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Beyond Straight Lines: Contextualizing Lobachevsky's Parallel Postulate Through the Geometry of the "bubu" Fishing Gear

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Abstract

The axiom of Lobachevsky's parallelism is one of the topics that students often find difficult. The purpose of this research is to design a learning trajectory about the Lobachevsky axiom of parallelism using the context of a valid, practical and effective traditional fishing gear "bubu". This research applies development studies which consist of three main phases: preliminary study: analysis and exploration; prototype development: design and construction; and the last stage of assessment: evaluation and reflection. The results of the study show that the learning trajectory of the Lobachevsky equation axiom using the context of traditional fishing gear "bubu" is valid, practical and effective to improve problem-solving skills for mathematics education students. The conclusion is that there are six steps to the learning trajectory, namely: First, identification of problems with the local cultural context; Second: representation of problems; Third: make a settlement plan; Fourth: implementing the plan; Fifth: evaluate the solution of the problem; and lastly, make a conclusion about the axiom of Lobachevsky's parallelism.

Keywords: bubu, axiom of parallelism, fishing gear, learning trajectory, lobachevsky geometry

Introduction

Non-Euclidean geometry, especially the Lobachevsky axiom of parallelism, is one of the topics that students often find difficult. Based on the author observation at the the universities in Bengkulu showed that mathematics education students had difficulty understanding the axioms of parallelism in Lobachevsky's Geometry. It is found that almost all students to difficulty visualizing and understanding the properties of hyperbolic space. Students already have a mature schematic of the concepts and principles of flat plane geometry. As through the point P outside the g line, there is exactly one line that is parallel to the g line.

A similar thing was also found in the author's observations at the universities in Lubuklinggau, South Sematera. Students' cognitive processes tend not to accept Lobachevsky's geometric properties, such as "through the point P outside the g line, there are at least two lines parallel to the g line"; "The large number of angles in a triangle is less than 180^{o}.” Research by Sukestiyarno et al. (2023), Nugroho et al. (2022) and Nugroho et al. (2021) found many students who had difficulty in understanding geometry, especially problem-solving related to non-Euclidean geometry.

Lobachevsky geometry is one of the areas of study of non-Euclid geometry. The geometry is also known as hyperbolic geometry which was discovered by the Russian mathematician Nikolai Ivanovich Lobachevsky in the 19th century (Ramírez et al., 2023). This geometry challenges Euclid's parallel postulate, proposing that more than one parallel line can be drawn through a given point outside a particular line. This concept brought about radical changes in the understanding of space and geometry, opening the door to new explorations in mathematics and the natural sciences (Rachid and Telesphore, 2021). Therefore, problem-solving skills are an important aspect of Lobachevsky's geometry learning. Students must be able to visualize hyperbolic spaces, apply geometric concepts to analyze situations, find solutions, and test the results. Lobachevsky's geometry requires students to think critically and logically, connect abstract concepts, and apply creative problem-solving strategies.

To overcome these difficulties, the complexity of Lobachevsky's geometry requires an innovative and contextual approach to learning. One way that can be done is to use local cultural contexts, such as traditional “bubu” fishing tools. In general, "bubu" are made from readily available natural materials, such as woven bamboo, rattan, or palm fronds (sticks). The structure of the bubu has a specific geometric shape, often resembling a truncated cone or a hollow box. Its main characteristic is the body, which consists of parallel, tightly packed ribs. This tool is not only relevant to the daily life of students, but can also be used to explain complex concepts of Lobachevsky geometry (Nugroho et al.,2022, 2021; Sukestiyarno et al.,2023).

By using a context that is close to the student's thoughts and daily lives, it makes it easier for students to understand and solve problems in Lobachevsky Geometry. In addition, the results of the study Hwang et al. (2020) states that the use of online application technology (such as Liveworksheets) can help develop students' problem-solving skills in an interactive and visual way. Liveworksheets is an online platform that allows teachers to transform traditional worksheets (PDF, DOCX, JPG) into interactive, self-paced worksheets digitally. The app also provides structured exercises, challenging geometry problems, and opportunities for students to practice different problem-solving strategies. Meanwhile, students' difficulties in understanding the basic concepts of Lobachevsky geometry due to the use of curved lines, curved surfaces, and the concept of distance that are different from Euclidean geometry, can be overcome by using real-time information technology media, such as Liveworksheets (Nugroho et al., 2021). Points, lines, planes and angles in Lobachevsky's geometry have different properties than Euclidean geometry, resulting in different concepts of space. For example, the sum of interior angles in a triangle in Lobachevsky's geometry is always less than 180 degrees (Niekel & Niela, 2018). It can all be visualized through the help of Liveworksheets application media. These concepts challenge students' intuition about traditional geometry and pave the way for applications in a variety of fields, including cosmology, relativity, and computing.

On the other hand, one of the effective strategies to improve problem-solving skills in Lobachevsky geometry learning is to guide students to identify relevant information, help them formulate problem-solving strategies, encourage them to explore various solutions, and provide constructive feedback. This is done through application rocky learning Liveworksheets (Tagaeva et al., 2024). Liveworksheets can help facilitate this process by providing step-by-step instructions, visualizations, and exercises designed to test students' understanding and encourage them to apply geometry concepts in real-world situations. Liveworksheets plays an important role in Lobachevsky's geometry learning by providing an interactive and adaptive platform for students (Hwang et al., 2020). Liveworksheets allowing students to practice and test their understanding of geometry concepts, solve problems, and get hands-on feedback.

Important Liveworksheets lies in its ability to minimize *Cognitive Load* by presenting information in a clear and structured format, offering interactive exercises to test students' understanding, and providing hands-on feedback to help students identify mistakes and improve their understanding (Jamshid et al., 2021). Liveworksheets can be used for a variety of Lobachevsky geometry learning activities, including visualizing abstract concepts, performing geometric constructions, solving geometry problems, and even collaborating with other students (Widyastuti and Retnowati, 2021). Interactive exercises at Liveworksheets can be designed to build understanding gradually, from basic concepts to more complex problems. Live feedback provided by Liveworksheets Help students identify their mistakes and provide opportunities for students to learn from mistakes, encouraging a more effective learning process.

The integration of local cultural contexts in the learning of Lobachevsky geometry is essential to motivate students and make them relevant. Students will be more engaged and eager to learn when they see the relevance of the subject matter to their own culture and lives (Widada et al., 2020). Integrating local examples, stories, or geometric patterns from their culture can help students connect abstract geometry concepts to everyday experiences (Agusdianita et al., 2021). Lecturers can introduce Lobachevsky's geometric concepts by using examples of local traditional architecture, handicraft patterns, or textile design. Students can be invited to analyze these patterns, identify the geometric elements involved, and relate them to Lobachevsky's concept of geometry. The integration of local cultural contexts not only increases students' interest, but also helps them develop a deeper understanding of the subject matter and encourages them to think critically about the relationship between mathematics and culture. By (Sukestiyarno et al., 2023) that Students who learn Non-Euclid Geometry through the ethnomathematical learning approach (local cultural context) have higher spatial abilities than students who learn through conventional learning approaches.

This study conducted integration of elements of local culture into mathematics problem-solving education, particularly within the framework of Lobachevsky Geometry. This approach not only enhances students' understanding of complex geometric concepts, but also fosters the relationship between geometry and their cultural heritage. This research emphasizes the use of local cultural contexts, such as dance and traditional tools, to create relevant math problems, thereby increasing student engagement and understanding (Rawani et al., 2023; Supiarmo et al., 2022). By incorporating familiar cultural references, this learning theory aims to make abstract concepts more real to students, thereby improving their problem-solving skills.

The urgency of this research lies in the fact that Lobachevsky's Parallel Axiom is one of the most difficult topics for students to understand. This difficulty arises because students have difficulty visualizing hyperbolic space and tend to reject its geometric properties. Therefore, this research urges the design of a valid, practical, and effective learning trajectory (HLT). The solution uses a contextual approach through local culture, namely the traditional fishing tool "bubu", to bridge the understanding of abstract concepts and improve problem-solving skills.

This research is built on the hypothetical learning trajectory framework on the subject of geometry that is difficult for mathematics education students in Southern Sumatra, by intervening using the unique cultural and educational context in South Sumatra and Liveworksheets media (Sagita et al., 2024; Simorangkir et al., 2023). Therefore, by using design research methodologies, which include iterative cycles of design, implementation, and evaluation, which ensure that learning materials are valid, practical and effective in improving Lobacvehsky's geometry problem-solving abilities. Thus, this study answers the question "how is the validity, practicality, and effectiveness of the learning trajectory of Lobachevsky's parallel axiom using the context of the traditional fishing tool "bubu"?".

