Journal of Honai Math, 8(1), 89-114, April 2025
https:/doi.org/10.30862/jhm.v8i1.848

Hypothetical learning trajectory on cylinder with Bloom's taxonomy perspective

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Received: 30 January 2025 | Revised: 22 April 2025 | Accepted: 29 April 2025 | Published: 30 April 2025

Abstract

Students' persistent difficulties in understanding three-dimensional geometric figures, particularly cylinders, due to limited spatial visualization and difficulty identifying relationships among their elements, such as cylinder nets. These difficulties are often rooted in traditional instructional practices that emphasize procedural tasks over conceptual development. Despite various interventions, there remains a lack of structured instructional models based on cognitive development frameworks to support students' conceptual growth in geometry. Addressing this gap, the present study aims to develop and evaluate a Hypothetical Learning Trajectory (HLT) grounded in Bloom's taxonomy to enhance students' understanding of cylinders. This study employed a design research methodology consisting of three phases: preliminary design, design experiments, and retrospective analysis. Two experimental cycles were conducted with 28 fifth-grade students, categorized into low, moderate, and high levels of understanding. Data were collected through classroom observations, student worksheets, tests, and interviews, and analyzed qualitatively. The HLT consisted of four key learning activities: modeling a cylinder, identifying its elements, constructing the net, and solving application problems, mapped to Bloom's cognitive levels of remembering, understanding, and applying. Findings revealed that students showed significant improvement in the first three activities, with increased spatial reasoning and conceptual clarity. However, difficulties persisted in the final activity involving reasoning and problem-solving. The results indicate that the proposed Bloom's taxonomy-based HLT offers a systematic framework for guiding geometry instruction. This study contributes a practical and theoretically grounded instructional model that can support teachers in designing adaptive learning experiences. Further research is recommended to explore its application across diverse topics and student groups.

Keywords: bloom's taxonomy, conceptual understanding, cylinder, hypothetical learning trajectory

Introduction

Conceptual understanding constitutes a foundational element in mathematics education, serving as a critical basis for bridging students' prior knowledge with meaningful engagement in mathematical problem-solving (Bangalan et al., 2023; Rittle-Johnson & Siegler, 2021). It facilitates the integration of existing knowledge structures with new learning, thereby promoting the development of coherent and transferable mathematical understanding (Ningrum et al., 2023). For instance, students who possess a solid grasp of the concepts of area and perimeter of two-dimensional figures are more likely to comprehend the notions of volume and surface area of three-dimensional shapes, such as cylinders. Moreover, conceptual understanding enables the evaluation of the depth and quality of students' mathematical thinking, as reflected in their ability to represent abstract concepts using visual or diagrammatic models (Ho, 2020). According to Magfirotin and Amir (2024), procedural knowledge plays a supportive role in reinforcing conceptual understanding, particularly in primary education, by allowing students to identify patterns and strengthen the interrelation between processes and underlying principles. For example, mastering the procedural steps for calculating volume enhances a student's ability to comprehend the logic underpinning the volume formula for a cylinder. The interdependence of procedural and conceptual knowledge has been well-documented, with numerous studies confirming their reciprocal influence on mathematical proficiency (Rittle-Johnson & Schneider, 2014).

Bloom's taxonomy, particularly its cognitive domain, provides a structured framework for understanding the progressive stages of student cognition and is closely aligned with the development of conceptual understanding. Empirical studies (Boles et al., 2015; Krathwohl, 2002) have emphasized that the foundational levels remembering, understanding, and applying are critical in cultivating students' conceptual comprehension. The remembering level enables students to recall essential mathematical facts and terminology, forming the initial cognitive engagement with content. The understanding level supports interpretation, explanation, and contextual linkage of concepts, facilitating the internalization of meaning. The applying level allows students to utilize learned concepts in problem-solving contexts, demonstrating their ability to transfer knowledge to novel situations. Collectively, these stages represent the cognitive scaffolding necessary for conceptual development from the perspective of Bloom's taxonomy (Akinboboye & Ayanwale, 2021; Arievitch, 2020; Çelik et al., 2022).

In primary mathematics education, these cognitive stages are manifested through various tasks. At the remembering stage, students recognize geometric terms and formulas related to three-dimensional figures such as cylinders, cuboids, and cubes. At the understanding level, they justify the use of specific formulas and connect mathematical concepts to everyday objects, such as associating the shape of a water bottle with a cylinder. The applying stage emerges when students solve contextual problems, such as calculating the capacity of a cylindrical container using the volume formula. These three stages form a progressive trajectory that underpins conceptual understanding and supports structured mathematics learning.

Despite its importance, conceptual understanding among primary students remains limited. Azis et al. (2023) report that students often struggle with problem-solving tasks due to inadequate mastery of essential facts, concepts, and procedures. Studies by Ho (2020) and

Kristidhika et al. (2020) indicate that students tend to exhibit stronger procedural skills than conceptual insights, particularly in tasks involving real-life applications, such as dividing objects equally or determining volume. Bangalan et al. (2023) further highlight those instructional practices in mathematics frequently prioritize procedural memorization over conceptual development. From a semiotic perspective, Milinia and Amir (2022) found that students encounter difficulties in interpreting mathematical terminology and symbols, especially in distinguishing between linear measurements and area units, which hinders their conceptual growth. These limitations often impede students from achieving deeper levels of understanding (Ayuningtyas et al., 2024).

To address these challenges, the implementation of instructional approaches such as Problem-Based Learning (PBL), inquiry-based methods, and Realistic Mathematics Education (RME) is recommended. These pedagogies emphasize student-centered learning by leveraging students' contextual experiences to construct mathematical knowledge (Wijaya et al., 2021; Wilson et al., 2015). Effective instructional design necessitates anticipating students' cognitive pathways and engaging in a conceptual analysis of potential learning trajectories (Amador & Lamberg, 2013). Simon (2020) underscores the importance of predicting students' thinking processes during task engagement to inform instructional decisions. In this regard, the Hypothetical Learning Trajectory (HLT) framework provides a systematic approach to designing and sequencing instruction based on anticipated student thinking (Clements & Sarama, 2009; Ebby, 2022). HLTs offer structured learning progressions that guide students from concrete to abstract understanding while addressing obstacles related to conceptual development.

An HLT typically comprises three core components: learning goals, instructional activities, and hypotheses regarding students' learning processes (Avenilde, 2015; Simon & Tzur, 2004). For example, an HLT for teaching volume might begin with exploratory activities such as filling containers, proceed to conceptual discussions on measurement units and volume formulas, and culminate in solving contextual problems. This progression ensures that students can anchor abstract concepts in real-world experiences, promoting deeper conceptual understanding. Hence, integrating Bloom's cognitive framework within HLT design can further align instructional activities with students' developmental needs and support the attainment of specific conceptual goals (Wilson et al., 2015).

