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Designing learning trajectory on ratio and proportion through PMRI with the Indonesian traditional food context

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Abstract

Mastery of the concepts of ratio and proportion is essential in mathematics education, as it underpins students' ability to develop logical reasoning and proportional thinking skills critical for advanced mathematical understanding. Despite its importance, students often struggle with these concepts due to the lack of learning activities that meaningfully connect mathematics to real-life contexts and cultural relevance. This disconnect highlights a gap in instructional approaches that fail to bridge abstract mathematical content with students' lived experiences. Addressing this issue, the present study introduces a novel integration of the Pendidikan Matematika Realistik Indonesia (PMRI) approach with local cultural context, specifically the traditional Sumatran dish, namely *lemang*, to enhance students' conceptual grasp of ratio and proportion. The study aims to design and implement a PMRI-based learning sequence centered on the *lemang* context to facilitate meaningful learning experiences among junior high school students. Employing a design research methodology, the study involved seventh-grade students at SMP IT An-Nida Lubuklinggau, with data collected through student activity sheets (SAS), classroom observations, interviews, and documentation. Findings indicate that the integration of PMRI with the *lemang* context significantly supports students' progressive understanding of ratio and proportion, fostering both cognitive engagement and cultural appreciation. This research contributes to the enrichment of mathematics education by demonstrating how culturally contextualized PMRI designs can promote deeper learning and affirm the relevance of mathematical knowledge in students' everyday lives.

Keywords: learning design, *lemang*, PMRI, ratio and proportion, traditional food

Introduction

Ratio and proportion constitute essential mathematical constructs that underpin the comprehension of more advanced mathematical topics, including percentages, fractions, measurement, probability, and algebraic reasoning (Lesh et al., 1988; Avcu & Avcu, 2010; Van de Walle, 2013; Sari et al., 2023; Bulut, 2023). Conceptually, a ratio represents a multiplicative comparison between two quantities, expressed through division or multiplication (Ben-Chaim et al., 2012). Consistent with this, proportional reasoning involves the recognition of multiplicative relationships between quantities (Cramer et al., 1993). Mastery of these concepts not only facilitates success in mathematics but also enhances students' capacity to engage with real-life problem-solving contexts (Karplus, 1983; Hamidah et al., 2018; Rismayantini et al., 2021). Moreover, proportional reasoning serves as a foundational bridge between elementary arithmetic and more abstract mathematical domains, such as algebra, thereby playing a critical role in students' cognitive progression toward higher-order mathematical thinking (Fuson & Abrahamson, 2005; Lamon, 2007; Lesh et al., 1988; Lamon, 2012). Within the secondary mathematics curriculum, proficiency in proportional reasoning is widely regarded as a key indicator of conceptual mathematical understanding (Tunç, 2020).

Despite its significance, the concepts of ratio and proportion are often identified as challenging for students, frequently leading to systematic misconceptions and procedural errors (Jitendra et al., 2009; Debreli, 2011; Nunokawa, 2012; Boudreaux et al., 2015; Dougherty et al., 2017). Difficulties arise primarily from students limited conceptual understanding and persistent misconceptions (Rahmasantika & Prahmana, 2019; Domondon et al., 2022). For instance, Wahyuningrum et al. (2019) reported that many students misconstrue the multiplicative relationship between quantities when engaging with fraction multiplication. Subsequent research indicates that learners struggle to differentiate between direct and inverse proportion and face challenges utilizing tables and graphs to solve proportion-related problems (Wijaya et al., 2019; Bintara & Suhendra, 2021). Furthermore, Çalışıcı (2018) observed that students often fail to recognize equivalent ratios and erroneously manipulate fractional components by treating numerators and denominators as discrete whole numbers. Misconceptions such as the belief that adding identical values to the numerator and denominator does not change the value of a fraction further complicate students' understanding. Errors in cross-multiplication procedures are also prevalent due to students' failure to simplify quantities prior to computation. Khasawneh et al. (2022) similarly documented persistent misunderstandings related to ratio and proportion, particularly students' difficulty in identifying proportional relationships as inherently multiplicative and their tendency to incorrectly apply additive reasoning. Confusion regarding the identification and application of direct versus inverse proportionality further exacerbates these difficulties. These conceptual challenges are shaped not only by students' cognitive misconceptions but also by pedagogical approaches that inadequately support the development of proportional reasoning (Deringöl, 2019).

Instructional materials on ratio and proportion presented in conventional textbooks are often characterized by limited conceptual depth, frequently focusing solely on procedural rules and algorithmic computations (Zulkardi, 2002). Compounding this issue is the tendency of educators to design tasks that emphasize rote procedures rather than fostering conceptual understanding, thereby impeding students' ability to engage meaningfully with the underlying mathematical structures (Andini & Jupri, 2017). As a result, students commonly resort to memorization strategies when solving problems, rather than constructing meaningful connections or employing flexible reasoning (Diba & Prabawanto, 2019). This reliance on surface-level learning becomes especially problematic when students are faced with novel or contextually different problem situations, where procedural knowledge alone proves insufficient.