Methods

This research is a Development Studies as a Design Research (Putrawangsa, 2019). The research is oriented to develop interventions as a solution to the complex learning problem of Lobachevsky geometry, which in this article is devoted to the axiom of Lobachevsky parallelism. This design research is the development of a comprehensive local instructional theory about how students learn Lobachevsky Geometry. It was a validation, practicality, and effectiveness study of hypothetical learning trajectory (HLT) design (Simon et al., 2018; Simon & Tzur, 2004)

The design research approach emphasizes students' problem-solving abilities about the rediscovery of the Lobachevsky axiom of parallelism using the context of traditional fishing gear."*fish trap*". These are Fishing gear woven from skewers made of bamboo or rattan. "bubu" is a traditional fishing tool that is very familiar to the people of Bengkulu, Indonesia. The focus of this research is to develop a learning trajectory design that allows the identification of effective strategies that increase students' knowledge of Lobachevsky Geometry (Fahrurozi et al., 2018; Gravemeijer & van Eerde, 2009).

This study involved 50 students of Mathematics Education at one of the universities in Bengkulu and 50 students at one of the universities in Lubuklinggau, Indonesia. The research subjects were selected at random from the population in the two campuses. The design research process consists of three main phases: preliminary studies (analysis and exploration); prototype development (design and construction); Assessment Stages (Evaluation and Reflection) (Plomp & Nieveen, 2013; Eerde, 2013).

Preliminary studies (analysis and exploration)

The preliminary study stage is a number of activities consisting of analysis and exploration of contexts and needs (problems), literature review, and development of theoretical frameworks for design activities. For the identification of the problem is the problem of learning the Lobachevsky parallel axiom which does not have a solution guideline to overcome the difficulties and mistakes of mathematics education students as the research subjects of the two campuses, so it is necessary to conduct a study to find a valid, practical and effective solution to the problem. In this paper, what is presented as the result of the preliminary study is A hypothetical learning trajectory consisting of three components: the main learning activity, the learning objective, and the hypothetical learning activity (Yuliardi & Rosjanuardi, 2021; Fahrurozi et al., 2018).

Based on needs analysis and exploration in preliminary studies on the research subject, students need to learn real-time animation and online. Based on literature review, the use of online application program media Liveworksheets proven to improve students' problem-solving skills and learning independence (Anh et al., 2023; Rudenko et al., 2021; Abadi et al., 2023). In addition, cognitive processes in geometry learning can be enriched by real-time learning through the app Liveworksheets using the local cultural context to help students understand complex concepts such as those contained in Lobachevsky and Riemann Geometry (Widada et al., 2020). Thus, the researcher developed a prototype of the learning trajectory, teaching materials and lesson plan using the context of local culture and the media of the online application program Liveworksheets.

Prototype Development (design and construction)

The prototype development stage is an intervention development activity that is still in the form of a prototype through trial activities that are carried out iteratively. During that process, formative evaluations are carried out as a basis for improving the quality of the intervention and also the theory of intervention. In this activity, the researcher paid attention to other intervention models and a review of relevant literature, to produce an initial intervention draft as Prototype 1. This initial intervention was then tested on a limited target through a pilot study of 15 students, and its impact was evaluated. The evaluation results became the basis for revising Prototype 1, which resulted in Prototype 2. Prototype 2 was retested on 25 students, then evaluated and revised to become Prototype 3. This prototype development process continued until the prototype was valid, practical, and had a positive impact in overcoming students' difficulties and errors in understanding Lobachevsky's parallelism axiom. The content validity test was conducted by five experts (1 Non-Euclidean Geometry content expert, 1 design research methodology expert, 1 mathematics learning expert, 1 assessment and measurement expert, and 1 language expert). Practicality was measured through an observation instrument for the implementation of HLT.

Assessment stages (evaluation and reflection)

The last stage of activities is the assessment stage. At this stage, a summative evaluation is carried out to test the intervention and intervention theory, namely to test whether the intervention and intervention theory that have been developed have been effective.The learning trajectory is said to be effective if the problem-solving ability of Lobachevsky's axioms of students who learn using the "bubu" context with the help of the Liveworksheets application is higher than that of students who learn with a conventional learning approach after taking into account the influence of students' initial abilities.

At this stage, an assessment is carried out through pseudo-experimental research to test the effectiveness of the prototype results in the design and construction stages. There were 100 students involved from two different campuses, each of which was 50 students. The randomly selected research sample was divided into four groups, each consisting of twenty-five students. The experimental design for this stage can be seen in Table 1.

Table 1. 2x2 Factorial Design for HLT Experiments

Learning media	Context type	
	Local Culture (A1)	Conventional (A2)
Liveworksheets (B1)	**A1B1**	**A2B1**
MFI (B2)	**A1B2**	**A2B2**

Description: A1: Types of local cultural contexts; A2: Conventional (without local cultural context); B1: Liveworksheets; B2: MFI (Student Worksheet in hard-copy form); X: Initial problem-solving ability about Lobachevsky's axioms (pretes), and Y: Problem-solving skills about Lobachevsky's axioms (postes)

At this stage, the researcher also compares the implementation of learning, namely between the actual learning trajectory (ALT) and the design of HLT (Ivars et al., 2018; Baroody et al., 2022). In addition, the researchers identified how students discover, understand, and use the context of traditional fishing gear"*fish trap*". During the learning process, the researchers collected data through learning observation videos, student answers on student worksheets, anecdotes, and interviews. It was to collect data on the implementation of HLT, and data on students' understanding of the Lobachevsky axiom of parallelism. The findings at this stage are used to explain the role of the context of traditional fishing gear"*fish trap*" in helping students find and understand the concept and principles of cone volume.

Experimental data were analyzed using inferential statistics, i.e. covariate analysis, with covariate being the student's initial ability about the Lobachevsky axiom of parallelism. The pre-post data on the ability to understand the Lobachevsky equivalence axiom was analyzed using inferential statistical analysis, namely covariate analysis. It was to determine the level of effectiveness of the final prototype of the students' learning trajectory in learning the Lobach parallelism axiomevsky.

Results and Discussion

Based on research data, there are three research results presented in this paper. It is in accordance with the three main phases of the study, namely preliminary studies (analysis and exploration); prototype development (design and construction); assessment stages (evaluation and reflection).

Preliminary study results

Based on the results of this initial stage, it was found that the problem of learning the Lobachevsky parallelism axiom was that it was difficult for students to develop an intuitive understanding of Lobachevsky geometry. In the initial tests, the introduction of Lobachevsky's geometry used the Poincaré disc model, but this model also has limitations, and has not been able to help students understand Lobachevsky's equation postulate easily. In addition, the mathematics curriculum in Indonesia tends to emphasize Euclid geometry, so the introduction and understanding of Lobachevsky geometry is a challenge i. Therefore, a learning approach to the Lobachevsky equivalence axiom that is familiar to students is needed.

Based on the analysis, and literature review, the appropriate approach for the context of learning the axiom of Lobachevsky's parallelism is the traditional fishing instrument “bubu”, the traditional wind instrument of the flute, the traditional wind instrument serunai, the tunnel of the fort of Fort Marlborough in Bengkulu, and the trumpet of the new year. In this paper, it is presented about the learning context of the axiom of Lobachevsky's parallelism is the traditional fishing gear “bubu”.

Results of the literature review, Liveworksheets is instrumental in achieving this integration by providing a flexible and adaptive platform for personalized learning. With Liveworksheets, students can learn at their own pace, find additional learning resources if needed, and practice problem-solving with hands-on feedback (Anh et al., 2023; Rudenko et al., 2021; Abadi et al., 2023).

Therefore, in the initial study (need assessment) of the research subject, the following student statements were obtained:

1. Liveworksheets can help integrate the local cultural context by presenting problems, exercises, and illustrations that reflect the student's culture.
2. Students find it easier to understand the concept of geometry by connecting it with everyday experience.

Thus, a prototype of learning trajectory, teaching materials and lesson plans was developed using the context of local culture and the media of the Liveworksheets online application program.

Based on preliminary analysis, literature review and need assessment, a hypothetical Learning trajectory was obtained which consisted of three components: the main learning activity, the learning objective, and the hypothetical learning activity (Yuliardi & Rosjanuardi, 2021; Fahrurozi et al., 2018). The result is Hypothetical Learning Activities and Trajectories learn students find the axiom of Lobacheksy alignment presented in Table 2.

