connecting the center point with A and C. They should draw a straight line directly connecting A and B. This is where the teacher was needed to guide the students. The teacher gave instructions to students in drawing and defining chords.

![The chords by Group 2](image)

**Figure 6.** The chords by Group 2

The following is a snippet of the conversation between the teacher and Group 2.
Dialogue 2
Teacher : Show me, where is the bowstring?
Student : Here (* while pointing at the picture they have made)
Teacher : Please read the instructions again. Draw a straight-line connecting A and B. Where are A and B located?
Student : Here and here (* pointing at A and B)
Teacher : Meaning Which bowstring it is? Try to draw it.
Student (*the students then draw the bowstring according to the instructions given by the teacher.

Dialogue 2 shows the existence of communication between teachers and students. This is another characteristic of the IRME approach, namely interactivity. Likewise, with Group 1, Group 2 could also list the parts of a circle correctly. They can draw elements according to the instructions given in the table. Furthermore, they can define the elements. They explain that a chord is a straight line connecting adjacent points. Meanwhile, an arc is a curved line that connects one point to the other. Figure 7 shows parts of a circle listed by Group 2.

Groups 3 and 4 provided some parts of a circle correctly. Group 3 can draw the parts correctly according to the instructions given. However, they could not define the circle's area. Meanwhile, Group 4 could draw and define lines correctly. However, they were less precise in describing the method used in the activity.

Figure 8 shows a table of the parts of circles listed by Group 4. They defined an arc as a line bounded by two radii and one arc. Furthermore, they defined a line as a line bounded by a
chord and arc. Sectors and segments are an area, so a more precise definition of a sector is an area bounded by two radii and an arc of a circle.

Figure 8. Parts of a circle listed by Group 4

Meanwhile, a segment is an area bounded by a chord and an arc of a circle. Students’ activities, including the circle section, can be seen in Figure 9.

Figure 9. Student activities in listing parts of a circle

**Activity 4: Solving problems related to the parts of the circle**

The teacher asked students to examine the problems in the student worksheet. Students are asked to solve problems related to the parts of a circle. First, they were asked to determine the diameter using the given radius. They discussed with their group members strategies to solve the assigned problem. Based on the previous activity table, Group 1 understood that a radius is
half of the diameter, as seen in Figure 10. If the length of the radius is 2 cm, then the diameter is $2 \times 2 = 4 \text{ cm}$.

In the second activity, students were asked to determine the difference in diameter and the chord of a circle. Group 4 explained that the diameter is twice the radius (Figure 11), while the chord is a line connecting one point to another and is shaded.

Finally, students could identify the parts of a circle. Figure 12 shows that students could draw the parts of a circle, such as the center point, radius, diameter, arc, chord, section, and segment.
The final stage in designing and developing the learning trajectory results contributes to several activities to understand the concept of a circle for sixth graders. The activity explains the steps that students must pass using the IRME approach through the context of a tambourine. The steps are divided into four learning activities: making a tambourine model, illustrating a tambourine, listing parts of a circle, and solving problems related to circle elements.

Finally, the researchers administered a test consisting of seven problems to the students. These problems were used to determine students' understanding after participating in the circle learning activities using a tambourine context. Of the 19 sixth graders, 15 students scored above the Minimum Criteria of Mastery Learning (known as KKM), and the remaining four students scored below the KKM. Figure 13 shows students working on the test. The results of these tests are discussed further in a retrospective analysis.

Figure 13. Students working on the test

The results of final test given to the students indicate that the overall average score is 3.07 (good category). It means that students have understood the concepts related to the parts of a circle. The results showed that the circle learning design with a tambourine context could provide students with an understanding of the circle concept. The tambourine context developed from an informal to a formal level through 4 activities, namely making a tambourine model, drawing a tambourine illustration, listing the parts of a circle, and solving problems related to the characteristics of the circle. Those activities deliver students' thinking strategies from real situations to formal knowledge. This is as explained by Zulkardi (2002), which states that these informal processes can provide students' thinking strategies to reach the formal level.

This research design provides empirical evidence that learning mathematics in the context of RME and ethnomathematics can make it easier for students to understand mathematical concepts of the circle. The design of this study is also supported by previous studies, such as using traditional Indonesian games, “Tepuk Bergambar” in learning number operations (Prahmana, Zulkardi, & Hartono, 2012), “Kubuk Manuk” in learning social arithmetic concept (Risdiyanti, Prahmana, & Shahrill, 2019), “Gasing” in measuring time (Jaelani, Putri, &
Hartono, 2013), and “Macanan” in understanding the idea of squares and rectangles (Angraini, Prahmana, & Shahrill, 2021).

Furthermore, the tambourine, which plays a role in understanding the concept of circle and understanding students about cultural values, is in line with Peni's research (2021), which explains that Ethnomathematics in learning mathematics can help students appreciate cultural diversity and various mathematical ways of thinking. Learning that teaches about real-world problems and instills in them the desire to seek and work to solve these problems for the good of humanity can make students more able to trust, appreciate and recognize their cultural roots and can easily assimilate the culture and critically reflect values in it (D'Ambrosio, 2007; Rosa et al., 2017). The role of the tambourine in this learning design can be of more value than learning mathematics which is not just understanding mathematical concepts but equipping students with values and ethics that can make students wiser in using the mathematics they have learned and understood. Values that can be explored and reflected from the culture raised in Ethnomathematics can maintain the ethics of using mathematics to humanize humans and not use mathematics for actions that harm other people and other civilizations (D'Ambrosio, 2007; D'Ambrosio, 2016; Supiyati, Harun, & Jailani, 2019).

The results of this study contribute to complementing previous studies on the RME approach and the context of ethnomathematics. Furthermore, this research enriches mathematics and cultural education in Indonesia regarding mathematics learning with the RME approach and ethnomathematics context. It can be a reference for further researchers to develop similar learning designs with different topics.

**Conclusion**

The Indonesian Realistic Mathematics Education (IRME) approach using the Tambourine context has an essential role in producing a learning trajectory. The learning trajectory can support students' understanding of the concept of the parts of the circle in four activities. Firstly, in the informal stage, they were introduced to the circle through the model of a tambourine. Secondly, students could illustrate the four jingles on a tambourine. Thirdly, students listed the parts of the circle. Lastly, students could identify the parts of the circle, determine the relationship between the radius and the diameter, also examine the difference between a diameter and a chord. Furthermore, the results can be used to implement a learning trajectory designed more broadly. It can also be compared with the results of other activities using different approaches to generalize its effectiveness to improve students' understanding of circles.
Acknowledgment

The authors express their gratitude to Universitas Ahmad Dahlan and the University of South Africa for supporting facilities and providing opportunities to develop this research to completion in collaboration.

Conflicts of Interest

The authors declare that no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely by the authors.

References