Recent studies have applied HLTs to various mathematical topics. Deciku et al. (2022) proposed an HLT for systems of linear equations using the RME approach, while Ali et al. (2024) developed learning trajectories for squares and rectangles. Although these studies demonstrate the utility of HLTs in fostering meaningful learning, they often lack explicit integration of cognitive frameworks such as Bloom's taxonomy. Research by Bangalan et al. (2023), Rittle-Johnson and Siegler (2021), and Kristidhika et al. (2020) explored strategies to enhance conceptual understanding, while Nurmatova and Altun (2023) examined the cognitive demands of mathematical problems using Bloom's taxonomy. Studies by Magdalena et al. (2023), Murtiyasa and Sari (2022), and Rodrigues (2023) also investigated cognitive domains using Bloom's framework, yet did not directly relate it to the structure of HLTs.

While these contributions have expanded the knowledge base regarding HLTs and Bloom's taxonomy, their integration remains underexplored. For instance, Ali et al. (2024) and

Deciku et al. (2022) incorporated elements of cognitive development in their instructional sequences through emergent modeling, yet without explicit reference to Bloom's taxonomy. The present study addresses this gap by explicitly aligning the initial three levels of Bloom's taxonomy remembering, understanding, and applying with each stage of the HLT. This integration is intended to support the systematic development of conceptual understanding, particularly in the context of teaching volume in primary mathematics.

The rationale for developing an HLT grounded in Bloom's taxonomy lies in the taxonomy's utility as a structured cognitive framework that guides students from basic recall to complex reasoning. Embedding this framework into HLT design ensures that instruction moves beyond procedural competence to foster conceptual insight. Compared to traditional teaching approaches that emphasize rote learning, this integration promotes a cognitively coherent and purposeful learning experience (Magdalena et al., 2023; Murtiyasa & Sari, 2022; Wilson et al., 2015). Furthermore Simon and Tzur (2004) emphasize that an effective HLT comprising learning goals, instructional activities, and conjectures about student thinking can meaningfully support conceptual development. In the present study, conceptual understanding is fostered through learning activities mapped onto the first three cognitive processes of Bloom's taxonomy (Boles et al., 2015; Krathwohl, 2002). Accordingly, the aim of this research is to design a Hypothetical Learning Trajectory based on Bloom's taxonomy to enhance primary students' conceptual understanding in mathematics. This study contributes to the field by offering a theoretically grounded and practically applicable framework to guide instructional design, thereby addressing students' learning obstacles and aligning teaching practices with their cognitive development.

Methods

This study adopted a design research methodology as proposed by van den Akker et al. (2006), encompassing three core phases: preliminary design, design experiment, and retrospective analysis. This methodology was selected due to its iterative nature and its capacity to facilitate the contextualized development of instructional interventions while simultaneously enabling an in-depth examination of students' mathematical thinking processes. In particular, the study utilized design research to develop a Hypothetical Learning Trajectory (HLT) aimed at fostering primary students' conceptual understanding of three-dimensional geometric figures, with a specific focus on the cylinder. The construction of the HLT was anchored in three cognitive processes from Bloom's taxonomy remembering, understanding, and applying to scaffold students' mathematical cognition progressively from foundational recall to application in novel contexts.

Bloom's taxonomy was employed as the guiding theoretical framework due to its systematic classification of cognitive domains, which supports the alignment of instructional goals with students' levels of reasoning. The integration of Bloom's framework into the design process allowed for the development of instructional sequences that were cognitively calibrated and pedagogically structured. The implementation site was Sekolah Dasar Negeri Punggul 2, involving 28 purposively selected fifth-grade students. Selection criteria included: (1) the school's geographical accessibility to facilitate intensive classroom engagement; (2) the students' prior exposure to prerequisite mathematical concepts relevant to solid figures, particularly the cylinder; and (3) the school's willingness to collaborate and adopt instructional innovations.

Preliminary Design Phase

The preliminary design phase commenced with the formulation of an initial Hypothetical Learning Trajectory (HLT), which was further developed during the experimental design stage. Data collection at this stage involved a narrative literature review, interviews with the classroom teacher, and an analysis of three mathematics test results drawn from the teacher's gradebook. These tests assessed students' understanding of prerequisite concepts essential for learning cylinder volume, including unit measurement, two-dimensional figures, and introductory volume concepts. The test items, constructed by the classroom teacher, were validated by two university-level experts in mathematics education, specifically in the context of primary school learning. The validation process ensured alignment with the national curriculum's basic competencies and the cognitive levels articulated in Bloom's Taxonomy, with agreement levels reaching ≥80%, thereby affirming the content's appropriateness for assessing students' prior knowledge. The resulting test scores were categorized into three levels low, moderate, and high according to percentage-based intervals aligned with the school's assessment criteria and minimum mastery standards. This classification was utilized to develop initial skill profiles that informed the design of the HLT, enabling the tailoring of instructional activities to students' cognitive readiness (Ayuningtyas et al., 2024).

The literature review adopted a narrative approach, drawing from key sources related to hypothetical learning trajectories (Avenilde, 2015), conceptual understanding (Ho, 2020), and Bloom's taxonomy (Boles et al., 2015; Krathwohl, 2002). These references were selected based on their relevance to elementary mathematics education and their utility in guiding the integration of cognitive development principles within the instructional design. The review served to establish both a theoretical framework and practical foundation for the development of the HLT. Subsequently, the HLT was constructed by integrating data from the students' initial skill assessments with insights from the literature through a needs analysis and skill mapping aligned with Bloom's cognitive taxonomy. This integration aimed to identify students' levels of readiness and common misconceptions, allowing for a systematic progression of instructional activities from lower-order to higher-order cognitive skills (i.e., remembering, understanding, and applying). The resulting HLT represented a hypothesized sequence of learning steps, incorporating structured learning activities and pedagogical strategies responsive to student needs and designed to promote conceptual understanding. Importantly, the HLT retained a degree of flexibility, permitting iterative refinement during the trial implementation phase. Table 1 presents the categorization of students' initial mathematical skill levels, which formed the basis for the HLT design.

Table 1. Subject selection results

Categories	Score intervals	Total	Score
Low	$0 \leq x < 59$	6 students	71,11
moderate	$60 \leq x < 79$	7 students	78,81

High	$80 \leq x < 100$	15 students	83,73

Analysis of the data presented in Table 1 indicates that the majority of students were categorized within the moderate and high skill levels. However, the presence of students in the low skill category underscores the need for targeted instructional interventions to bridge gaps in their conceptual understanding. The development of the HLT design was carried out collaboratively by the researcher and the classroom teacher, ensuring that pedagogical decisions were grounded in both research-based practices and contextual classroom knowledge. This collaborative process involved a series of structured discussions aimed at designing learning activities, formulating instructional plans, and preparing student worksheets, all of which were carefully aligned with the students' assessed skill levels. Table 2 provides an overview of the finalized HLT design developed during this phase.