Table 2. Activities in a hypothetical Learning trajectory to find the axioms of Lobacheksy alignment

Main Activities	Main objectives	Hypothetical activity
Activity-1: Identifying Problems with Local Cultural Contexts	Strengthening students' understanding of the concept of line alignment based on the traditional measuring instrument “bubu”	• Students identify the problem of the Lobachevsky Axiom of Parallelism through the context of traditional fishing gear “bubu”;
Activity-2: Problem Representation	Represents the relationship between “bubu” and line alignment.	• Students represent the problem of the Lobachevsky Axiom of Parallelism through the context of traditional fishing gear “bubu”;
Activity-3: Creating a Completion Plan	Students found a problem-solving plan about Lobachevsky's axioms of parallelism using the context of “bubu”.	• Students make a plan to solve the problem of the Lobachevsky Parallelism Axiom through the context of traditional fishing gear “bubu”;
Activity-4: Implementing the Plan	Train students' ability to solve unfamiliar line alignment problems, because they are different from Euclid alignment.	• Students carry out the problem-solving plan of the Lobachevsky Axiom of Parallelism through the context of traditional fishing gear “bubu”;
Activity-5: Evaluating Problem-solving	Improve students' ability to evaluate and compare different axioms of alignment with Euclid Geometry.	• Students evaluate the problem-solving of the Lobachevsky Parallelism Axiom through the context of traditional fishing gear “bubu”;
Activity-6: Making Conclusions	Students draw conclusions about Lobachevsky's Axioms of Parallelism.	• Students draw conclusions about the Lobachevsky Axiom of Parallelism through the context of traditional fishing gear “bubu” into the formal Lobachevsky Axiom.

Thus, students' difficulties in understanding Lobachevsky's Parallelism Axiom are caused by the intuitive challenges of non-Euclidean geometry and the dominance of Euclidean curricula. Therefore, a familiar learning approach is needed. Based on the analysis, local cultural contexts, such as the "bubu" fishing gear, are proposed as a relevant approach. Liveworksheets are used to integrate this local context, as they are considered to facilitate understanding.

Prototype Development Result

The results of the prototype development stage activities are prototypes through trial activities that are carried out iteratively. The resulting prototype is LT, Lobachevsky Geometry Teaching Materials using Local Cultural Context assisted by Liveworksheets, and lesson plans. The HLT Prototype is listed in Table 1, the cover of the teaching material can be seen in Figure 1, and the lesson plan can be seen in Figure 2.

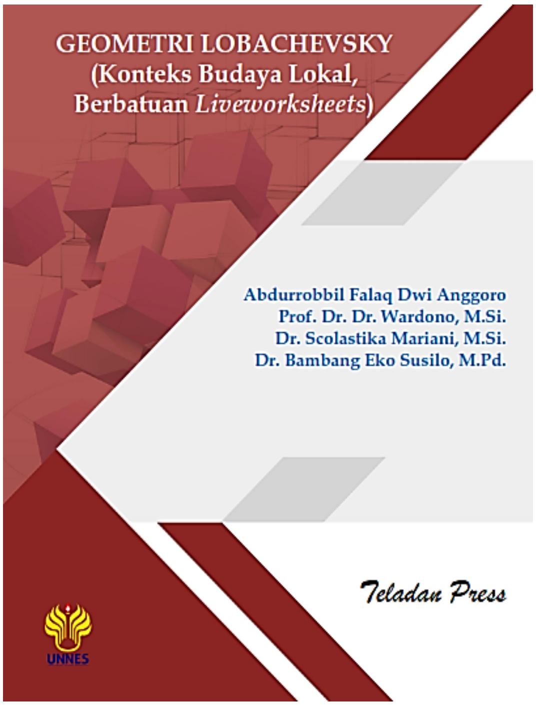

Figure 1. Lobachevsky Geometry Teaching Material Cover with Local Cultural Context with the help of Liveworksheets

UNNES	UNIVERSITAS NEGERI SEMARANG FAKULTAS MATEMATIKA DAN IPA S-3 PENDIDIKAN MATEMATIKA	
RENCANA PELAKSANAAN PEMBELAJARAN 1		
MATA KULIAH	Nama	Geometri Lobachevsky
	Kode	MPM – P10
	Kredit	2 SKS
	Semester	Pilihan
	Materi Perkuliahan	**Aksioma Kesejajaran Lobachevsky**
	Pendekatan Pembelajaran	**Konteks Budaya Lokal berbantuan Liveworksheets**
	Sistem Perkuliahan	**Luring**
Tujuan Pembelajaran		
Memahami aksioma kesejajaran Lobachevsky menggunakan konteks budaya lokal berbantuan aplikasi online liveworksheets.		
Skenario Kegiatan Pembelajaran		

Langkah-langkah	Uraian Kegiatan	Waktu (menit)	Metode	Media
1) Aktivitas-1: Identifikasi Permasalahan dengan Konteks Budaya Lokal	Mahasiswa melakukan identifikasi permasalahan Aksioma Kesejajaran Lobachevsky melalui konteks alat tangkap ikan tradisional "bubu";	5	Pemecahan Masalah	**Liveworksheets**
2) Aktivitas-2: Representasi Permasalahan	Mahasiswa merepresentasikan permasalahan Aksioma Kesejajaran Lobachevsky melalui konteks alat tangkap ikan tradisional "bubu";	15	Pemecahan Masalah	**Liveworksheets**
3) Aktivitas-3: Membuat Rencana Penyelesaian	Mahasiswa membuat rencana penyelesaian masalah Aksioma Kesejajaran Lobachevsky melalui konteks alat tangkap ikan tradisional "bubu";	40	Pemecahan Masalah	**Liveworksheets**
4) Aktivitas-4: Melaksanakan Rencana	Mahasiswa melaksanakan rencana penyelesaian masalah Aksioma Kesejajaran Lobachevsky melalui konteks alat tangkap ikan tradisional "bubu";	20	Pemecahan Masalah	**Liveworksheets**
5) Aktivitas-5: Melakukan Evaluasi terhadap Penyelesaian Masalah	Mahasiswa melakukan evaluasi terhadap penyelesaian masalah Aksioma Kesejajaran Lobachevsky melalui konteks alat tangkap ikan tradisional "bubu";	15	Pemecahan Masalah	**Liveworksheets**
6) Aktivitas-6: Membuat Simpulan	Mahasiswa menyusun kesimpulan tentang Aksioma Kesejajaran Lobachevsky melalui konteks alat tangkap ikan tradisional "bubu" menjadi Aksioma Kesejajaran Lobachevsky secara formal.	5	Dialog	**Liveworksheets**

Penilaian: 1) Penilaian Sikap dan Aktivitas (Rubrik Penilaian terlampir)
2) Tugas Penemuan Aksioma dalam Liveworkheests (Rubrik Penilaian terlampir)

Figure 2. Lobachevsky's Axiom Lesson Plan with Local Cultural Context with the help of Liveworksheets

Based on Table 1, Figure 1, and Figure 2, the three HLT components are prototypes, so their validity was tested by experts. The validation results of the three HLT components from the content aspect by the validators showed a very valid category. Several minor inputs (typos) have been revised. Based on the language aspect assessment, all three components are included in the very valid category. Details of the validation data can be analyzed with the following results:

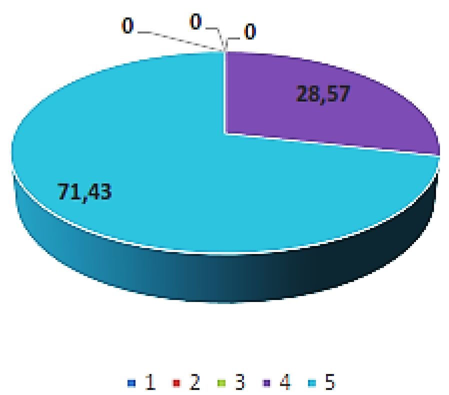

Figure 3. Percentage of Assessment of the 3 Components of HLT by Experts from the Objective Aspect (Source: Authors' own elaboration)

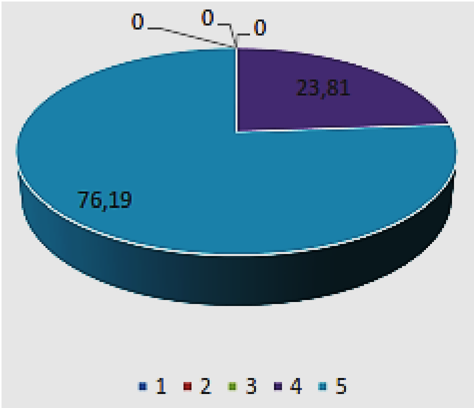

Figure 4. Percentage of Assessment of 3 HLT Components by Experts from the Aspect of Material Presentation (Source: Authors' own elaboration)

Based on Figure 3, the percentage of HLT assessments by experts from the objective aspect was obtained that it was 71.43% who stated that it was very valid, and 38.57% stated that it was valid, and 0% that it was quite below. Thus, it is concluded that LT, Lobachevsky Geometry Teaching Materials using the Local Cultural Context with the help of Liveworksheets, and the lesson plan from the aspect of categorized objectives are very valid.

