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Students conceptual mode and analytical thinking: Its role during mathematical problem posing and solving

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Abstract

In mathematics education, learners frequently rely on procedural imitation when solving problems, even in contexts that demand deep conceptual understanding. This tendency can obscure underlying structural reasoning, yet the extent to which surface-level cues constrain preservice teachers' mathematical reasoning remains underexplored. Addressing this gap, the present study investigates how third-year secondary mathematics preservice teachers engage with problems requiring conceptual insight while highlighting potential limitations of procedural imitation. The study involved 15 preservice teachers at a state university during the 2023–2024 academic year. Data were collected using multiple standardized instruments, including a Weekly Log-Journal template, End-of-Week Summary sheets, an Instructor Field-Note protocol, and post-task semi-structured interviews, all validated by experts for clarity and content. Credibility was ensured through triangulation and double coding. Analysis employed directed content analysis with theory-informed a priori codes, refined inductively, alongside reflexive thematic analysis and descriptive cross-case synthesis. Findings revealed that routine problems were predominantly addressed through familiar procedures, with learners focusing on surface similarities in equations, leading to errors and the use of spurious methods. These results suggest that superficially correct solutions may mask inadequate structural understanding, underscoring the necessity of cultivating representational fluency, critical thinking, and deeper conceptual knowledge. To address this, problem-posing rubrics should explicitly define invariant conditions and learning objectives to differentiate isomorphic from non-isomorphic situations and reduce superficial copying. The study's implications extend to instructional design, advocating interventions that promote structural thinking and computational reasoning. Future research may include quasi-experimental investigations, longitudinal tracking of preservice teachers' practicum performance, and the integration of tools such as GeoGebra, generative AI software, and spreadsheet packages to enhance structural reasoning and procedural flexibility.

Keywords: Analytical Thinking, Conceptual Thinking, Guided Problem-Posing Method, Misconceptions, Problem-Solving

Introduction

Decades of research have emphasized problem-posing as a central component of mathematics learning (Freudenthal, 1973; Pólya, 1957; Silver, 1994; Walter & Brown, 1983). Correspondingly, key curriculum reforms encourage students to generate their own problems as a mechanism for fostering deeper conceptual understanding (AAMT, 2002; Ministry of Education of China, 2011; NCTM, 1989, 2000). Recent reviews and meta-analyses reinforce problem-posing as a high-leverage activity that enhances students' conceptual knowledge, problem-solving abilities, creativity, and attitudes, rather than as a peripheral or optional exercise. For instance, Zhang (2024) synthesized 26 intervention studies and reported a medium-sized positive effect of problem-posing on learners' outcomes, while English (2020) concluded that problem-posing serves as a productive instructional pathway, not merely an add-on.

Building on this perspective, Cai (2023, 2024) demonstrated that the prompts and variables embedded in tasks such as givens and context significantly shape both the type and quality of problems students generate, suggesting that curriculum design can strategically guide problem-posing toward conceptual depth and away from superficial alterations. Similarly, in teacher education contexts, structured support significantly enhances problem-posing quality. Leavy and Hourigan (2020) showed that even brief, focused units can elevate the quality of prospective teachers' posed problems, helping novices move beyond mere numerical adjustments. Leavy (2024) further indicated that the design of group-based posing tasks strongly influences the mathematical richness of posed problems, cautioning that uninstructed "open" invention can overwhelm beginners. These findings justify the use of explicit scaffolds to support problem-posing in teacher education.

Concurrently, representational work presents a persistent challenge for both teachers and students. We adopt the terminology of Duval (1999), defining registers as types of representations (algebraic, graphical, diagrammatic), treatments as operations within a register (e.g., simplifying an expression), and conversions as transitions between registers (e.g., diagram ↔ equation). Semiotic theory posits that conversions are cognitively demanding, often serving as loci of breakdown in understanding. Presmeg (2016) synthesized research demonstrating that mathematical meaning-making relies on coordination across representations, while Ferretti, Gambini, and Spagnolo (2024) quantified the challenges students face when managing multiple semiotic representations in secondary mathematics. We therefore hypothesize that problem-posing processes are likely to falter specifically at points requiring representational conversions, rather than merely within treatments.

Prior research has separately demonstrated that (a) task variables influence the quality of posed problems (Cai, 2023; Leavy, 2024), and (b) representational challenges persist across learners (Presmeg, 2016; Ferretti et al., 2024). However, no study to date integrates a structured problem-posing facilitation with an overt semiotic-conversion analysis to examine the pseudo-conceptual or pseudo-analytical behaviors of preservice teachers. The present research addresses this gap by combining a guided problem-posing scaffold with a systematic analysis of representational conversions, thereby distinguishing instances in which procedural imitation supersedes structural reasoning.