Table 2. Structure of the HLT based on Bloom's taxonomy

Activity: Bloom's Taxonomy Cognitive to-n (Bcn)	**Bloom's Based Conceptual Understanding (CUB)**
Activity CU1. Re-express a concept	
Remembering (Bc1) Bc 1.1. Imitate	CU1Bc1.1 Modeling the 3-dimensional figure, especially on a cylinder, using cylinder-shaped objects in the vicinity.
Remembering (Bc1) Bc 1.2. Identifying	CU1Bc1.2 Identifying the definition of a 3-dimensional figure, especially a cylinder, is based on knowledge.
Remembering (Bc1) Bc 1.3 Mentioned	CU1Bc1.3 Mentioned two types of 3-dimensional figures, especially on the cylinder, according to what students know.
Activity CU2. Classify mathematical objects based on their properties.	
Understanding (Bc2) Bc 2.1 Elaborated	CU2Bc2.1 Elaborate on the elements in the 3-dimensional figure, especially on the cylinder.
Aktivtas CU3. Giving examples and non-examples of a concept	
Understanding (Bc2) Bc 2.1 Distinguishing	CU3Bc2.1 Distinguishing examples of the net of a cylinder.
Activity CU4. Present concepts in various forms of mathematical representations	
Applying (Bc3) Bc 3.1 Portraying	CU4Bc3.1 Estimating the cylinder net drawing for the problem (portraying the cylinder net without a cap).

Table 2 presents the instructional activities constituting the Hypothetical Learning Trajectory (HLT), with each activity designated as CU to signify stages of conceptual understanding and annotated with the symbol Bc to indicate the associated cognitive level within Bloom's taxonomy. The HLT comprises four core learning activities CU1 through CU4 each of which is subdivided into tasks categorized according to specific Bloom's taxonomy levels. These tasks are systematically organized to form an integrated instructional sequence aimed at developing students' conceptual understanding. Instructional activities are implemented through student worksheets designed to address conceptual understanding, structured in alignment with Bloom's cognitive dimensions. Students' levels of conceptual understanding were assessed using both conceptual tests and structured observation instruments. Each learning activity is paired with an HLT hypothesis that accounts for variations in students' skill levels (low, moderate, and high), enabling differentiated instruction. These hypotheses underwent expert validation by two mathematics education scholars and one primary school teacher to ensure theoretical soundness and practical relevance. The initial implementation of these hypotheses occurred during the first design cycle to evaluate their consistency with students' actual classroom responses. Furthermore, as part of the HLT design, a set of pedagogical conjectures and anticipated teacher interventions was developed to address the range of student thinking strategies observed across different skill levels for each instructional activity, as detailed in Table 3.

Table 3. The teacher's conjecture and response are based on student skills per activity

Activity	Teacher's Conjecture	Teacher's Response
CU1Bc1.1.1	Students with all skill levels observe cylinder-shaped objects and select one to model.	The teacher guides the observation and asks open-ended questions: "Which part of this object resembles a cylinder?"
CU1Bc1.1.2	High-skilled students can model the original shape and pay attention to cylinder elements.	The teacher gives a follow-up challenge to name the elements that make up the structure and function.
CU1Bc1.1.3	Moderate-skilled students may model the cylinder but not focus on the original elements or shapes.	The teacher provides scaffolding, such as cutout images of cylinder elements, as visual guides.
CU1Bc1.2.1	After creating the model, students are expected to identify the definition of a cylinder.	The teacher facilitates discussion on "What makes this model a cylinder?" and notes important terms from students.
CU1Bc1.3.1	Students name the types of cylinders based on the model.	The teacher provides context cards (e.g., cans) and asks students to classify them.
CU2Bc2.1.1	High-skilled students name, explain, and describe cylinder elements.	The teacher asks high-skilled students to explain to other groups to strengthen concept elaboration.
CU2Bc2.1.2	Moderate-skilled students can describe, but cannot explain.	The teacher provides help with trigger questions: "Why is this called a base?"
CU2Bc2.1.3	Low-skilled students only mention elements but do not describe or explain	The teacher provides pictures of cylinder parts and asks the students to assemble them like a puzzle.
CU3Bc2.1.1	High-skilled students can distinguish a net cylinder and explain its components.	The teacher gives an exploratory task: "Try making another similar net, is it still a cylinder?"
CU3Bc2.1.2	Moderate-skilled students choose a net that is not the right shape.	The teacher compares two similar nets and discusses the differences.

CU3Bc2.1.3	Low-skilled students find it difficult to distinguish the appropriate net cylinder.	The teacher shows a real net cylinder and demonstrates how to fold it into a 3D shape.
CU4Bc3.1.1	High-skilled students explain the function of a net without a lid	The teacher asks students to relate the shapes to real-life functions.
CU4Bc3.1.2	The moderate-skilled students described the net without a lid, but did not explain.	The teacher asked: "Without the lid, what would it be suitable for?"
CU4Bc3.1.3	Low-skilled students struggled to draw a net without a lid.	The teacher provides a concrete model of an open net and asks students to imitate.

Table 3 systematically outlines the learning conjectures using a structured coding system (e.g., CU1Bc1.1.1), which represents the hierarchical connection between specific learning activities, the conceptual learning goals aligned with Bloom's taxonomy, and the anticipated variations in student responses based on their initial skill levels. These conjectures serve as predictive frameworks that inform the planning of instructional strategies and teacher interventions. Each conjecture is designed to capture students' potential learning pathways and common difficulties, thus enabling a responsive instructional approach. Teacher interventions, including scaffolding techniques, reflective questioning, and the use of visual representations, are integrated within each learning activity to accommodate students' individual characteristics and conceptual needs.

During implementation, the teacher engages dynamically with students, applying the conjectured strategies not only in real-time classroom interactions but also during post-lesson reflection sessions. These reflections aim to assess the congruence between predicted and observed student behaviors, allowing for iterative refinement of the instructional approach. The HLT is operationalized through instructional plans and student worksheets that are derived directly from the conjectures and conceptual learning objectives. Consequently, the conjectures function as both predictive and evaluative instruments guiding instructional design, managing classroom learning processes, and supporting the assessment of students' conceptual understanding. They also provide a foundation for ongoing improvements in the subsequent design and implementation cycles of the learning trajectory.

Design Experiment Phase

The experimental design phase was structured into two interconnected cycles, in accordance with the iterative methodology outlined by Gravemeijer and Cobb (2006). The first cycle functioned as a preliminary trial to identify instructional shortcomings and student responses to the initial HLT design, while the second cycle focused on refining and re-evaluating the revised instructional framework. This dual-cycle structure enabled a systematic approach to iterative design development tailored to the classroom context. During the first cycle, a limited trial implementation of the HLT was conducted to assess its feasibility. Quantitative criteria were used to evaluate the trial's success, including the active engagement of at least 75% of students, task completion by a minimum of 70% of students at the targeted cognitive level of Bloom's taxonomy, and adherence of at least 80% of instructional sequences to the original HLT predictions. These indicators were aligned with minimum competency standards and accepted practices in design-based research. Observational data and student performance during this cycle informed necessary revisions to the instructional activities, particularly in terms of conceptual clarity, time management, and student attainment of cognitive learning goals.

In the second cycle, the revised HLT was implemented in a broader classroom setting to assess its effectiveness in supporting conceptual understanding. Data collection during this phase included classroom observations, analysis of student worksheets, and post-tests. These data were used to evaluate the extent to which the enhanced design facilitated learning and to refine the trajectory based on observed patterns of student engagement and achievement. The effectiveness of the revised HLT was judged by three primary indicators: (1) at least 80% of students attaining the "understood" category in conceptual understanding assessments, (2) consistent positive student responses to instructional activities, and (3) alignment between the anticipated conjectures and actual classroom practices. Although this study encompassed only two design cycles, the HLT is conceptualized as an evolving framework that may require further development when applied in different educational contexts or with varying student demographics. As such, the HLT presented in this study is open to continued refinement and adaptation by future researchers or practitioners to enhance its relevance and applicability in broader instructional settings.