Based on Figure 4 of the LT assessment percentage, Lobachevsky Geometry Teaching Materials using the Local Cultural Context assisted by Liveworksheets, and lesson plans by experts from the aspect of material presentation were obtained that 76.19% stated very valid, 33.81% stated valid, and 0% stated that it was sufficient. Thus, it is concluded that LT, Lobachevsky Geometry Teaching Materials using the Local Cultural Context with the help of Liveworksheets, and lesson plans from the aspect of presenting categorized material are very valid. Notice next Figure 5 and Figure 6.

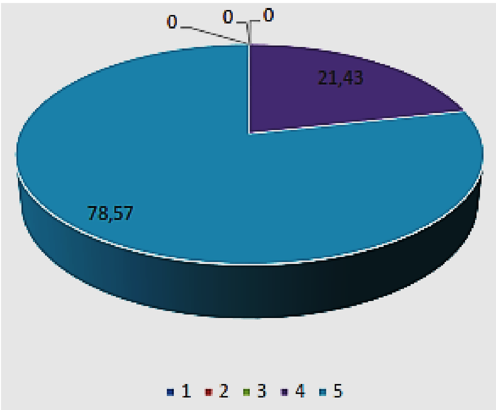

Figure 5. Percentage of Assessment of 3 HLT Components by Experts from Language Aspects (Source: Authors' own elaboration)

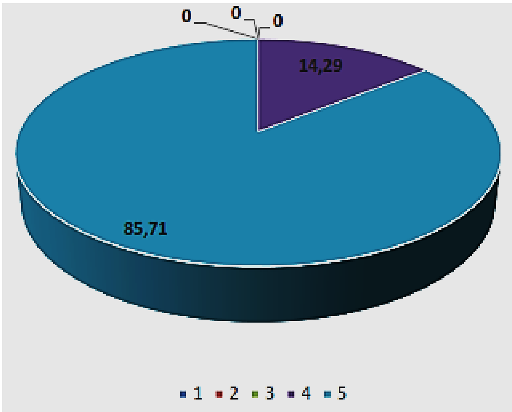

Figure 6. Percentage of Assessment of the 3 Components of HLT by Experts from the Time Aspect (Source: Authors' own elaboration)

Based on Figure 5 of the percentage of LT assessments, Lobachevsky Geometry Teaching Materials using Local Cultural Context assisted by Liveworksheets, and lesson plans by experts from the language aspect were obtained that 78.57% stated that it was very valid, and 21.43% stated valid, and 0% stated that it was sufficient. Thus, it is concluded that LT, Lobachevsky Geometry Teaching Materials using Local Cultural Context assisted by Liveworksheets, and lesson plans from the language aspect are categorized as very valid.

Based on Figure 6. the percentage of LT assessments, Lobachevsky Geochemistry Teaching Materials using Local Cultural Context assisted by Liveworksheets, and lesson plans by experts from the time aspect obtained that 85.71% stated very valid, and 14.29% stated valid, and 0% stated sufficient below. Thus, it is concluded that LT, Lobachevsky Geometry Teaching Materials using the Local Cultural Context assisted by Liveworksheets, and lesson plans from the time aspect are categorized as very valid.

Thus, based on the results of the analysis of the percentage of LT assessments, Lobachevsky Geometry Teaching Materials using the Local Cultural Context assisted by Liveworksheets, and lesson plans, it is concluded that the 3 components of HLT from all aspects are categorized as very valid.

Furthermore, the results of the pilot study on 15 students, and continued with a practicality test on 25 students as a formative test, the results of the analysis were obtained as follows. The practicality of the 3 components of HLT can be explained as follows. The group consisted of 15 and 25 students with learning using the local cultural context with Liveworksheets. Students are active in the classroom in a very fun atmosphere. They were very enthusiastic about learning about the axioms of Lobachevsky's parallelism using the context of "bubu". The learning atmosphere in the classroom is illustrated in Figure 7.

Figure 7. Student learning atmosphere in the classroom
(Source: Authors' own elaboration)

The learning process by applying the Learning trajectory of the Lobachevsky equivalence axiom is observed by independent observers to obtain accurate and reliable data. Observations were made within a period of four meetings. The data of the results of the observation is presented in the diagram in Figure 8.

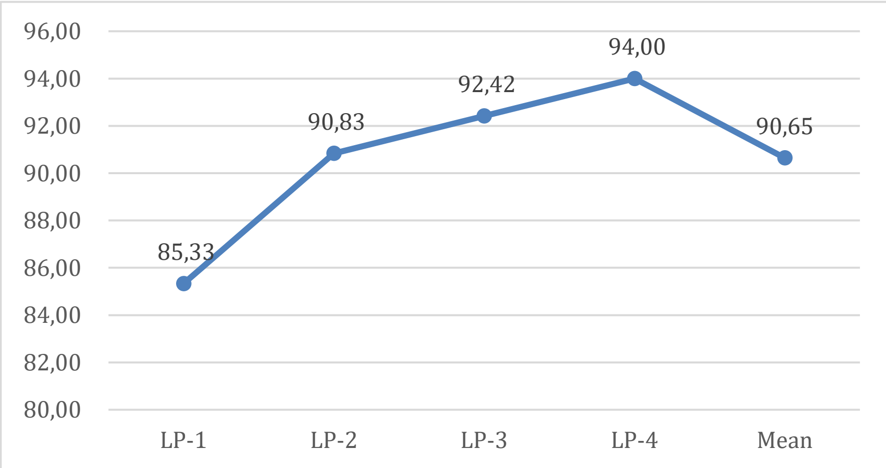

Figure 8. HLT Implementation Graph in each Lesson Plan (LP)
(Source: Authors' own elaboration)

Figure 8 shows that the implementation of *the* Learning trajectory for the FI student group with an ethnomathematics approach is very well implemented. This is shown based on the implementation of *learning trajactory* which was implemented 85.86% in RPP-1, increasing continuously in the implementation of 90.50% in RPP-2, 91.36% in RPP-3, until at the 4th meeting, namely 93.43% RPP-4 was carried out very well. This shows that the average implementation of HLT from 1-4 has reached 90.65%, which is more than 85%, which means that *the* Learning trajectory for the 3 components of HLT is carried out very well.

For each stage of HLT activities are also carried out very well for each step, it can be seen from Figure 9.

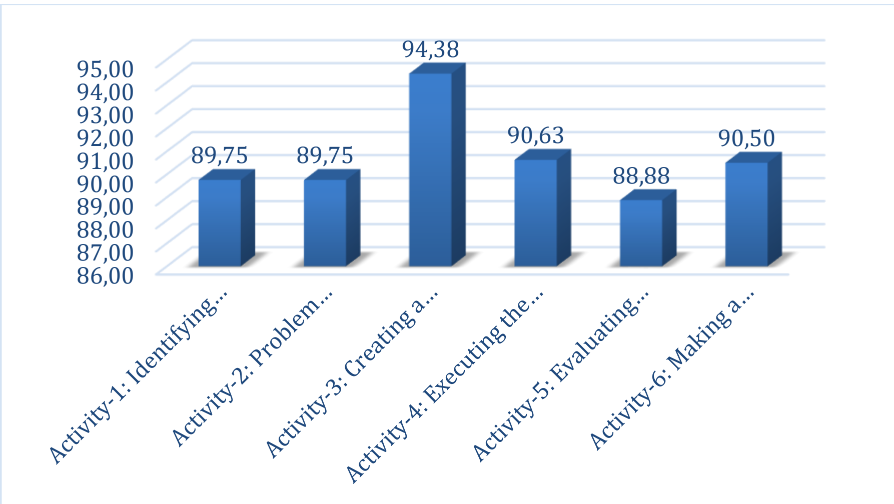

Figure 9. Liveworksheets Liveworksheets Step-by-step Implementation Chart
(Source: Authors' own elaboration)

Figure 9 shows that students who carried out the learning of the Lobachevsky alignment axiom using the context of the "bubu" fishing gear with the application of Liveworksheets were carried out very well, when reviewed from each step of HLT. It was shown that there were 89.75% of students who did Activity-1, namely identifying problems using culturalcontext; there were 89.75% doing Activity-2, namely presenting problems; as many as 94.38% of students doing Activities.3 namely making a settlement plan; there are 90.63% of students doing Activity-4, which is implementing the plan; there are 88.88% of students doing Activity-5, which is evaluating problem-solving; and as many as 90.50% of students carry out Activity-6, which is making conclusions. This shows that 90.65% of students are able to carry out the Learning trajectory stages very well.