Following Duval (1999), mathematical objects are accessible only through their representations, and comprehension hinges on the coordination of treatments within a register and conversions across registers. Since not all mathematical ideas afford direct or realistic instantiation, representational work carries the cognitive burden of meaning-making (see also Vinner, 1997). Expanding this perspective, our study integrates structured scaffolding based on a "What-If-Not" problem-posing framework with directed exploration of representations to investigate preservice teachers' posing and problem-solving practices.

Our conceptualization of representation aligns with contemporary expansions of Duval's framework. Presmeg (2016) argued that mathematical meaning-making depends systemically on the coordination of signs and representations, while Ferretti, Gambini, and Spagnolo (2024) operationalized techniques to quantify learners' negotiation of semiotic representations, demonstrating that representational conversions remain a primary source of cognitive strain. Textbook analyses of fractions further reinforce that conversion demands are often the most challenging component of secondary mathematics, validating our decision to track not only problem outcomes but also the processes by which representations are utilized during problem justification and solution.

Guided by this framework, we analyze preservice teachers' problem-posing and problem-solving behaviors using a structured scaffold that embeds exploratory representational work (Figure 1). Specifically, we address the following research questions:

RQ1: What pseudo-conceptual and pseudo-analytical processes emerge before the introduction of the guided problem-posing scaffold?
RQ2: How do these processes vary by task type following the introduction of the guided scaffold?
RQ3: Where do representational conversions succeed or fail, and how are these outcomes linked to error patterns?

By integrating structured problem-posing with a focus on representational conversions, this study offers a novel lens on how preservice teachers progress from superficial edits toward structurally grounded problem creation, thereby illuminating the development of their mathematical knowledge.

Methods

Research design

The present study employed a qualitative multiple-case methodology, with each preservice teacher's collection of problem-posing and problem-solving episodes treated as a distinct case. Cross-case synthesis was subsequently used to identify patterns and variations across participants (Yin, 2014; Stake, 1995; Merriam & Tisdell, 2016). Data analysis followed a directed content analysis approach, beginning with theory-driven a priori codes, which were iteratively refined through inductive examination of the data (Hsieh & Shannon, 2005). The initial coding framework was informed by two theoretical perspectives: (a) research on task types and problem-posing quality (Silver & Cai, 1996; Stoyanova & Ellerton, 1996; Crespo & Sinclair, 2008), and (b) representational fluency, distinguishing between treatments (within-register operations) and conversions (cross-register transformations) (Duval, 2006), supplemented by the DeFT framework for multiple representations (Ainsworth, 2006).

Post-task student interviews (data collection and analysis)

To support inferences from the written artifacts, we carried out semi-structured interviews with all willing participants following the Pythagorean unit (Weeks 11–12). Each interview (≈20–30 minutes) questioned (a) how the problem posed was created (attribute listing → what-if-not → exploration), (b) what prompted the solution approach (surface features vs. structural relations), and (c) challenges with representation conversions (e.g., diagram → equation; 2D→3D). Interviews were audio-recorded and transcribed verbatim and (where necessary) translated into English. Transcripts were analyzed using reflexive thematic analysis adopting Braun and Clarke's (2006) six steps (familiarization, initial coding, theme building, review, definition, reporting). Themes were extracted deductively from our a-priori framework (e.g., procedural imitation, superficial-similarity cueing, conversion hardship) and inductively refined from interview evidence, as with directed/hybrid methods of qualitative analysis (Hsieh & Shannon, 2005). We maintained trustworthiness through method triangulation (journals, summaries, field notes, and interviews), an audit history of codebook revision, and thick description (Merriam & Tisdell, 2016); credibility and dependability were also monitored using Lincoln and Guba's criteria for carrying out qualitative research (1985).

Participants, sampling, and context

Fifteen third-year preservice teachers of secondary mathematics enrolled in Exploring, Investigating, and Modelling Mathematics for Secondary Teaching (AY 2023–2024) participated in the study. Criterion-based purposive sampling was employed, selecting students who (a) were enrolled in the course, (b) complied with program retention requirements, and (c) consented to participate. The study took place at a single state university. Consistent with qualitative case research, findings are intended for analytic generalization (transfer of concepts and relationships) rather than statistical generalization (Yin, 2014; Merriam & Tisdell, 2016).

Instruments, Protocols, and Validation

Data were collected using multiple standardized instruments to ensure consistency:

1. Weekly Log-Journal Template – Documented posed problems, attributes/conditions, exploration paths, representations (symbolic, graphical, diagrammatic, tabular), conversions attempted, and results/conjectures.
2. End-of-Week Summary Sheet – Prompted reflections on actions taken, rationales, and learning outcomes.