Retrospective Analysis Phase

The retrospective analysis phase involved a systematic comparison between the initially designed Hypothetical Learning Trajectory (HLT) and its actual implementation during the instructional experiment. This comparison was facilitated through data triangulation, incorporating multiple sources such as classroom observations, semi-structured interviews with both the teacher and students, and the analysis of student worksheets and conceptual assessment results. Observational and interview data were employed to detect discrepancies between the planned instructional activities and their real-time execution, thereby revealing areas where the HLT required refinement. Simultaneously, students' performance on conceptual understanding assessments served to evaluate the extent to which the HLT effectively fostered the intended cognitive outcomes.

Following data collection, a comprehensive analysis was conducted to assess the design's validity and inform possible improvements for future iterations. This included identifying instructional factors that contributed to either the success or failure of specific learning activities and formulating targeted strategies for refinement. Qualitative data from interviews and observations were subjected to thematic coding to uncover recurrent patterns, while student work was analyzed descriptively to trace levels of conceptual understanding as defined by Bloom's taxonomy. The outcomes of this analysis not only provided insights into the efficacy of the HLT but also offered a foundation for iterative enhancements, reinforcing the dynamic and adaptable nature of the design-based research process.

Results and Discussion

The results of this study pertain to students' conceptual understanding of the cylinder topic, analyzed through the lens of Bloom's taxonomy. This understanding was explored through a structured sequence of activities: CU1 (restating mathematical concepts), CU2 (categorizing mathematical objects based on their attributes), CU3 (identifying examples and non-examples of a concept), and CU4 (representing a concept using multiple mathematical representations). Each activity was implemented and observed in accordance with the instructional framework proposed by Amir and Wardana (2017). Systematic classroom observations were jointly conducted by the researcher and the classroom teacher, employing structured field notes and an analytical rubric aligned with Bloom's conceptual understanding indicators. The rubric served as a tool for evaluating students' performance in each instructional task, while the field notes documented classroom interactions, including students' cognitive engagement and the teacher's pedagogical responses. These observations formed the basis for the analysis of students' conceptual understanding across the four activities, as presented in Table 4.

Table 4. The outcomes of classroom observations

Activity CU	K1	K2	K3	K4	Total	Score	Categories
CU1Bc1	12	12	9	11	44	91,6	Highly satisfactory
CU2Bc2	8	9	9	7	33	91,6	Highly satisfactory
CU3Bc2	5	5	5	5	20	83,3	Satisfactory enough
CU4Bc3	4	6	9	4	23	63,8	Unsatisfactory

Table 4 presents the outcomes of classroom observations across four key learning activities (CU1Bc1 to CU4Bc4), each designed to foster students' conceptual understanding of cylinders based on Bloom's taxonomy. These activities targeted various cognitive levels, including restating concepts (CU1), classifying mathematical properties (CU2), identifying examples and non-examples (CU3), and translating concepts into multiple representations (CU4). The rubric-based evaluation revealed differential levels of student performance across the activities, which were categorized as "highly satisfactory," "satisfactory enough," and "unsatisfactory."

The activities CU1Bc1 and CU2Bc2 demonstrated the highest effectiveness, each achieving a 91.6% success rate, classified as "highly satisfactory." Several pedagogical strategies contributed to these results. In CU1Bc1, the use of tangible objects such as cylindrical cans served as effective 3D models, allowing students to visually and physically identify the defining elements of a cylinder. This hands-on approach enhanced students' spatial reasoning and conceptual clarity. Meanwhile, CU2Bc2 engaged students in collaborative discussions and visualizations of cylinder components, promoting meaningful connections between abstract mathematical ideas and real-world representations. These strategies also improved students' ability to articulate mathematical ideas, aligning well with Bloom's C2 (understanding) and C3 (application) cognitive domains.

Additional factors that supported successful outcomes included explicit instruction through well-structured worksheets, proactive teacher scaffolding, and guided group discussions. These instructional supports created opportunities for structured exploration and conceptual reinforcement, facilitating deeper student engagement and understanding.

In contrast, the performance in CU3Bc3 and CU4Bc4 was less satisfactory. CU3Bc3 yielded a success rate of 83.3%, categorized as "satisfactory enough," whereas CU4Bc4 achieved only 63.8%, falling into the "unsatisfactory" category. The diminished outcomes in CU3Bc3 were primarily attributed to students' difficulty in interpreting and mentally transforming 2D cylinder nets into corresponding 3D structures. Many students exhibited limited spatial visualization abilities and struggled with abstract representations due to insufficient guidance provided in the worksheet instructions.

The challenges in CU4Bc4 were even more pronounced. This task involved analyzing a special case of a cylinder net without a lid, which required higher-order thinking at Bloom's C4 (analyzing) and C5 (evaluating) levels. However, most students demonstrated cognitive performance only at the understanding level (C2). Limited teacher facilitation, particularly for students with lower spatial reasoning skills, further constrained the development of conceptual understanding. The lack of targeted scaffolding such as probing questions or visual aids hindered students' ability to engage in deeper cognitive processing.

Overall, the lower achievement in CU3Bc3 and CU4Bc4 can be attributed to a misalignment between instructional strategies and students' cognitive readiness. The abstract nature of visual materials and the absence of concrete instructional tools impeded students' capacity to transition between two-dimensional and three-dimensional representations. These findings underscore the need for improved instructional designs that account for learners' developmental stages. Integrating concrete manipulatives, such as foldable cylinder nets, verbal scaffolding, and teacher-mediated group discussions, is recommended to enhance students' conceptual understanding and foster higher-order mathematical reasoning. Thus, instructional refinement is essential to optimize the effectiveness of learning activities, particularly those currently yielding suboptimal outcomes. The following synthesis elaborates on how students' conceptual understanding of cylinders evolved through the implementation of activities CU1Bc1 to CU4Bc4, as structured within Bloom's taxonomy framework.

Activity CU1Bc1. Model, identify, and mention types of 3-dimensional figures, such as a cylinder

In Activity CU1Bc1.1, students engaged with the concept of a cylinder using a real-world object, specifically a can, and illustrated its structure in alignment with worksheet instructions. The use of a tangible object provided students with the opportunity to directly observe and manipulate the elements of a cylinder, fostering greater engagement and a deeper understanding compared to abstract methods such as verbal explanations or image-based representations. This approach is consistent with findings from Ali et al. (2024) and Deciku et al. (2022), who emphasized that the use of concrete materials enhances spatial reasoning and strengthens conceptual mastery. The performance data (91.6, as shown in Table 4) from CU1Bc1 indicate that this hands-on method proved particularly effective for students with low to moderate skill levels, outperforming more traditional approaches.