Thus, the 3 components of HLT, namely learning trajectory, Lobachevsky Geometry Teaching Materials using Local Cultural Context assisted by Liveworksheets, and lesson plan are very practical. It is shown from the data of the above performance, that the learning trajectory, the Lobachevsky Parallels Axiom Teaching Materials using the "bubu" Context with the help of Liveworksheets, and the lesson plan were carried out very well.

The description can be concluded that *the* Learning trajectory of the Lobachevsky parallel axiom using the context of traditional "bubu" fishing gear has six steps of activities that are carried out very well. The six steps of the practical Learning trajectory are Activity-1: identification of problems with the local cultural context; Activity-2: representation of problems; Activity-3: make a completion plan; Activity-4: implement the plan; Activity-5: evaluate problem-solving; and Activity-6 is to draw conclusions. These results provide the conclusion that the prototype testing iteration has been achieved, namely a valid and practical HLT prototype, so that it is continued with the assessment stage, namely evaluation and reflection.

Assessment Level Results

A valid and practical HLT prototype was applied in a pseudo-experiment with a 2x2 factorial design. The result of the experiment is a Learning trajectory using the context of traditional fishing gear "bubu" which is evaluated and reflected so that an effective Learning trajectory is obtained. This research was conducted on students of the Mathematics Education study program at one of the universities in Bengkulu, Indonesia. The research data on the implementation of HLT and supported by an in-depth interview between Researcher (R) and Student (S) about the Discovery of the Lobachevsky Parallelism Axiom Using the Context of Traditional Fishing Gear "bubu". The researcher explored students' understanding of the Lobachevsky Axiom of Parallelism, R asked questions, and S gave explanations and views. The research data was analyzed the actual Learning trajectory stages in the implementation of the hypothetical learning trajectory. The following is a description of the results of this research.

Activity-1: Identifying Problems with Local Cultural Contexts

In Activity-1, students observed the traditional fishing gear "bubu" directly, thenidentified the fishing gear associated with the axiom of line alignment in Lobachevsky's geometry. The "bubu" can be seen in Figure 10.

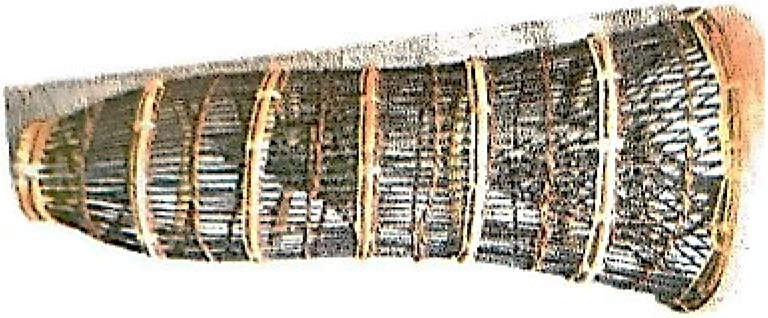

Figure 10. Traditional Fishing Gear “bubu”
(Source: Authors' own elaboration)

Figure 10 is a “bubu” fishing tool identified by S students. Students are very familiar with the traditional tool, so students are able to identify it well. This is supported by the results of the interview with S by R, as follows.

R: “Good morning. Thank you for taking the time to do this interview. We will discuss your interesting discoveries regarding non-Euclidean geometry, in particular the Lobachevsky Axiom of Parallelism, which you find in the context of the “bubu” fishing gear. Can you explain your invention in detail?”

S: “Good morning, sir. It was a pleasure to share my observations. I observed the “bubu” structure made of woven skewers. In particular, I focus on the spatial relationship between the skewers that make up the “bubu” skeleton. You can see it in Figure 1. I found the geometric patterns interesting.”

This interview excerpt provides confirmation of what the student has done in Figure 9. That means that students can identify the “bubu” fishing gear, is a build made of woven skewers, and S is able to connect spatially between the skewers that make up the “bubu” skeleton. This indicates that S identified the problem of the Lobachevsky alignment axiom problem through the context of traditional fishing gear “bubu” well.

Activity-2: Problem Representation

The student activity in Activity-2 is that student S is able to visualize the g-line on the “bubu” that he chooses from one skewer located at the base of the “bubu”. This can be seen in Figure 11.

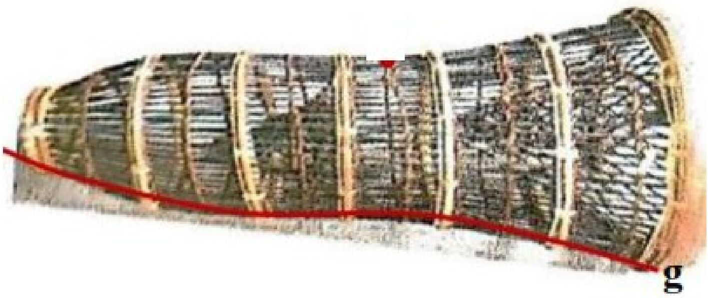

Figure 11. One skewer at the base of “bubu” (line g)
(Source: Authors' own elaboration)

Figure 11 is that the student represents the "bubu" as a wake made from many skewers woven in a curved manner and each of the lids does not intersect. Student S also chose one skewer at the base of the "bubu" which he called the g-line. This is supported by the following interviews.

R: "Can you explain the pattern?"

S: "Let's imagine a simple "bubu". It is made from many curved woven skewers where each of the skewers does not intersect. At the base of the "bubu" there is a skewer that can be called the main support, call it the g line (see Figure 11.)."

Based on this interview excerpt, it confirms that students can accurately represent the problem of Lobachevsky's parallel axiom through the context of traditional fishing gear "bubu".

Activity-3: Creating a Completion Plan

Activity-3 is that students are able to develop a problem-solving plan. It is that S takes the point P which is at the top, associating it with the position of the g-line. He showed a plan to make lines that pass through point P outside the g line. This can be seen in Figure 12.

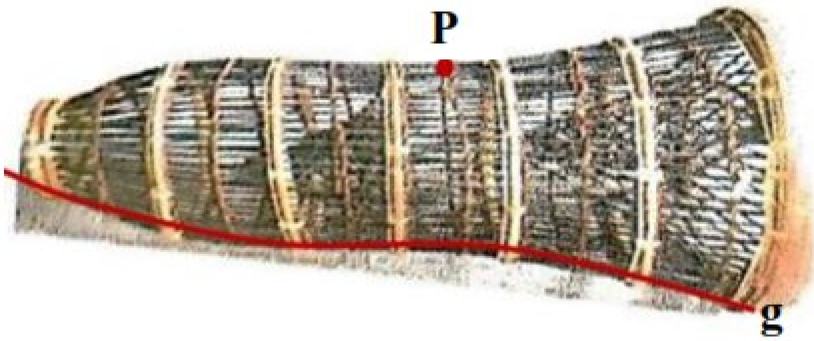

Figure 12. One skewer (g-line) at the base and Point P at the top "bubu"
(Source: Authors' own elaboration)

Figure 12 shows that student S is able to make a plan to solve the problem of the Lobachevsky equivalence axiom by using the context of "bubu" well. This is supported by the results of the interview, as follows.

R: "What would you do about the skewer pattern on "bubu":?"

S: "Based on the skewers on "bubu", ... and e... if we are going to take one main support skewer, and a P point at the top of the "bubu". Let's try how the sticks that pass through that point."

The excerpts from the interview and Figure 12 confirm each other that S can make a plan to solve the problem of the Lobachevsky alignment axiom through the context of the traditional fishing gear "bubu" appropriately.

Activity-4: Implementing the Plan

In activity 4, students executed a problem-solving plan about the Lobachevsky axiom of alignment through the context of traditional fishing gear "bubu". In this activity, students can determine that the number of lines is parallel to a certain line. This can be seen in Figure 13.