Effective instruction in ratio and proportion necessitates more than the transmission of formulas and mechanical steps; it requires the integration of meaningful, real-world contexts that resonate with students' lived experiences. Such contextualization not only mitigates the abstractness of these concepts but also enhances students' appreciation of their practical relevance (Siegler et al., 2010; Domondon et al., 2024). It is important to acknowledge, however, that even before formal instruction, many students possess intuitive and informal strategies related to ratio and proportional reasoning (Lamon, 1995). For instance, Resnick and Singer (1993), through a fair-sharing task involving feeding fish of varying sizes, demonstrated that children as young as 5 to 7 years old could engage in proportional reasoning by distributing more food to larger fish despite the absence of numerical values. Similarly, Confrey (1995) identified that children often utilize what she termed "small recipes" strategies based on identifying and reasoning with the smallest integer pairs that maintain a given ratio to solve proportion problems. To support the development of robust proportional reasoning, it is therefore essential to provide learners with concrete, exploratory, and contextually rich learning experiences that enable them to transition from intuitive strategies to formal mathematical understanding (Lachance & Confrey, 2002).

To overcome the challenges associated with students' misconceptions and procedural reliance in learning ratio and proportion, there is a pressing need for instructional designs that are both contextual and grounded in students' real-life experiences. One promising pedagogical framework is Realistic Mathematics Education (RME), and its Indonesian adaptation known as Pendidikan Matematika Realistik Indonesia (PMRI). As a domain-specific heuristic, PMRI guides the design of instructional activities that promote conceptual understanding by aligning learning trajectories with students' cognitive development and lived realities (Bustang et al., 2013). In particular, PMRI emphasizes the use of real-world contexts to support students in transitioning from informal reasoning to formal mathematical thinking a process referred to as horizontal and vertical mathematization (Fredriksen, 2021; Franchini et al., 2017; Tien Da, 2022). By integrating students' intuitive strategies into learning designs, PMRI fosters deeper engagement and understanding, particularly in complex topics such as ratio and proportion (Sumarto et al., 2014).

A core tenet of PMRI is the contextualization of mathematical concepts through cultural or environmental situations familiar to students. This approach not only diminishes the abstractness of mathematical ideas but also facilitates knowledge construction through meaningful exploration (Wahyudi et al., 2016). Culturally embedded contexts such as traditional practices, crafts, or food offer authentic learning experiences and serve as effective mediators between everyday experiences and mathematical reasoning (Putri et al., 2015; Risdiyanti & Prahmana, 2021). Traditional foods, which carry rich cultural significance, are particularly effective in this regard (Guerrero et al., 2009). In the context of Sumatra, *lemang*, a traditional rice dish cooked in bamboo, presents a culturally relevant and mathematically rich context through which ratio and proportion can be explored meaningfully (Siregar & Asrul, 2024). Engaging students with the preparation of *lemang* as a learning activity not only enhances conceptual understanding but also fosters appreciation for local heritage, making mathematics both relevant and applicable (Sembiring et al., 2010).

Moreover, such contextual integration supports the development of both cognitive and affective learning domains by presenting mathematics in an enjoyable and skill-oriented manner (Rahayu & Putri, 2020). This pedagogical strategy aligns with broader principles advocating for mathematics education that is problem-centered, relevant to daily life, and grounded in students' experiences (Siegler et al., 2010). Consequently, a PMRI-based instructional design utilizing the context of *lemang* is anticipated to create a more engaging, meaningful, and culturally responsive learning environment for junior high school students. Beyond facilitating conceptual mastery of ratios and proportions, this approach nurtures a deeper recognition of mathematics as an integral part of students' everyday lives and local culture, equipping them with valuable skills and a stronger sense of cultural identity.

Several prior studies have explored the development of instructional designs for teaching ratio and proportion using the PMRI approach in various contextual settings. For example, Muttaqin et al. (2017) utilized the natural landscape of East OKU as a context for proportional reasoning, incorporating ratio tables and graphical representations as core strategies. Similarly, Zuhra et al. (2018) constructed a learning trajectory on direct proportion based on the timpan recipe, emphasizing ratio tables as the principal tool. Nursa et al. (2020) designed learning activities grounded in the Musi Tour water attraction in Palembang, again applying ratio tables as the primary problem-solving method. Wardani and Prahmana (2021) developed instructional activities targeting both direct and inverse proportions using a contextual scenario involving pens and currency, while Sugiarni et al. (2022) employed the context of *Pandanwangi* rice farmers in Cianjur to design learning trajectories for direct proportion, also centered around the use of ratio tables.

This study offers a distinct contribution by introducing a novel context *lemang*, a traditional food indigenous to Sumatra which has not previously been explored in the literature on ratio and proportion learning through the PMRI framework. Unlike earlier studies that often began directly with proportional reasoning, this research prioritizes a foundational understanding of ratio as a relational concept between two quantities. Through activities rooted in everyday experiences, students are first encouraged to conceptualize ratios before progressing to proportional reasoning. The instructional strategies applied in this design include diagrams, unit rates, and ratio tables, providing a diversified set of tools that not only enhance cognitive engagement but also foster flexible and critical problem-solving approaches.

This research significantly advances the design of contextual mathematics learning by integrating cultural relevance within the PMRI approach. The utilization of *lemang* as an authentic and culturally rich context expands the diversity of real-world settings applied in mathematics education. More importantly, this study contributes an innovative learning trajectory that emphasizes the rediscovery of mathematical ideas through guided inquiry and exploration, rather than the direct transmission of formal concepts. By embedding mathematical reasoning in culturally familiar practices, the instructional design promotes deeper understanding, strengthens students' mathematical thinking and problem-solving capabilities, and reinforces cultural identity offering a holistic and meaningful approach to mathematics education.