To construct the cylinder model, students followed a four-step procedure: sketching two circles and a rectangle, cutting out the shapes, assembling them, and attaching the components using glue. The incorporation of real-world objects, such as the can, significantly supported students' understanding of the cylinder's structure. This was demonstrated in Activity CU1Bc1, where students accurately identified and described the elements of the cylinder based on their physical model (observation score: 91.6, Table 4). These findings corroborate the work of Ali et al. (2024), who concluded that object-based learning facilitates spatial comprehension and reduces misconceptions. Kristidhika et al. (2020) further highlighted the value of realia in bridging abstract mathematical concepts with concrete experiences, enhancing both student engagement and retention. Figure 1 illustrates students' activity as they constructed their cylinder models.

Figure 1. Student activity in constructing the cylinder model

In Activity CU1Bc1.2, students were tasked with explaining the definition of a three-dimensional figure, specifically a cylinder. All students were able to accurately articulate the definition, describing the cylinder as a space with a circular base. This outcome suggests that students had successfully developed a conceptual understanding of the cylinder, consistent with the conjecture outlined in the Hypothetical Learning Trajectory (HLT). The ability to rephrase the concept of a cylinder demonstrates the internalization of its key characteristics, confirming that the instructional approach employed effectively facilitated their conceptual growth. Figure 2 provides a visual representation of students' engagement as they identified and described the three-dimensional attributes of a cylinder.

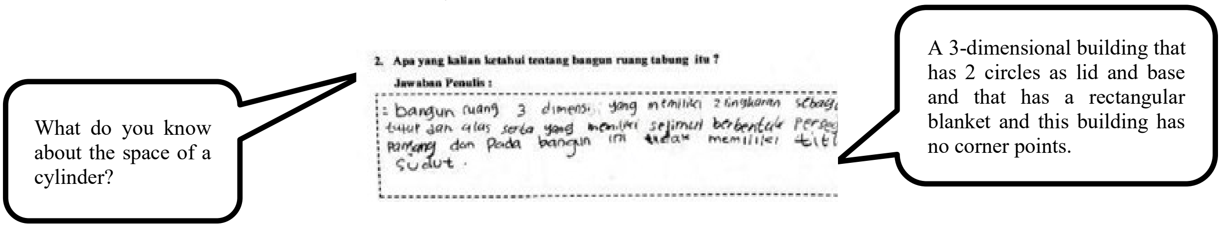

Figure 2. Definition of a cylinder according to group 2

In Activity CU1Bc1.3, students were asked to identify the types of space associated with a cylinder using the worksheet provided. Three student groups successfully identified the expected concepts, while one group struggled to provide accurate responses. Students with lower levels of conceptual understanding were unable to independently answer the question and required scaffolding from the teacher to continue. This observation underscores the necessity of teacher intervention in helping students recognize and comprehend the spatial characteristics of a cylinder. The teacher's support was pivotal in guiding these students through the cognitive challenges they faced. Figure 3 illustrates students' engagement as they worked to identify the types of three-dimensional space associated with the cylinder.

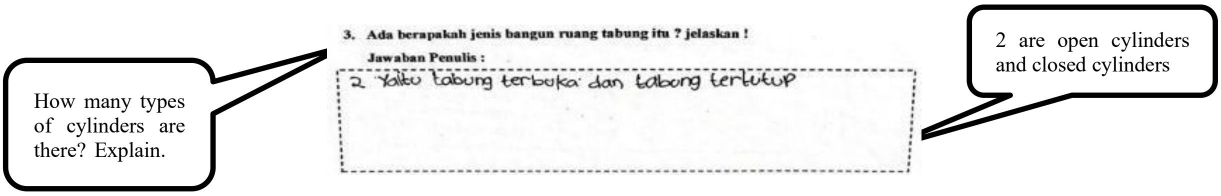

Figure 3. Identifying types of cylinders according to group 2

Activity C2Bc2. Elaborating on the Elements of a Cylinder

In Activity CU2Bc2, students were instructed to observe a problem presented in the worksheet and identify the elements of a cylinder by completing a structured table. This task required students to engage in group discussions, during which they worked collaboratively to define and visualize the cylinder's elements, following guided instructions. The collaborative nature of this activity encouraged peer interaction, which played a crucial role in fostering a deeper conceptual understanding of the cylinder's characteristics.

According to the post-activity conceptual understanding test, 86% of students (24 out of 28) successfully identified and defined the key elements of a cylinder: the base, top, and lateral surface. The breakdown of success rates revealed that 100% of high-skilled students, 86% of moderate-skilled students, and 50% of low-skilled students demonstrated correct understanding. This outcome suggests that the structured group discussions effectively supported students across varying skill levels, helping them articulate the geometric elements of the cylinder. These findings are consistent with Kristidhika et al. (2020), who highlighted that well-structured group discussions are instrumental in enhancing students' abilities to express and clarify geometric concepts, as they facilitate the exchange of ideas and minimize misconceptions.

Figure 4 depicts Group 2's accurate illustrations and definitions of the cylinder's elements, showcasing their clear understanding of the table instructions. The group represented the lateral surface as the curved side connecting the base and the top, while the circles illustrated the base and top with matching diameters. This representation reflects a comprehensive understanding of the cylinder's components, which aligns well with the learning objective of identifying the cylinder's parts.

However, not all groups achieved the same level of accuracy. For example, Group 4 initially confused the lateral surface with the base, mistakenly thinking that a cylinder consists of three separate parts rather than a continuous surface. This misunderstanding arose from challenges in visualizing the 3D object from its 2D representation. To address this issue, the teacher used real objects, such as cans, and prompted students with guiding questions like, "Which part can you wrap around?" and "Which part touches the table when standing upright?"

These prompts helped to refine students' understanding and promote a more accurate conceptualization of the cylinder's structure. The differing responses across groups underscore the importance of visual aids and guided questioning in the teaching and learning process of geometric concepts.