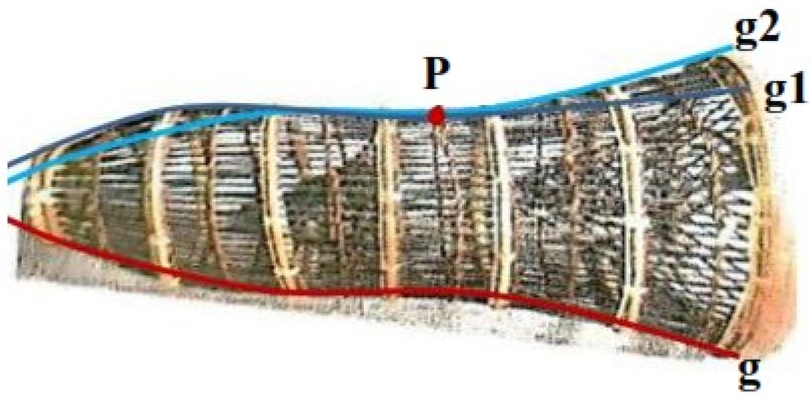

Figure 13. The skewers at the base (line g) are aligned with the two skewers that pass through the point P (lines g1 and g2) (Source: Authors' own elaboration)

Figure 13 shows that student S is able to implement the plan to solve the problem of the Lobachevsky alignment axiom by using the context of "bubu". This is supported by the results of the interview, as follows.

A: *"Ok... What is the next step?"*

S: *"In keeping with what I said earlier that we took one main support skewer at the base, and a dot at the top of the "bubu", I found that there were at least two other skewers that were parallel to the base skewer, without ever intersecting (see Figure 12). This is in contrast to Euclidean geometry, where there is only one parallel line that can be drawn through a point outside the other lines"*

R: *"So, you find that in the context of the "bubu" structure, Euclid's Axiom of Parallelism does not apply?"*

S: *"Right. The structure of "bubu" shows a clear example of non-Euclidean geometry, in particular Lobachevsky's hyperbolic geometry"*

R: *"What Do You Mean?"*

S: *"Yes, I found in "bubu" Lobachevsky's axiom of parallelism which states that through a point outside the line, there are at least two lines parallel to that line. That is the result of my observation of "bubu" which supports the axiom of Lobachevsky's parallelism"*

The interview excerpt confirms Figure 12 that S can carry out the plan to solve the problem of Lobachevsky's Parallelism through the context of the traditional fishing gear "bubu". It is that S states "through the point at the top of the "bubu" ditemu at least two other skewers that are parallel to the skewer at the base of the "bubu".

Activity-5: Evaluating Problem-solving

In Activity-5, students evaluated the solution of the problem of Lobachevsky's alignment axiom with the context of traditional fishing gear "bubu", by re-testing directly, about the alignment of two skewers on top of "bubu" with one line at the bottom of "bubu". It is the alignment of the lines in Lobachevsky's Geometry that can be seen in Figure 14.

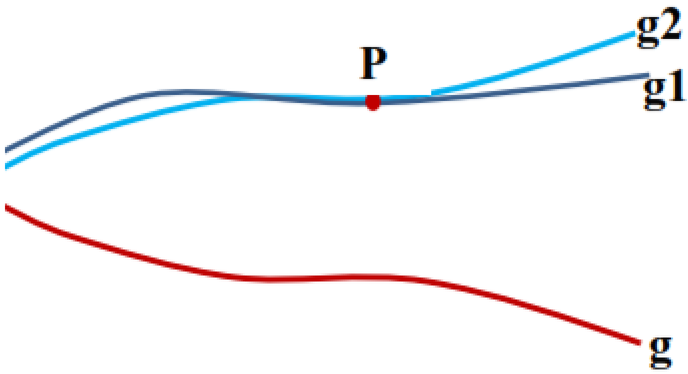

Figure 14. Two lines g1 and g2 pass through the point P outside the g line
(Source: Authors' own elaboration)

Figure 14 shows that students S are able to evaluate the solution of the problem about the Lobachevsky equation axiom by using the context of "bubu" in the form of visual images. This issupported by the results of the following interview.

R: "Can you explain more about how the structure of "bubu" represents hyperbolic geometry?"

S: "The woven structure of the skewers on "bubu" creates a curved surface. Lobachevsky's geometry, in contrast to Euclidean geometry which applies to flat surfaces, applies to negative curved surfaces. The curved surface of the "bubu" allows the existence of more than one parallel line passing through points outside the other lines. The skewers on the "bubu" can be thought of as the lines on the surface of this curve, see Figure 13."

Based on the excerpt of the interview, student S confirms Figure 13 that the woven structure of the skewer on "bubu" creates a negative curved surface that makes the existence of more than one parallel line passing through a point outside the other line. Students S evaluated the solution of the problem of Lobachevsky's Parallelism through the context of traditional fishing gear "bubu", and produced a visualization of the Lobachevsky Parallelism axiom clearly visible spatially.

Activity-6: Making Conclusions

Activity-6 is the last activity in the Learning trajectory of the Lobachevsky parallelism axiom using the context of traditional fishing gear "bubu". In this activity, students make conclusions about Lobacvehsky's equation assioma in the form of mathematical statements. It can be seen in Figure 15.

Saya dapat menyimpulkan bahwa melalui suatu titik P yang berada di luar garis g, ada dua garis g1 dan g1 yang sejajar dengan g.	Translate: *I can conclude that through a point P which is outside the line g, there are two lines g1 and g1 which are parallel to g.*

Figure 15. Conclusion from S (Source: Authors' own elaboration)

Figure 15 shows that student S can make a statement about Lobachevsky's axiom of parallelism, namely "through a point P outside the g line there are two lines g1 and g2 that are parallel to the g line. This is supported by the results of the following interview.

R: "What can you conclude from your experiments on the Lobachevsky axiom of alignment using the context of the "bubu" fishing gear"

Q: "I can draw a conclusion that "Through a single point P outside the g-line there are at least two lines g1 and g2 parallel to g."

Based on Figure 15 which is confirmed with the interview excerpt, the student was able to draw conclusions about the Lobachevsky Parallelism Axiom through the context of traditional fishing gear "bubu" into the formal Lobachevsky Parallelism Axiom. The axiom is "Through a single point P outside the g-line there are at least two lines g1 and g2 that are parallel to g."

HLT Effectiveness Testing through Test-Hypothesis (H1)

Hypothesis test (H-1) on the problem-solving ability of Lobachevsky axioms students who learn using the context of "bubu" with the help of Liveworksheets applications and students who study with conventional learning approaches to Lobachevsky's axiom material after controlling the influence of students' initial abilities. The hypothetical pairs tested were Ho: $\delta 1 \leq 0$; H1: $\delta 1 > 0$. The description is Ho: The problem-solving ability of the Lobachevsky axiom of students who learn using the context of "bubu" assisted by the Liveworksheets application is not higher than that of students who learn with a conventional learning approach after controlling for the influence of students' initial abilities; H1: The problem-solving ability of Lobachevsky's axioms students who learned using the context of "bubu" assisted by the Liveworksheets application was higher than students who studied with a conventional learning approach after controlling for the influence of students' initial abilities. The statistics used are t-tests for row A1B1 in the *Parameter Estimates* table. Accept Ho, if the t value for row A1B1 in the *Parameter Estimates* table has a p-value > 0.05. The results of the analysis are presented in Tabel 3.

Table 3. Parameter Estimates

Parameters	B	Std. Error	T	Sig.
Intercept	36,991	3,776	9,796	0,000
X	0,623	0,081	7,691	0,000
A1B1	27,273	1,275	21,391	0,000

Dependent Variable: Problem-solving Ability

Based on Table 3 in the third row, it was found that the t-test with a count of t = 21.391 and p-value = 0.000 < 0.05 means that Ho is rejected. This means that the Lobachevsky axiom problem-solving ability of students who learn using the context of "bubu" assisted by the application of Liveworksheets is higher than that of students who study with a conventional learning approach after controlling for the influence of students' initial abilities. The results of this test stated that the six *steps of the* Learning trajectory experimented were effective.

Based on the complete results of this study, it is concluded that *the* Learning trajectory of the Lobachevsky alignment axiom using the context of traditional fishing gear "bubu" with the help of Liveworksheets is valid, practical and effective. There are six Learning trajectory activities, namely Activity-1: identification of problems with the local cultural context; Activity-2: representation of problems; Activity-3: make a completion plan; Activity-4: implement the plan; Activity-5: evaluate problem-solving; and Activity-6 is to draw conclusions.

Dicussion

The results of the study show that Percentage of Assessment expert on learning trejectory, Lobachevsky Geometry Teaching Materials using Local Cultural Contexts with the help of Liveworksheets, and lesson plans by Whole Aspects be Very valid. The formative test in the development of the design obtained the results that the 3 components were practical, and the summative test through evaluation and reflection obtained the effectiveness of the designed learning trajectory. This result is supported by several other studies, namely that teaching materials based on local culture are valid and practical (Widada et al., 2019), ethnobra model by integrating Geogebra applications is valid and practical for Geometry learning. Research results Sukestiyarno et al. (2023) concludes that learning non-Euclidean geometry through learning paths with an ethnomathematical approach has a positive impact on improving students' spatial abilities.