Methods

This study adopts a design research methodology, with the central aim of constructing a learning trajectory for the mathematical concepts of ratio and proportion by integrating the cultural context of *lemang*, a traditional food. Design research, as articulated by Gravemeijer (2004), is an iterative and cyclical process encompassing the systematic design and empirical testing of instructional activities and associated pedagogical elements. It serves as a methodological framework for developing instructional designs and learning tools that target specific mathematical concepts by addressing students' learning difficulties, anticipating a range of possible student responses, and promoting meaningful mathematical understanding.

The research was conducted with seventh-grade students at SMP IT An-Nida, located in Lubuklinggau, Indonesia. The selected participants demonstrated varied levels of mathematical proficiency categorized as high, moderate, and low. Following the design research framework outlined by Gravemeijer and Cobb (2006) and Akker et al. (2009), the study was implemented in three key phases: preliminary design, design experiment, and retrospective analysis.

In the preliminary design phase, a comprehensive literature review was conducted to establish the pedagogical relevance of ratio and proportion. This was followed by a series of focus group discussions (FGDs) involving mathematics educators, content experts, and instructional designers. These deliberations led to the development of a preliminary Hypothetical Learning Trajectory (HLT) and a set of Student Activity Sheets (SAS). The HLT, grounded in the theoretical framework proposed by Simon (1995) and further elaborated by Gravemeijer (2004), outlines a predictive sequence of learning experiences, including instructional goals, proposed learning activities, and conjectured student thinking and reasoning processes within specific learning contexts.

The design experiment phase began with a pilot study aimed at eliciting initial feedback to refine the instructional sequence involving the context of *lemang* in teaching ratio and proportion. This pilot served as a preparatory step for the subsequent teaching experiment. The final phase, retrospective analysis, was conducted after both the pilot and teaching experiments were completed. This phase involved a systematic analysis of the empirical data collected, focusing on comparing the initially constructed HLT with the Actual Learning Trajectory (ALT) that emerged during classroom implementation.

Data collection techniques encompassed SAS, classroom observations, semi-structured interviews, and documentation. The SAS were employed to evaluate students' conceptual understanding and application of ratio and proportion through the designed learning tasks. Interviews were conducted to triangulate student responses and to gain further insights not readily observable during classroom instruction. Observational data captured student interactions and participation, while documentation both visual and textual provided supplementary evidence to support the analysis and refinement of the HLT.

Results and Discussion

The initial phase of this study involved experimental preparation, which served as a critical foundation for ensuring that all research components were systematically designed and aligned with the intended learning objectives. This phase began with an extensive literature review, where the researcher analyzed relevant sources, including the 2013 Curriculum and the Merdeka Curriculum, specifically focusing on the content of ratios and proportions. Following the literature review, classroom observations were conducted, complemented by in-depth discussions with mathematics teachers. These activities informed the development of the HLT and the associated student worksheets.

The HLT functions as a predictive framework for how students are expected to engage with and progress through the instructional activities. This framework remains dynamic and is subject to revision based on insights gained during implementation and design trials (Juniarti et al., 2022). Table 1 outlines the sequence of learning activities along with conjectures regarding student responses at each stage.

Table 1. Overview of Learning Activities and Conjectures Related to Student Thinking

Main Goal	Activities	Conjecture
Students are able to understand the concept of ratio, how to write it, and how to simplify it into its simplest form.	Activity 1: Understanding the Context of *Lemang* through Images and Videos to Introduce the Concept of Ratio	Students are able to identify key information such as the main ingredients (glutinous rice and coconut milk) and their respective quantities. Students are confused in identifying key information in the images and video related to the ingredient measurements for making *lemang*.
	Activity 2: Colouring Ingredient Measurements in a Visual Representation	Students are able to shade the circles with different colours for glutinous rice and coconut milk according to the respective quantities used by each character. They can accurately distinguish parts of the image and visually represent the ratio. Some students have difficulty shading according to the given quantities. They may use the wrong number of circles,

Main Goal	Activities	Conjecture
		choose incorrect colours, or use the same colour for both ingredients. They have not yet understood that the difference in quantity in the visual representation reflects the comparison between two quantities.
	Activity 3: Writing and Simplifying Ingredient Ratios	Students are able to write ratios in the correct order based on the given information. Students write the ratios incorrectly, for example by switching the order or using numbers that do not match the information in the task. Students can simplify ratios by dividing both numbers by a common factor. Students are not yet able to simplify ratios because they struggle to find a common factor or apply incorrect steps. Students use visual models (such as bar models) to help simplify ratios. Students are unable to connect visual models with correct ratio forms and have difficulty using diagrams to support simplification.
	Activity 4: Determining the Best Ratio for *Lemang* Quality	Students are able to analyse several ingredient ratios and relate them to the quality of the *lemang*. Students show initial understanding but are not yet able to connect the ratio with the final result. Students have difficulty understanding how the ratio affects the quality of *lemang*.