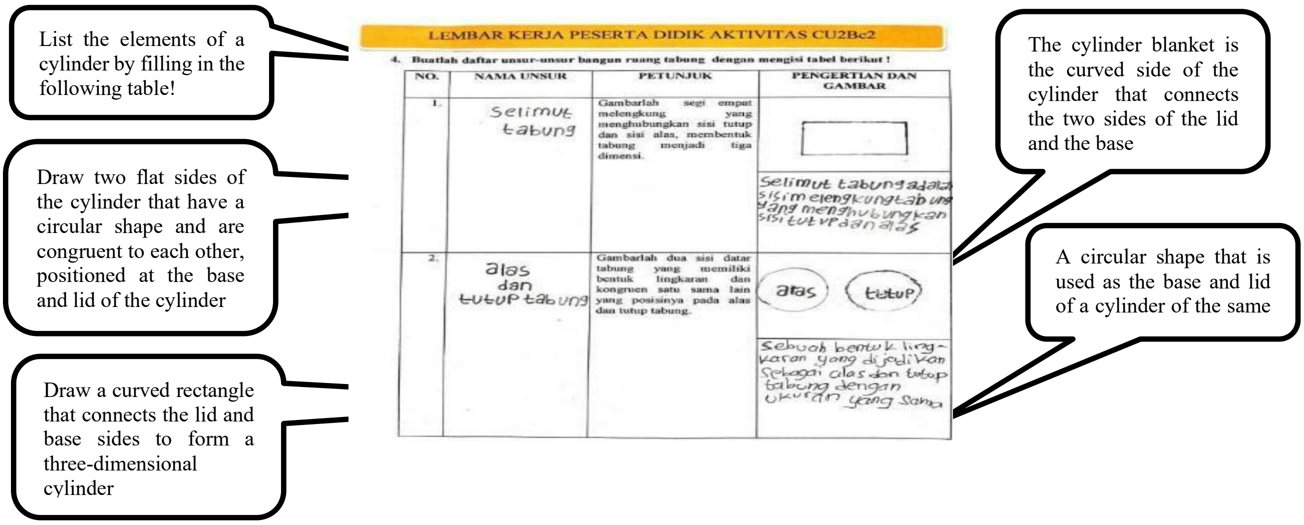

Figure 4. Table of elements of a cylinder listed by group 2

In Figure 5, Group 2 initially misunderstood the concept of cylinder height, equating it with the measurement of the lateral surface, which they interpreted as a curved line. Their first attempt involved drawing a straight line from the top (cap) to the base of the cylinder, which is incorrect in the context of cylinder geometry. This misunderstanding highlights the importance of teacher intervention at this stage. The teacher's guidance was crucial in helping the students recognize the correct definition of height as the perpendicular distance between the base and the top of the cylinder, not a measurement along the lateral surface. Following the teacher's instructions, the students revised their drawings, leading to a more accurate understanding of the cylinder's height.

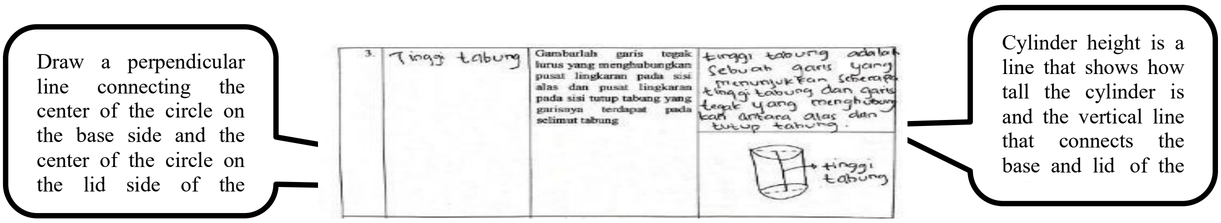

Figure 5. Height of the cylinder drawn by group 2

The following is a transcript of the teacher and Group 2 conversation.

Teacher	:	*Show me which one is the height of the cylinder?*
Low student	:	*This one. (Points to the curved part on the cylinder's lateral surface.*

Teacher	:	*Please reread the instructions. Draw a straight line connecting the lid and base of the cylinder.*
Low student	:	*This way.*
Teacher	:	*Try to portray it. A straight line, not a curved one (the student then draws a straight line according to the instructions given by the teacher).*

The dialog in Activity CU2Bc2 exemplifies how teacher-guided interaction can support low-achieving students in identifying and understanding the components of a three-dimensional figure, such as a cylinder. By engaging in focused, step-by-step guidance, these students were able to enhance their conceptual grasp of the figure's elements, particularly in terms of visualizing and classifying geometric objects. This aligns with the idea that structured, teacher-led interventions can significantly help in correcting misconceptions and reinforcing accurate geometric reasoning.

In the case of Groups 2 and 3, their success in identifying and representing the cylinder's elements according to the instructions demonstrates the effectiveness of the activity in fostering understanding. In contrast, Group 1 produced accurate visual representations but struggled to articulate clear definitions of the elements, indicating a gap in their conceptual clarity. Group 4 faced challenges both in their drawings and verbal explanations, suggesting that additional support might be necessary for them to refine their understanding of the cylinder's properties. Figure 6 showcases the elements of the cylinder as listed by Group 2, reflecting their ability to correctly identify and define the components based on the activity's structured approach.

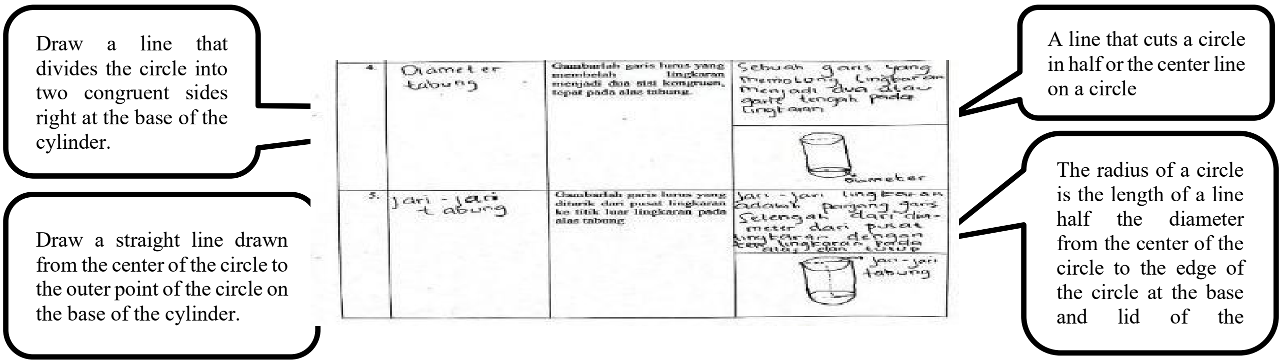

Figure 6. Table of elements of a cylinder listed by group 2

Activity CU3Bc3. Elaborated on the net 3-dimensional shape, especially on the cylinder

In Activity CU3Bc3, all students were engaged in constructing a cylinder net, demonstrating varying levels of proficiency based on their skill levels. High-skilled students were able to independently visualize and accurately depict the net, reflecting their strong spatial reasoning and prior understanding. In contrast, students with moderate and low skill levels required peer support to complete the task, indicating that collaborative learning played a crucial role in scaffolding their conceptual development.

Figure 7. Illustration of the cylinder net by group 7

The construction process involved three main stages: first, students drew a rectangle to represent the lateral surface of the cylinder (commonly referred to as the "blanket"); second, they drew two circles freehand using a pencil to represent the base and top of the cylinder; and third, they decided on the correct positioning of the circles either aligning them parallel to each other or placing them at opposite corners of the rectangle. This process highlights a key feature of the instructional approach, which emphasizes student contributions and active participation in constructing geometric understanding. The students' collaborative efforts and resulting work are illustrated in Figure 7.

Activity CU4Bc3. Solving problems related to the circle net

In Activity CU3Bc3, the teacher facilitated students' application of their understanding by presenting worksheet problems related to constructing and reasoning about a cylinder net. These tasks aligned with the "applying" level (C3) of Bloom's taxonomy, as students were required to synthesize prior knowledge of a cylinder's parts (base, lid, and lateral surface) and apply it to generate a two-dimensional net representing a three-dimensional object. The complexity of the task was moderate to high, requiring students to connect theoretical understanding (C2) with practical construction (C3). Observational data revealed that high-skilled students completed the activity independently, while moderate- and low-skilled students benefited from visual aids, peer collaboration, and teacher scaffolding to support their reasoning processes.