The Ethnomathematics-Based Trigonometry Project Worksheets through the Project-Based Learning model have met the valid criteria, and the validator states that the project worksheets are worth using (Husna and Abidin, 2021). Ethnomathematics-based geometry modules are categorized as effective for use in learning activities, and with a scientific approach have a positive effect on metacognition abilities (Mutaqin et al., 2021). Ethnomathematics-based e-Modules are valid, practical, and effective to improve students' metacognitive abilities on spatial materials, so that students can access them in real-time for remote learning (Muzaki et al., 2022; Widada et al., 2019, 2020). The Learning trajectory of Non-Euclid Geometry is to convey learning objectives; provide ethnomathematics-based visual problems; students conduct exploration; students make conclusions and summarise the results of the exploration; and ended with students sharing conclusions/summaries of concepts and principles in Non-Euvlid geometric systems (Sukestiyarno et al., 2023).

The spatial ability of students who are given the Ethnomathematical Learning Approach is higher compared to students who are given a conventional learning approach on Non-Eucluded Geometry material (Nugroho et al., 2022). The ornaments of the Bandung Grand Mosque are rich in geometric concepts. Therefore, mosque ornaments can be used as an alternative learning medium to overcome students' mathematical difficulties, especially in geometry materials (Purniati et al., 2022). There are principles and concepts of geometry used by the ancestors of the Sasak tribe in making beleq drums. Existing geometric concepts include two-dimensional and three-dimensional geometric construction, the principle of translation, and the principle of dilation (Novitasari et al., 2023).

The category of studies on ethnomathematics and ethnomodelling, the most studied sequentially is geometry, followed by algebra, numbers, sets, and finally arithmetic. These results overall support that learning trajectory, Lobachevsky Geometry Teaching Materials using Local Cultural Contexts with the help of Liveworksheets, and lesson plans by whole aspects be very valid, practical and effective to improve the problem-solving ability of Lobachevsky's geometry.

The results of another study that is very close to this study found that students make two or more properties in the Lobachevsky equivalence axiom through ethnomathematics in the form of “bubu”. The cognitive process of these students is that they can construct an object about infinite lines that are parallel to a certain line. Also, encapsulate geometric objects so as to produce a correct understanding based on the properties of “bubu” weaving. Mathematics education students who are taught through an ethnomathematical approach can achieve a high level of cognitive processes (Herawaty et al., 2020), also its spatial abilities are significantly improved (Nugroho et al., 2022). Thus, it is very convincing that the Learning trajectory of the axiom of Lobachevsky's parallelism by using Local Cultural Contexts can improve problem-solving skills for mathematics education students.

Based on the results of the study and discussion, this study contributes to the literature. That “bubu” as a cognitive bridge: This study maps the development of understanding of Lobachevsky's axioms in mathematics education students. By combining local wisdom “bubu” and mathematical thinking, students undergo a six-step learning trajectory towards a deep understanding of Lobachevsky's parallel axioms. The results of the study indicate that local instructional theories that integrate “bubu” are valid, practical, and effective in empowering Lobachevsky's geometry problem-solving abilities.

The novelty highlighted is the unique contribution of this study: the use of bubu as an ethnomathematical context for visualizing and understanding the properties of Lobachevsky's parallel lines. This approach has not been widely explored in the literature, making it groundbreaking in the development of a culture-based non-Euclidean geometry curriculum. This study also paves the way for further research on the integration of other cultural objects into higher mathematics.

Conclusion

Mathematics education students have difficulty in understanding the axiom of Lobachevsky's parallelism as a result of the absence of a mathematics curriculum in universities in Indonesia that requires studying Lobachevsky Geometry courses. Also, many students already have a frame and mind-set that each geometry is based on Euclid's geometry. The study concluded that the Learning trajectory of the Lobachevsky equation axiom using the context of traditional fishing gear “bubu” is valid, practical and effective to improve problem-solving skills for mathematics education students.

There are six steps of the learning trajectory, namely: First, identifying problems with the local cultural context; Second: representation of problems; Third: make a settlement plan; Fourth: implementing the plan; Fifth: evaluate the solution of the problem; and lastly, make a conclusion about the axiom of Lobachevsky's parallelism.

The finding of this research is recommended that this learning trajectory be widely implemented in non-Euclidean geometry learning, especially in mathematics education study programs. The Learning trajectory can be used as an innovative learning approach that integrates the local cultural context with abstract mathematical concepts. Further research can be conducted to explore the effectiveness of this Learning trajectory in other local cultural contexts.

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Conflicts of Interest

The authors declare no conflict of interest.

References

Abadi, A. P., Aisyah, D. S., Lestari, K. E., & Yudhanegara, M. R. (2023). Digital Puzzle Worksheet for Identifying Metacognition Level of Students: A Study of Gender Differences. *European Journal of Educational Research*, *12*(2), 795–810. https://doi.org/10.12973/eu-jer.12.2.795

Agusdianita, N., Widada, W., Afriani, N. H., Herawati, H., Herawaty, D., & Nugroho, K. U. Z. (2021). The exploration of the elementary geometry concepts based on Tabot culture in Bengkulu. In *Journal of Physics: Conference Series* (Vol. 1731). IOP Publishing Ltd. https://doi.org/10.1088/1742-6596/1731/1/012054

Anh, T. T. N., Nguyen, N. T., & Nguyen, A. T. T. (2023). The Impact of Online Self-Assessment on Learning Outcomes and Self-Assessment Skills Among Grade 11 Students in Vietnam. *International Journal of Learning, Teaching and Educational Research*, *22*(4), 21–35. https://doi.org/10.26803/IJLTER.22.4.2

Bakhtiyarovich, J., Burkhanovna, M., & Bakhtiyorovna, G. (2021). Application of information and communication technologies in solving geometric problems. *Annals of the Romanian Society for Cell Biology*, *25*(3), 4191–4197. Retrieved from https://www.annalsofrscb.ro/index.php/journal/article/view/1909/1593

Baroody, A. J., Clements, D. H., & Sarama, J. (2022, March 1). Lessons Learned from 10 Experiments That Tested the Efficacy and Assumptions of Hypothetical Learning Trajectories. *Education Sciences*. MDPI. https://doi.org/10.3390/educsci12030195

Rachid, C.Y., & Telesphore, B.T. (2021). Modelling, Conception and Simulation of a Digital Watermarking System based on Hyperbolic Geometry. *Signal & Image Processing : An International Journal*, *12*(4). https://doi.org/10.5121/sipij.2021.12401

Eerde, H. (2013). Design research: Looking into the heart of mathematics education. *Proceeding The First South East Asia Design/ …*, 1–11. Retrieved from http://scholar.google.com/scholar?hl=en&btnG=Search&q=intitle:Design+research:+looking+into+the+heart+of+mathematics+education#0

Fahrurozi, A., Maesaroh, S., Suwanto, I., & Nursyahidah, F. (2018). Developing Learning Trajectory Based Instruction of the Congruence for Ninth Grade Using Central Java Historical Building. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, *3*(2), 78. https://doi.org/10.23917/jramathedu.v3i2.6616

Gravemeijer, K., & Van Eerde, D. (2009). Design research as a means for building a knowledge base for teachers and teaching in mathematics education. *Elementary School Journal*, *109*(5), 510–524. https://doi.org/10.1086/596999

Hamidah, K, J. W., & Auliana, S. (2024). Development of Discovery-Based Etnobra (Ethnomathematics Geogebra) Geometry Learning Model to Improve Geometric Skills in Terms of Student Learning Styles and Domicile. *Mathematics Teaching-Research Journal*, *16*(3), 25–57.