The HLT developed during this phase was refined to incorporate recent advancements in mathematics education and learning design presented in Table 1. It was iteratively reviewed through a FGD involving two mathematics education specialists and one instructional design expert. The purpose of this collaborative review was to ensure theoretical alignment, contextual relevance, and instructional clarity. Based on expert feedback, the HLT was revised and finalized, resulting in a model that adheres to the pedagogical principles of the PMRI framework.

The finalized HLT fosters meaningful learning by enabling students to connect their prior knowledge with real-world contexts, consistent with Ausubel's theory of meaningful learning. This approach facilitates cognitive bridging, wherein students relate newly introduced mathematical concepts specifically ratios to familiar experiences and intuitive understanding (Ningsih, 2014; Hasbi et al., 2019). Subsequently, the pilot implementation phase was conducted with a sample of six seventh-grade students representing diverse academic abilities. This phase involved the application of the HLT in a controlled instructional setting to observe its effectiveness and gather preliminary insights regarding student engagement and learning progression.

Activity 1. Understanding the Context of *Lemang* through Images and Videos to Introduce the Concept of Ratio

Activity 1 commenced with students analyzing visual representations and a video depicting the process of making *lemang*, a traditional dish from Sumatra. The primary learning objective (TP1) was to facilitate students' comprehension of ratios in the context of a real-life scenario.

(a)

(b)

Figure 1. Students observe images and videos in the Pilot Experiment

Figure 1 illustrates the activity, wherein students focused on observing the proportions of ingredients used in *lemang* preparation and the corresponding quantity of *lemang* produced. Students were tasked with recording critical information regarding the composition of the ingredients, as well as the ratios between them, in addition to noting the final outcome of the cooking process. This exercise aimed to enhance students' observational skills, helping them to identify relevant details and engage with the mathematical concept of ratio in a meaningful, context-based manner.

The scenario presented in the activity outlined the quantities of ingredients used by various individuals to prepare *lemang*, including Mr. Budi, a well-known *lemang* seller, and other participants in the traditional food preparation process. The ingredients included glutinous rice and coconut milk, with specified amounts for each participant.

<table>
<tr>
<td>
Diskusikan dengan teman kelompokmu! Perhatikan situasi gambar berikut!

Bahan dasar yang digunakan:

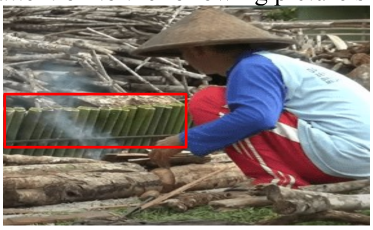
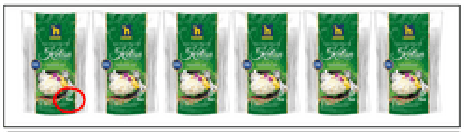
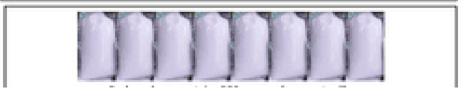

Masing-masing Santan Setiap Bungkus 500gr
Lemang termasuk salah satu makanan tradisional dari Provinsi Bengkulu. Tradisi lemang tapai untuk ritual adat seperti acara lamaran, pernikahan dan khitanan selain itu juga sebagai lambang rasa syukur masyarakat dalam menyambut panen padi. Pak Budi, sebagai salah satu penjual lemang, selalu mempersiapkan dagangannya. Ia terkenal dengan ketepatannya dalam menakar bahan, yaitu beras ketan dan santan, sehingga menghasilkan lemang yang pulen dan gurih sesuai takaran pada gambar 1. Dalam acara adat, Bu Yuni, Bu Eka dan Bu Ani akan mempersiapkan memasak lemang dengan takaran masing-masing 8 kg beras ketan dan 6 kg santan, 10 kg beras ketan dan 5 kg santan dan 12 kg beras ketan dan 8 kg santan.
</td>
<td>
Discuss with your group members! Pay attention to the following picture situation!

The basic ingredients used


Each package contains 500 grams of coconut milk.
Lemang is one of the traditional foods from Sumatra. The tradition of serving lemang is commonly practiced in cultural rituals such as engagement ceremonies, weddings, and circumcisions. In addition, it also symbolizes the community's gratitude during the rice harvest season. Mr. Budi, one of the lemang sellers, always prepares his products carefully. He is known for his accuracy in measuring the ingredients—glutinous rice and coconut milk—which results in lemang that is soft and savory, according to the measurements shown in the picture. During a traditional ceremony, Mrs. Yuni, Mrs. Eka, and Mrs. Ani will prepare lemang with the following measurements: Mrs. Yuni: 8 kg of glutinous rice and 6 kg of coconut milk, Mrs. Eka: 10 kg of glutinous rice and 5 kg of coconut milk, Mrs. Ani: 12 kg of glutinous rice and 8 kg of coconut milk.
</td>
</tr>
<tr>
<td>
a. Amati takaran bahan yang digunakan oleh Pak Budi dan yang lainnya, dan tulislah masing-masing takarannya!
Diketahui : 8 kg ketan dan 6 kg santan } : bu yuni
: 10 kg ketan dan 5kg santan } : bu eka
: 12 kg ketan dan 8kg santan } : bu ani
Pak budi : 6 kg ketan dan 4kg santan , 20 lemang
Jawaban siswa a
Dik: takaran Pak budi : 6 kg ketan dan 4 kg santan
takaran Bu yuni: 8kg ketan dan 6 kg santan
takaran Bu eka: 10kg ketan dan 5kg santan
takaran Bu ani: 12 kg ketan dan 8kg santan
Jawaban siswa b
</td>
<td>
Observe the measurements of the ingredients used by Mr. Budi and the others and write down each of their measurements!
Student A's Answer:
Given: Mrs. Yuni:
8 kg of glutinous rice and 6 kg of coconut milk
Mrs. Eka: 10 kg of glutinous rice and 5 kg of coconut milk
Mrs. Ani: 12 kg of glutinous rice and 8 kg of coconut milk
Mr. Budi: 6 kg of glutinous rice and 4 kg of coconut milk to produce 20 lemang
Student B's Answer:
Given:
Mr. Budi's measurements: 6 kg of glutinous rice and 4 kg of coconut milk
Mrs. Yuni's measurements: 8 kg of glutinous rice and 6 kg of coconut milk
Mrs. Eka's measurements: 10 kg of glutinous rice and 5 kg of coconut milk
Mrs. Ani's measurements: 12 kg of glutinous rice and 8 kg of coconut milk
</td>
</tr>
</table>