During group discussions, students employed several strategies to complete the task: (1) replicating the net structure based on previously observed real-world objects (such as cans), (2) clearly identifying cylinder components before assembly, and (3) creating physical models by cutting and assembling 2D representations into 3D forms. These strategies were especially effective for moderate- and high-skilled students, who successfully connected abstract concepts to tangible representations. Although low-skilled students primarily relied on imitation, teacher support helped them demonstrate foundational understanding. This was evident in the overall score of 91.6 for CU3Bc3 (see Table 4), reflecting a "highly satisfactory" level of performance. Interestingly, Group 3 concluded that a cylinder net could still exist without a lid, reasoning that the remaining base and lateral surface were sufficient. Conversely, Groups 1, 2, and 4 disagreed, stating that the net was incomplete without both circular components. Group 3's unique reasoning and illustrative response are captured in Figure 8 and the subsequent group-teacher dialogue.

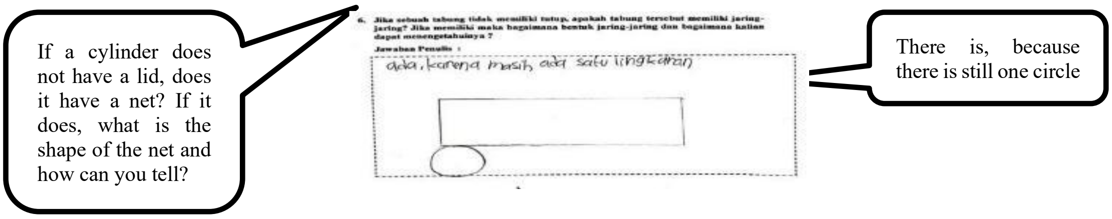

Figure 8. Answer by group 3

The following is a transcript of the teacher and Group 3 conversation.

Low student	:	*Mom, is there a net form of problem number 6*
Teacher	:	*(The teacher gives an illustration of a tin can object), If I remove this lid, will it be empty? There is no lid; now, do you think this object is perfect, and is there a possibility of a net?*
Low students	:	*There is, but is it the same shape, Mom?*
Teacher	:	*Try to think about whether the net is drawn whether it can form this object*

In Activity CU4Bc3, significant learning disparities were observed among students of varying skill levels. Low-skilled students struggled to comprehend the context and demands of the problem, resulting in minimal participation and limited contributions within their groups. Concurrently, many moderate- and high-skilled students misinterpreted the problem, assuming it did not pertain to a cylinder net, as illustrated in Figure 9. These misunderstandings revealed conceptual gaps in interpreting geometric representations and applying prior knowledge to new problem contexts.

To support struggling learners, the teacher implemented a series of targeted scaffolding strategies. These included providing additional visual aids such as illustrations of cylinder nets, posing guided questions to help students identify and relate the cylinder's components, and conducting hands-on demonstrations using physical objects like cans. The teacher also encouraged peer-assisted learning by asking higher-skilled students to restate the problem using simpler terms. These scaffolding efforts were designed to activate the students' zone of proximal development (ZPD), gradually guiding them toward independent problem-solving. Despite the overall outcome of CU4Bc3 being categorized as "unsatisfactory," a positive shift in individual participation was noted. This suggests that, while conceptual mastery was not fully achieved, the interventions promoted cognitive engagement and should inform the refinement of future instructional designs to better support conceptual representation in multiple mathematical forms.

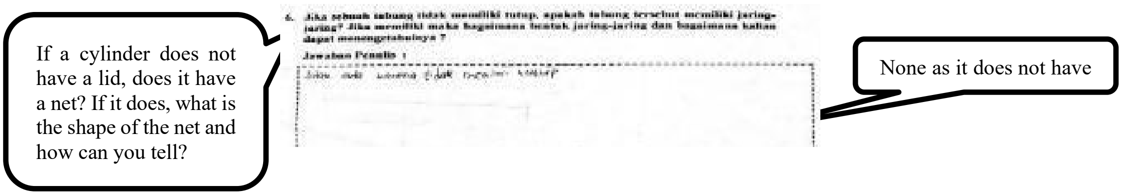

Figure 9. Answer by group 2

Table 5 reveals that although high-skilled students generally exhibited strong performance across the activities, about 28% still required additional instructional support, particularly during activity CU4Bc3, which involved identifying an incomplete cylinder net. This indicates that even among the high-achieving group, the complexity of interpreting partial geometric representations posed challenges. In comparison, approximately 60% of moderate- and low-skilled students consistently required more intensive scaffolding to navigate higher-order tasks, such as recognizing varied forms of cylinders or articulating the logic behind a lidless cylinder net.

Table 5. HLT as a result of retrospective analysis

Objective Code	Conjecture of Students' Thinking	Students' Thinking in the Actual Classroom	Teachers' Response
CU1Bc1.1	Students with low, moderate, and high skills can make a model according to the original shape by considering the cylinder elements.	Students with low-skilled, moderate-skilled, and high-skilled can solve the cylinder model perfectly.	The teacher facilitates modeling by using real cans and providing clear step-by-step instructions.
CU1Bc1.2	Students with all skill levels are expected to identify the definition of a cylinder.	Students define cylinders correctly.	The teacher confirms answers through guided discussion and clarifies unclear terms.
CU1Bc1.3	Students can mention the types of 3-dimensional figures, especially on cylinders.	The moderate-skilled and low-skilled students could not name the types of cylinders correctly.	The teacher provides comparison visuals and provides examples.
CU2Bc2	Students will describe the cylinder elements by naming, explaining and describing the elements.	Moderate-skilled and high-skilled students need additional assistance.	The teacher uses visual aids and probing questions to support student articulation.
CU3Bc2	Students will describe the net cylinder with clear steps.	All students solve the problem of making a net cylinder	The teacher confirms the solution and prompts to explain their steps.
CU4Bc3	Students will explain the situation of the net cylinder without a cover.	Low-skilled and moderate-skilled students cannot solve the problem; high-skilled students need help.	The teacher introduced a real-life example (e.g., an open-top container) and guided small group discussions to re-establish understanding.

To address these learning needs, the teacher implemented several targeted pedagogical interventions. These included whole-class discussions for conceptual reinforcement, clarification of task instructions, and guided questioning strategies aimed at stimulating deeper thinking. The use of visual aids such as three-dimensional cylinder models and real-life concrete examples further helped students make meaningful connections between abstract and tangible representations. Additionally, a think-aloud strategy was employed to model reasoning processes, while heterogeneous group arrangements allowed high-skilled students to support their peers, fostering collaborative learning environments. These strategies proved particularly effective in activity CU4Bc3, where initial performance was low, but student engagement and comprehension improved as a result of the scaffolded support.

To evaluate the impact of these interventions, a post-test comprising 10 multiple-choice items and three open-ended questions was administered to 28 fifth-grade students. This assessment aimed to measure students' conceptual understanding of cylinders following the instructional sequence. The outcomes of this test are presented in Table 6 and further examined in the retrospective analysis to inform instructional refinement.