Hendriyanto, A., Priatna, N., Juandi, D., Dahlan, J. A., Hidayat, R., Sahara, S., & Muhaimin, L. H. (2023). Learning Mathematics Using an Ethnomathematics Approach: A Systematic Literature Review. *Journal of Higher Education Theory and Practice*, *23*(7), 59–74. https://doi.org/10.33423/jhetp.v23i7.6012

Herawaty, D., Khrisnawati, D., Widada, W., Mundana, P., & Anggoro, A. F. D. (2020). The cognitive process of students in understanding the parallels axiom through ethnomathematics learning. In *Journal of Physics: Conference Series* (Vol. 1470). Institute of Physics Publishing. https://doi.org/10.1088/1742-6596/1470/1/012077

Husna, N., Saminan, & Abidin, Z. (2021). Development of student worksheets on ethnomathematics-based trigonometry through Project-Based Learning models. In *Journal of Physics: Conference Series* (Vol. 1882). IOP Publishing Ltd. https://doi.org/10.1088/1742-6596/1882/1/012071

Hwang, W. Y., Zhao, L., Shadiev, R., Lin, L. K., Shih, T. K., & Chen, H. R. (2020). Exploring the effects of ubiquitous geometry learning in real situations. *Educational Technology Research and Development*, *68*(3), 1121–1147. https://doi.org/10.1007/s11423-019-09730-y

Ivars, P., Fernández, C., Llinares, S., & Choy, B. H. (2018). Enhancing noticing: Using a hypothetical learning trajectory to improve pre-service primary teachers' professional discourse. *Eurasia Journal of Mathematics, Science and Technology Education*, *14*(11). https://doi.org/10.29333/ejmste/93421

Mutaqin, A., Syaifuddin, M., & Cholily, Y. M. (2021). Ethnomathematics Based Geometry Module Development With a Scientific Approach to Improve Students' Metacognition Ability. *IndoMath: Indonesia Mathematics Education*, *4*(1), 11. https://doi.org/10.30738/indomath.v4i1.8867

Muzaki, A., Hastuti, I. D., Fujiaturrahman, S., & Untu, Z. (2022). Development of an Ethnomathematics-Based e-Module to Improve Students' Metacognitive Ability in 3D

Geometry Topic. *International Journal of Interactive Mobile Technologies* , *16*(3), 32–46. https://doi.org/10.3991/IJIM.V16I03.24949

Nickel, M., & Kiela, D. (2018). Learning continuous hierarchies in the Lorentz model of hyperbolic geometry. In *35th International Conference on Machine Learning, ICML 2018* (Vol. 9, pp. 6071–6080). International Machine Learning Society (IMLS).

Novitasari, D., Andi, T. M. S., Risfianty, D. K., Triutami, T. W., & Lu'Luilmaknun, U. (2023). Ethnomathematics in Sasaknese musical instruments: Exploration of geometry concept in Gendang Beleq. In *AIP Conference Proceedings* (Vol. 2619). American Institute of Physics Inc. https://doi.org/10.1063/5.0122581

Nugroho, K. U. Z., Sukestiyarno, Y. L., & Nurcahyo, A. (2021). Weaknesses of Euclidean Geometry: A Step of Needs Analysis of Non-Euclidean Geometry Learning through an Ethnomathematics Approach. *Edumatika : Jurnal Riset Pendidikan Matematika*, *4*(2), 126–149. https://doi.org/10.32939/ejrpm.v4i2.1015

Nugroho, K. U. Z., Sukestiyarno, Y., Sugiman, & Asikin, M. (2022). Students' Spatial Ability in Learning Non-Euclid Geometry Through Ethnomathematics Approach. *Journal of Positive School Psychology*, *6*(10), 10–35. Retrieved from https://journalppw.com/index.php/jpsp/article/view/12834

Plomp, T. (2013). Educational design research: An introduction. *Educational design research*, *1*, 11-50.

Purniati, T., Turmudi, Juandi, D., & Suhaedi, D. (2022). Ethnomathematics Study: Learning Geometry in the Mosque Ornaments. *International Journal on Advanced Science, Engineering and Information Technology*, *12*(5), 2096–2104. https://doi.org/10.18517/ijaseit.12.5.17063

Putrawangsa, S. (2019). *Design Research sebagai Framework Desain Pembelajaran*. Mataram: Reka Karya Amerta.

Ramírez, K. L., Rodríguez, D. S. M., & Mateus, C. H. (2023). Problem-based Learning Activities as a Promotion of Local Cultural Awareness in A1 English Students. *Mextesol Journal*, *47*(4), 1–10. https://doi.org/10.61871/mj.v47n4-10

Rawani, D., Putri, R. I. I., Zulkardi, & Susanti, E. (2023). RME-based local instructional theory for translation and reflection using of South Sumatra dance context. *Journal on Mathematics Education*, *14*(3), 545–562. https://doi.org/10.22342/jme.v14i3.pp545-562

Rudenko, N., Palamar, S., Nezhyva, L., Bondarenko, G., & Shyrokov, D. (2021). The Use of ICT Tools to Organize Distance Learning of Mathematics for Primary School Students under COVID-19 Pandemic Conditions. In *CEUR Workshop Proceedings* (Vol. 3013, pp. 371–380). CEUR-WS.

Sagita, N. K. D. L., Nurhanurawati, N., & Noer, S. H. (2024). Efektivitas Bahan Ajar Berbasis Local Instruction Theory dengan Pendekatan Pendidikan Matematika Realistik untuk Meningkatkan Kemampuan Penalaran Matematis. *MATHEdunesa*, *13*(2), 335–349. https://doi.org/10.26740/mathedunesa.v13n2.p335-349

Simon, M. A., & Tzur, R. (2004). Explicating the Role of Mathematical Tasks in Conceptual Learning: An Elaboration of the Hypothetical Learning Trajectory. *Mathematical Thinking and Learning*, *6*(2), 91–104. https://doi.org/10.1207/s15327833mtl0602_2

Simon, M. A., Placa, N., Kara, M., & Avitzur, A. (2018). Empirically-based hypothetical learning trajectories for fraction concepts: Products of the Learning Through Activity research program. *Journal of Mathematical Behavior*, *52*, 188–200. https://doi.org/10.1016/j.jmathb.2018.03.003

Simorangkir, F. M. A., Saragih, S., & Napitupulu, E. E. (2023). Pengembangan Local Instruction Theory Berbasis Model Pace untuk Melatihkan Kemampuan Pemecahan Masalah Matematis Siswa. *PRISMA*, *12*(2), 370. https://doi.org/10.35194/jp.v12i2.3267

Sukestiyarno, Y. L., Nugroho, K. U. Z., Sugiman, S., & Waluya, B. (2023a). Learning trajectory of non-Euclidean geometry through ethnomathematics learning approaches to improve spatial ability. *Eurasia Journal of Mathematics, Science and Technology Education*, *19*(6). https://doi.org/10.29333/ejmste/13269

Sukestiyarno, Y. L., Nugroho, K. U. Z., Sugiman, S., & Waluya, B. (2023b). Learning trajectory of non-Euclidean geometry through ethnomathematics learning approaches to improve spatial ability. *Eurasia Journal of Mathematics, Science and Technology Education*, *19*(6). https://doi.org/10.29333/ejmste/13269

Supiarmo, M. G., Harmonika, S., Utama, M. W., & Tarmuzi, T. (2022). Cultural-Based Contextual Learning Design Material Area and Circumference of the Square Through Tools Ancak. *Kontinu: Jurnal Penelitian Didaktik Matematika*, *6*(1), 48. https://doi.org/10.30659/kontinu.6.1.48-63

Tagaeva, D., Talipov, A., & Saipbekova, S. (2024). Formation of Schoolchildren Competencies when Teaching Geometry using Geometric Tasks with Life Content. *Bulletin of Science and Practice*, *10*(6), 670–674. https://doi.org/10.33619/2414-2948/103/79

Widada, W., Herawaty, D., Andriyani, D. S., Marantika, R., Yanti, I. D., & Anggoro, A.F.D (2020). The thinking process of students in understanding the concept of graphs during ethnomathematics learning. In *Journal of Physics: Conference Series* (Vol. 1470). Institute of Physics Publishing. https://doi.org/10.1088/1742-6596/1470/1/012072

Widada, W., Herawaty, D., Jumri, R., Wulandari, H., Aliza, F., & Anggoro, A. F. D. (2020). Students of the extended abstract in proving Lobachevsky's parallel lines theorem. In *Journal of Physics: Conference Series* (Vol. 1470). Institute of Physics Publishing. https://doi.org/10.1088/1742-6596/1470/1/012098

Widada, W., Nugroho, K. U. Z., Sari, W. P., & Pambudi, G. A. (2019). The ability of mathematical representation through realistic mathematics learning based on ethnomathematics. In *Journal of Physics: Conference Series* (Vol. 1318). Institute of Physics Publishing. https://doi.org/10.1088/1742-6596/1318/1/012073

Widyastuti, B. W., & Retnowati, E. (2021). Effects of Worked Example on Experts' Procedural Skills in Solving Geometry Problems. In *Proceedings of the 7th International Conference* *on Research, Implementation, and Education of Mathematics and Sciences (ICRIEMS 2020)* (Vol. 528). Atlantis Press. https://doi.org/10.2991/assehr.k.210305.049

Yuliardi, R., & Rosjanuardi, R. (2021). Hypothetical learning trajectory in student's spatial abilities to learn geometric transformation. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, 174–190. https://doi.org/10.23917/jramathedu.v6i3.13338