Figure 2. Example of answers during Activity 1

Figure 2 shows examples of students' responses during Activity 1. Student (a) provided a complete and accurate record of the ingredient quantities, including the amount of *lemang* produced by Mr. Budi. This response demonstrates a thorough understanding of the task, with Student (a) successfully identifying and documenting the full range of relevant information from the video and images. The integration of video as a visual aid significantly contributed to enhancing students' understanding and motivation, as it helped clarify the concepts and provided teachers with additional tools for reinforcing the content (Lalian et al., 2019; Rasiman et al., 2020; Rachmavita, 2020).

Student (a) also successfully concluded the results of the process depicted. Conversely, Student (b) recorded the ingredient quantities with clarity but omitted the critical piece of information regarding the amount of *lemang* produced, despite this detail being explicitly stated in the image. This omission suggests that, although Student (b) captured part of the required information, there was a lapse in the attention to detail necessary to fully engage with the problem. In an interview, Student (b) explained that their focus was on the ingredients, leading to the oversight. When asked about what could improve their attention to detail, the student suggested that providing more specific questions or visual cues, such as a box highlighting the relevant information, could prompt more careful observation. This response highlights the importance of fostering careful observation and guiding students to recognize and record all relevant data during learning activities mentioned in the following discussion.

Teacher	:	*"Could you explain why you did not include the amount of lemang produced, even though it was clearly depicted in the image?"*
Student b	:	*"I focused more on the ingredients used, Sir, and overlooked noting the quantity of lemang."*
Teacher	:	*"Was there any part of the task that you found difficult to understand?"*
Student b	:	*"No, Sir. The images and video were clear; I simply did not pay attention to the amount of lemang."*
Teacher	:	*"What suggestions do you have for improving your ability to record all the relevant information?"*
Student b		*"Perhaps if there were a highlighted section or more specific questions regarding the quantity of lemang, it would help me focus on that detail."*

Activity 2: Colouring Ingredient Measurements in a Visual Representation

In Activity 2, students were tasked with shading circles using distinct colors to represent the quantities of glutinous rice and coconut milk used by Mr. Budi, Mrs. Yuni, Mrs. Eka, and Mrs. Ani. The goal of this activity was to assist students in concretely visualizing the concept of ratio through a visual medium.

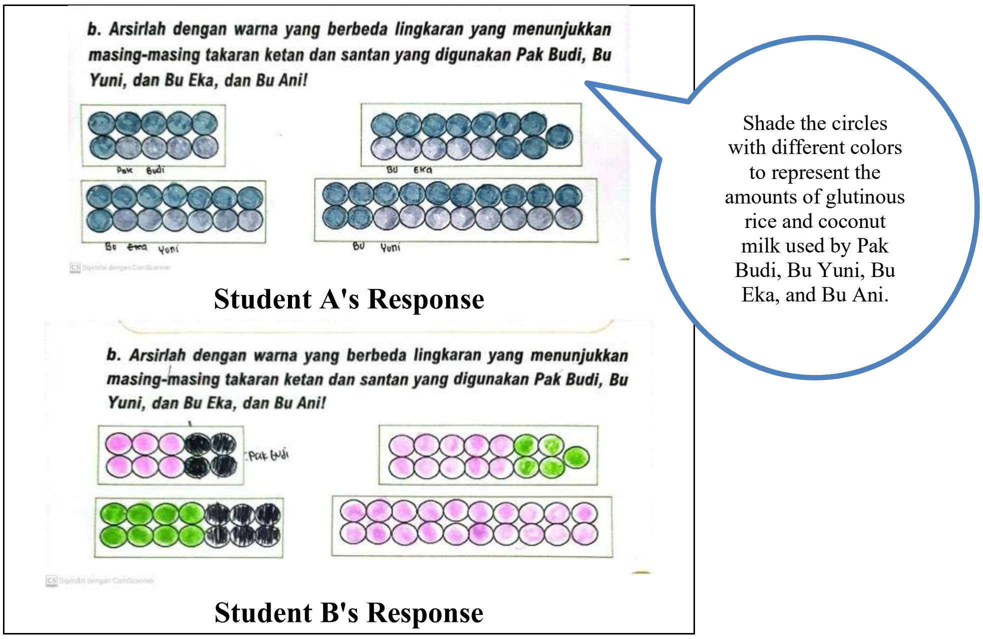


Figure 3. Example of students' responses during Activity 2

In the work displayed in Figure 3, both Student A and Student B applied two distinct colours to represent glutinous rice and coconut milk. However, Student B made a minor error by shading the final set of circles entirely in pink, which resulted in ambiguity regarding whether this portion corresponds to glutinous rice or coconut milk. This suggests that, although Student B grasped the concept of using colour to differentiate between ingredients, there is still a need for greater precision and consistency when visually representing ratios.