Table 6. Result of conceptual understanding

Categories	Score Intervals	Total
Less	$(0 \le x < 60)$	3 students
Fair	$(60 \le x < 75)$	5 students
Good	$(75 \le x < 85)$	10 students
Excellent	$(85 \le x \le 100)$	10 students

To obtain a more nuanced understanding of students' conceptual grasp of three-dimensional figures, specifically curved-surface solids, the post-test scores were categorized into four achievement levels: less $(0 \le x < 60)$, fair $(60 \le x < 75)$, good $(75 \le x < 85)$, and excellent $(85 \le x \le 100)$. This classification enabled a systematic evaluation of learning outcomes and facilitated an in-depth analysis of the effectiveness of the implemented Hypothetical Learning Trajectory (HLT). According to the classification results, three students were categorized as less, five as fair, ten as good, and ten as excellent, indicating that the majority demonstrated a satisfactory to advanced conceptual understanding. Nevertheless, a subset of students continued to exhibit difficulties that warranted additional pedagogical support.

A total of 20 students achieved scores in the good to excellent range (75–100), suggesting a strong level of conceptual understanding related to cylinders. This was evidenced by their ability to meet the following indicators: (1) correctly identifying the structural components of a cylinder (base, top, and lateral surface), (2) accurately constructing and proportioning a cylinder net, (3) articulating the interrelations among its geometric elements, and (4) solving contextual problems involving volume and cylinder nets. Quantitative analysis revealed that 88% of students satisfied the first indicator, 84% the second, 76% the third, and 72% the fourth. Notably, a substantial improvement was observed in the fourth indicator, compared to the initial cycle, in which only 48% of students successfully addressed contextual problem-solving tasks. These findings indicate that the HLT revisions introduced in the second cycle contributed significantly to enhancing students' ability to apply geometric concepts, particularly at the Understanding (C2) and Applying (C3) levels of Bloom's taxonomy. Therefore, it can be inferred that the instructional activities grounded in Bloom's framework were effective in cultivating deeper mathematical comprehension among learners.

The developed HLT consisted of four sequential learning activities: (1) Constructing a cylinder model students engaged in hands-on construction using familiar objects to conceptualize the geometric features of cylinders concretely; (2) Describing cylinder elements learners identified and described key components such as the base radius, height, and lateral surface, fostering foundational knowledge; (3) Illustrating the cylinder net this activity emphasized the transformation of three-dimensional figures into two-dimensional representations, deepening spatial reasoning; and (4) Solving problems involving cylinder nets students applied their understanding to real-world tasks, reinforcing conceptual knowledge through problem-solving.

The implementation of this Bloom's-based HLT demonstrated its efficacy in guiding elementary students through the progressive stages of conceptual development in the topic of cylinders. The learning trajectory emphasized structured reasoning, starting from concrete construction to abstract problem-solving. For example, classroom discussions explored questions such as whether a cylinder net can be identified in objects lacking caps, which contextualized learning within students' lived experiences. These contextual engagements not only made the learning process more meaningful but also fostered the development of lateral mathematical thinking, consistent with the theory of schema evolution (Steffe, 2004). This result aligns with Kristidhika et al. (2020), who reported enhanced conceptual understanding when instruction was situated in contexts familiar to students, as opposed to traditional textbook-driven methods.

Further, the use of real-world objects such as cans to introduce and generalize geometric concepts exemplifies the instrumental role of teaching aids in facilitating the abstraction process. These tools bridge informal understanding and formal mathematical reasoning, allowing students to grasp generalized ideas from indirect experiences, including objects lacking a top surface. This finding supports Doorman and Gravemeijer's (2009) assertion regarding the beneficial interaction between manipulative tools and students' conceptual mastery, as well as Shanty's (2016) conclusion that informal strategies can evolve into formal mathematical understanding through well-designed instruction.

A central tenet of the HLT design is the alignment of instructional sequences with anticipated student thinking and potential learning obstacles. While students with moderate understanding were often able to interpret and solve problems correctly, not all could satisfy the full range of conceptual indicators. Students exhibiting lower conceptual understanding generally operated within the lower tiers of Bloom's taxonomy Remembering and Understanding. However, the second design cycle offered critical insights into students' conceptual development and served to validate and refine the instructional trajectory. In line with Bakker (2018), iterative design research facilitates the evaluation and optimization of localized instructional theories. The second-cycle data provided valuable feedback for improving pedagogical strategies and refining the HLT.

Although students at the lower performance level primarily demonstrated competencies in basic recall and comprehension, the overall improvement in their conceptual understanding was evident. This trend corroborates the findings of Sulfiah et al. (2021), who emphasized that conceptual understanding entails the ability to establish conceptual connections, select appropriate strategies, and apply mathematical principles in diverse problem-solving contexts.

According to the framework proposed by Kholid et al. (2021), students' understanding can be classified into high, moderate, and low levels. High-level understanding is demonstrated through accurate problem-solving; moderate-level understanding is characterized by partial success across the indicators, while low-level understanding is evidenced by an inability to solve the problems correctly. These categorizations reinforce the idea that conceptual understanding is a critical objective in mathematics instruction and can be systematically assessed using Bloom’s taxonomy. Consequently, the study illustrates how the structured alignment between instructional activities and Bloom’s cognitive levels supports the progressive modeling process from contextual to formal mathematical understanding, as described by Gravemeijer and Cobb (2006). Thus, this study reaffirms that instructional designs based on Bloom’s taxonomy can effectively foster students’ conceptual development in mathematics.

Conclusion

The four instructional activities grounded in Bloom’s taxonomy were systematically designed and implemented to enhance students’ understanding of cylinders as three-dimensional geometric figures. The first three learning activities—constructing a cylinder model from real-world objects (CU1Bc1), identifying its essential components (CU2Bc2), and illustrating its net (CU3Bc2) proved to be effective in fostering students’ conceptual comprehension. These activities aligned respectively with the cognitive processes of remembering, understanding, and applying, and demonstrated the potential of a hierarchical learning trajectory in supporting students’ engagement with abstract geometrical concepts. The tasks allowed students to connect mathematical ideas with tangible experiences, thereby facilitating a more meaningful learning process. In contrast, the fourth activity (CU4Bc3), which focused on problem-solving related to cylinder nets without caps, presented greater challenges and yielded less satisfactory outcomes. This suggests that tasks requiring higher-order thinking skills, such as analyzing and evaluating, may necessitate additional scaffolding and more extended instructional time to support students in transitioning from procedural fluency to deeper mathematical reasoning.

Despite its contributions, this study is not without limitations. The design of the learning activities was limited to the first three cognitive levels of Bloom’s taxonomy, and the investigation primarily centered on the development and initial implementation of these activities without long-term follow-up or broader classroom validation. Furthermore, the scope of the study was constrained by the specificity of the context, sample size, and content focus limited to cylinder geometry. Future research is recommended to extend the learning trajectory to include higher-order thinking skills, such as analyzing, evaluating, and creating, in order to fully explore the impact of Bloom’s taxonomy across all levels of cognitive development. Additionally, subsequent studies could investigate the effectiveness of similar activities across other three-dimensional shapes and diverse student populations to examine the generalizability of the instructional approach. Incorporating mixed-methods research designs and longitudinal data may also provide deeper insights into students' conceptual growth and sustained understanding in geometry learning.

Acknowledgment

The authors would like to express their sincere gratitude to Universitas Muhammadiyah Sidoarjo for the institutional support provided throughout the research and publication process. Appreciation is also extended to Sekolah Dasar Negeri Punggul 2 for granting permission to conduct the study within the school environment.

Conflicts of Interest

No conflict of interest is reported.

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