Teacher	:	*"You used green for rice and pink for coconut milk, correct? Mr. Budi's set is accurate. But for Mrs. Ani's, why are all the circles pink?"*
Student b	:	*"It should be 12 green and 8 pink, but I was in a hurry and ended up coloring everything pink."*
Teacher	:	*"So, you understood that the colors should be distinct?"*
Student b	:	*"Yes, I just forgot to separate them for Mrs. Ani."*

Activity 3: Writing and Simplifying Ingredient Ratios

In Activity 3, students were tasked with writing and simplifying the ratio of the key ingredients used in making *lemang*, based on contextual scenarios. They utilized bar models to visually represent the distribution of ingredients according to the given ratio.

Mr. Budi prepares lemang using a total of 10 kg of ingredients, with a ratio of glutinous rice to coconut milk of 3:2. If Mr. Budi receives an order on the first day for double the usual amount of lemang, how much total ingredients are required? Additionally, how much of each ingredient is needed?"

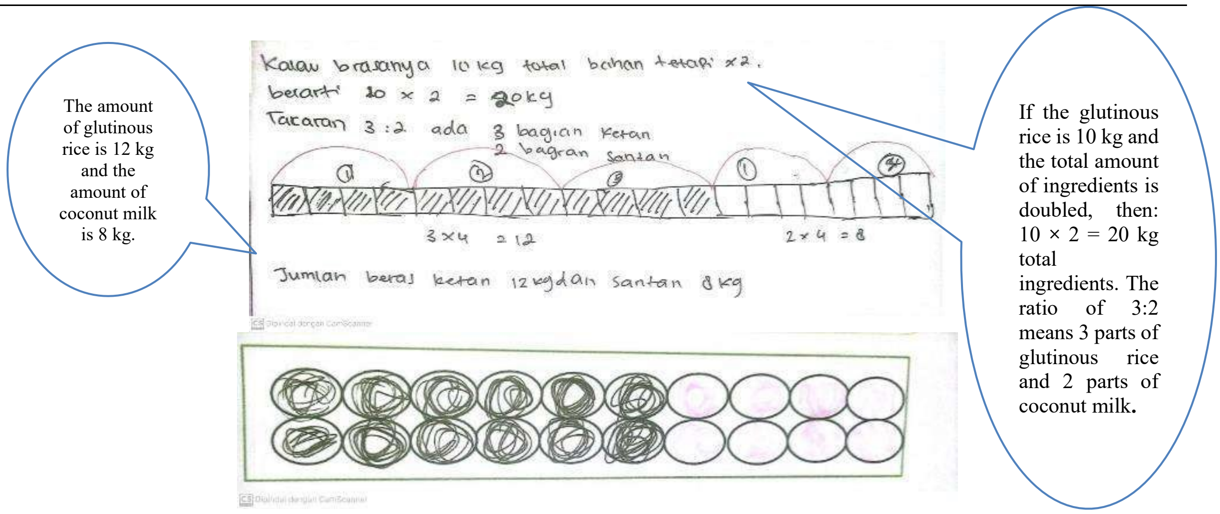
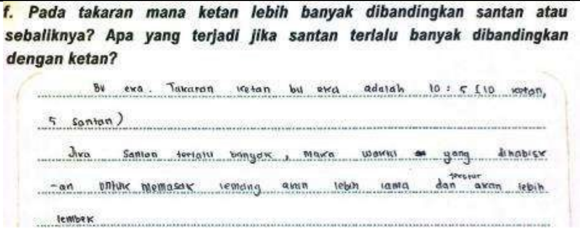


Figure 4. Example of student responses in Activity 3

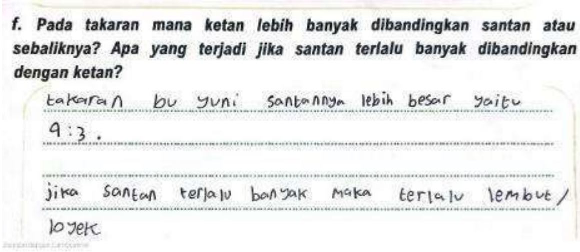
The students' responses indicate their comprehension of determining the proportions of each ingredient according to the specified ratio, as well as their application of bar models as a visual tool. This reflects their capability to apply the concept of ratios in a visual manner. A notable feature of the students' solutions to ratio and proportion problems with bar models is that the transformation does not alter the mathematical concept at the core of the problem. Instead, it reframes the problem into a visually understandable format using whole numbers (Baysal & Sevinc, 2021). This visual representation proves to be an effective tool for emphasizing the conceptual foundations of proportional reasoning (Büscher, 2021). By employing the bar model, students are able to explicitly map the relationship between the ratio of ingredients and the total quantity of materials used, thus simplifying the process of calculating the proportion of each ingredient.

Activity 4: Determining the Best Ratio for *Lemang* Quality

Students engaged in analysing and evaluating the previously identified ingredient ratios to determine which composition would most effectively produce a soft and flavourful *lemang*. Additionally, they were required to provide justification for identifying the most disproportionate ratio, based on their understanding of the effects of ingredient proportions on *lemang* quality.



Student A's Response



Student B's Response

In which measurement is the amount of glutinous rice greater than the coconut milk, or vice versa? What happens if the coconut milk is too much compared to the glutinous rice?

Student A's Answer:
Mrs. Eka's measurement is 10:5 (10 kg of glutinous rice and 5 kg of coconut milk). If the coconut milk is too much, the cooking time for *lemang* will be longer, and the texture will become softer.

Student B's Answer:
Mrs. Yuni's measurement has more coconut milk, with a ratio of 4:3. If the coconut milk is too much, the *lemang* will become too soft or mushy.

Figure 5. Example of answers during Activity 4

As depicted in Figure 5, Student A accurately identified the most disproportionate ingredient ratio as 2:1 (glutinous rice to coconut milk), corresponding to Mrs. Eka's recipe. This ratio is considered imbalanced due to the significantly larger quantity of glutinous rice compared to coconut milk, which may influence the texture and flavour of the *lemang*. In contrast, Student B's response was incorrect, as they misinterpreted the order of the ingredients in the ratio. Specifically, the ratio 4:3 should denote 4 parts glutinous rice and 3 parts coconut milk. However, Student B reversed this, interpreting it as 3:4, and consequently selected it as the most unbalanced ratio. This indicates a misunderstanding of both the contextual meaning and the syntactic structure of the ratio representation.

This study revealed a noteworthy misconception among students when analysing ingredient ratios for *lemang* preparation. Many students selected the 4:3 ratio (glutinous rice: coconut milk) as the most unbalanced, whereas the 2:1 ratio actually represents a more extreme disparity between ingredients. The root of this misconception appears to be students' lack of attention to the correct sequencing of ingredients within the ratio. Instead of focusing on the contextual relevance and mathematical meaning, they tended to fixate on the numerical values alone. This highlights the importance of guiding students to interpret ratios not merely as abstract numerical relationships, but as representations tied to real-world contexts. Teachers play a critical role in identifying and addressing such misconceptions promptly, while also exploring the underlying reasons for students' misunderstandings. Furthermore, an interview with Student B was conducted to explore the reasoning behind their selection of the 4:3 ratio as the most disproportionate, as follows:

Teacher	:	*"Why did you choose the 4:3 ratio as the most disproportionate?"*
Student b	:	*"Because the number 4 is bigger than 3, so I thought it was the most unbalanced, ma'am."*

Teacher	*:*	*"But actually, the 2:1 ratio is more disproportionate because there's much more glutinous rice than coconut milk. Why didn't you choose that one?"*
Student b	*:*	*"Oh, I didn't think of that. I just looked at the bigger numbers and didn't pay much attention to the comparison."*
Teacher	*:*	*"Yes, that's the important part. But the 2:1 ratio means there's twice as much glutinous rice as coconut milk, so it's much more unbalanced. Also, you need to pay attention to the order of the ingredients in the ratio. The glutinous rice should be the first number, followed by the coconut milk."*
Student b	*:*	*"Ah, I see now, ma'am. I actually got it reversed, the 4:3 ratio had coconut milk first and then glutinous rice."*
Teacher	*:*	*"That's right, the order is important. If you get the order wrong, it can lead to misunderstanding the proportions of the ingredients. In this case, the correct ratio is 2:1 (glutinous rice: coconut milk) because there's more glutinous rice."*
Student b		*"Oh, now I understand. I need to be more careful about checking the order of the ingredients and the correct ratio."*
Teacher		*"Yes, it's an important lesson to be more careful in paying attention to the order and the comparison of ingredients."*

The results from the students' responses during the pilot experiment served as an evaluative basis for revising the activity design and refining the accompanying worksheet for use in the teaching experiment phase. To address the identified misconceptions, one proposed solution was the incorporation of more specific guiding questions aimed at helping students engage in a more detailed analysis of the visual and contextual information provided, such as the quantity of *lemang* produced and the correct sequence of ingredients. This refinement was considered essential to ensure that students not only noticed key information but also developed a sound understanding of how the sequence and proportion of ingredients particularly the ratio of glutinous rice to coconut milk affect the quality of the *lemang* produced.

In the teaching experiment phase, involving 29 students, the revised HLT and updated worksheets were implemented as shown in Figure 6. During this stage, students exhibited improved conceptual understanding and a greater capacity to engage with the learning activities. The revisions introduced based on the pilot experiment outcomes enabled students to more effectively analyze images, interpret video content, and respond to structured prompts. As a result, students demonstrated increased accuracy in identifying the relationship between ratios and outcomes in the *lemang*-making context. They were also more capable of articulating coherent and logical arguments grounded in the data provided. Notably, misconceptions observed during the pilot phase such as errors in interpreting ratio order or misunderstanding ingredient proportions were progressively corrected. The inclusion of well-targeted guiding questions proved effective in promoting conceptual clarity, enhancing students' attention to detail, and fostering a deeper understanding of the mathematical relationships embedded within real-world contexts.

The findings of this study indicate that students have begun to develop a solid conceptual understanding of ratios and equivalent proportions. A ratio is defined as a comparative relationship between two quantities with the same unit, while equivalent proportions refer to situations in which two quantities vary in a consistent and proportional manner. This understanding enables students to recognize how changes in one variable predictably influence another, as seen in the context of determining the required ingredients for varying quantities of *lemang*. Students demonstrated the ability to calculate proportional relationships and apply this knowledge systematically, suggesting that their reasoning has moved beyond intuitive estimations toward formal mathematical thinking.

Figure 6. Teaching Experiment Implementation in Large Groups

The sequence of learning activities implemented in this study was informed by the HLT and refined through a retrospective analysis comparing the HLT to the ALT. This comparison was supported by theoretical frameworks in mathematics education, including the principles of PMRI and relevant prior research. Cultural contextualization particularly through the use of *lemang*, a traditional food from Sumatra served as the entry point for the learning process. The use of culturally-relevant contexts aligns with research indicating that students' engagement and comprehension increase when learning is anchored in familiar and meaningful experiences (Zulkardi, 2013). This finding affirms the effectiveness of the HLT in facilitating students' conceptual development in line with PMRI principles.

The integration of PMRI with cultural elements has been recognized as an effective approach to improving students' mathematical abilities, particularly critical thinking skills (Domu & Mangelep, 2020; Shahidayanti et al., 2024). Furthermore, the learning trajectory progressed through stages consistent with the PMRI framework: beginning with a concrete and contextual phase, moving through horizontal and vertical mathematization, and culminating in abstract formalization (Zulkardi, 2013; Risdiyanti & Prahmana, 2021). The concrete phase engaged students with real-life experiences, such as watching *lemang* preparation videos or discussing traditional food practices, helping them connect mathematics with daily life. This contextual engagement allowed students to perceive the relationship between ingredients and product quantities, reinforcing the notion that mathematics is a tool for understanding and solving real-world problems (Mumu & Tanujaya, 2019). Students then advanced to using visual models, such as bar representations, to depict ratios and proportional relationships, transitioning from concrete experience to semi-abstract reasoning.

As students progressed, they entered the "model for" stage, where mathematical models were used to solve problems beyond the initial context. Here, students began applying their understanding of ratios to novel situations, including calculations involving adjusted ingredient quantities and equivalent proportions. They demonstrated the ability to group, simplify, and manipulate ratios, signifying a shift toward generalization and abstraction. Ultimately, in the formal stage, students were able to apply their knowledge of ratios and proportions to solve more complex mathematical problems, including those not directly linked to real-life contexts. This development illustrates the effectiveness of the PMRI approach in fostering mathematical reasoning and problem-solving abilities through meaningful learning experiences (Hakim & Setyaningrum, 2024; Khoirunnisa & Putri, 2022).

This research addresses a gap in the literature by developing and validating a learning trajectory for teaching ratios and proportions within a local cultural framework, which has been underutilized in previous studies. The use of *lemang* as a culturally-relevant context is a novel contribution, demonstrating how local traditions can serve as powerful pedagogical tools in mathematics education. The study contributes to the design and implementation of a PMRI-based HLT that integrates culturally-grounded, visual, and engaging activities to enhance students' conceptual understanding. Moreover, it offers practical implications for educators seeking innovative approaches to teaching proportional reasoning, suggesting that the alignment of learning with students' cultural backgrounds can enrich mathematical discourse and foster deeper learning outcomes.

Conclusion

This study developed a culturally contextualized learning design for ratios and proportions using the traditional *lemang* food context to support students' understanding through the stages of a structured learning trajectory. The trajectory comprised three key phases: beginning with the model of stage, where students engaged in contextual activities using bar models to represent ingredient ratios in making *lemang*; followed by the model for stage, where students applied these representations to perform more complex operations such as grouping and simplifying ratios to determine appropriate ingredient quantities. Finally, in the formal stage, students abstracted their understanding to solve equivalent proportion problems using formal mathematical reasoning. The implementation of this learning design demonstrated its effectiveness in bridging informal, real-life experiences with formal mathematical understanding, thereby fostering meaningful and conceptually grounded learning. The results suggest that integrating culturally relevant contexts, such as traditional foods, enhances student engagement and deepens comprehension in mathematics education.

However, the scope of this research is limited by its application within a single cultural context and a specific group of learners, which may restrict the generalizability of its findings. The study did not address the transferability of students' conceptual understanding to other mathematical topics or real-world situations beyond the immediate context of *lemang*.

Therefore, future research is recommended to explore and validate this learning trajectory across diverse cultural and regional settings, incorporating various traditional practices to develop broader ethnomathematical learning designs. Further studies involving larger and more heterogeneous samples, as well as longitudinal investigations into learning retention and transfer, are essential to determine the sustainability and scalability of culturally contextualized mathematics instruction. Such work will contribute to a more inclusive, culturally responsive, and pedagogically robust framework for mathematics education.

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