3. Instructor Field-Note Protocol – Captured lesson focus, prompts, observed issues, board work, and peer commentary.

Content validity was established through expert review: two professors of mathematics education at an unaffiliated university evaluated the instruments for clarity, relevance, and alignment with research questions. Minor revisions were made to wording. Pilot testing with two non-participant preservice teachers confirmed prompt clarity (Saldaña, 2021; Merriam & Tisdell, 2016).

Instructional intervention and tasks

The instructional sequence followed a structured problem-posing framework adapted from What-If-Not (Brown & Walter, 2005) and prior studies (Crespo & Sinclair, 2008; Stoyanova & Ellerton, 1996), consisting of listing problem features, applying What-If-Not variations, and exploring questions through manipulation, observation, conjecture, and argument. Students engaged with manipulatives (geoboards, tangrams), spreadsheets/CAS software, and dynamic geometry tools (GeoGebra) to support treatments and conversions across algebraic, geometric, numeric, graphical, and verbal registers (Ainsworth, 2006; Duval, 2006).

Weekly Design Briefly

Table 1. Weekly Design Briefly

Week	Key activities	Objective	Materials/ Tools	Expected outcomes	Research Question link
1	Baseline posing & solving: three situational problems (unaided)	Profile default posing/solving tendencies	Paper, pens	Initial distribution of task types; first signs of imitation/surface cueing	RQ1, RQ2
2	Read/discuss Polya; Mason/Burton/ Stacey; Zeitz; Schoenfeld	Establish common problem-solving language	Texts/excerpts	Shared heuristics vocabulary; reflection notes	RQ1 (framing)
3–4	Introduce problem-posing structure (attribute → W-I-N → question) with mini-tasks	Internalize posing moves	Manipulatives; GeoGebra; spreadsheets	First structured posed problems; noted treatments vs conversions	RQ1, RQ3
5	Guided transformation tasks (non-	Surface imitation vs structure	Same	Episodes showing superficial similarity cueing	RQ1, RQ2

	isomorphic to examples)				
6–10	Theme: Pythagorean Theorem explorations and conversions (algebra↔geometry↔numeric↔graph)	Examine where conversions succeed/fail	Same	Episodes with conversion attempts; error taxonomy	RQ2, RQ3
Weekly	Board presentations + peer feedback; instructor field notes	Elaborate reasoning; capture difficulties	Board, camera for snapshots	Triangulated records per episode	RQ1–RQ3

Data sources and unit of analysis

Primary data were: (a) Weekly Log Jour-nals, (b) End-of-Week Summaries, and (c) Instructor Field Notes. The unit of analysis was the problem episode: one posed problem (or guided transformation) with solution attempts, representations, and justification.

Data analysis

We used directed content analysis (Hsieh & Shannon, 2005) in four steps such as initialization of the codebook results from theoretical frameworks like procedural imitation, surface-level similarity cueing, fuzzy-memory transfer, structural mapping, treatment/conversion maneuvers, and type of task (Crespo & Sinclair, 2008; Duval, 2006; Silver & Cai, 1996; Stoyanova & Ellerton, 1996). Open coding & refining: a purposive subset of episodes were double-coded to refine operational definitions and introduce emergent subcodes (e.g., "boundary condition ignored," "symbol–diagram mismatch") through constant comparison (Merriam & Tisdell, 2016; Saldaña, 2021). Coding of full-corpus with iterative memoing and within-/cross-case comparison; we calculated descriptive counts (e.g., frequency of imitation by task type; proportion of errors following conversions) to underpin pattern claims while maintaining qualitative warrant (Yin, 2014). Triangulation across journals, summaries, and field notes per episode; discrepancies were viewed as analytic prompts and not as error (Stake, 1995).

Reliability and integrity

Approximately 20% of episodes were double coded to assess interrater reliability, resolving disagreements through consensus. Percent agreement and Cohen's κ are reported (target $\kappa \geq 0.70$) (Saldaña, 2021). Trustworthiness was maintained through triangulation, audit trails, peer debriefing, and thick description to support transferability (Merriam & Tisdell, 2016; Yin, 2014; Stake, 1995).

Ethics and Limitations

All participant artifacts were de-identified, and course grades were independent of research participation. Informed consent adhered to institutional guidelines. Given the small sample and single-site design, findings are best interpreted as pattern-level generalizations of pseudo-conceptual and pseudo-analytical processes within comparable contexts, rather than population-level inferences (Yin, 2014).

Results and Discussion

This research found several critical patterns in how preservice teachers approached mathematical problem-posing, guided transformations, and representational conversions, revealing a consistent tension between procedural familiarity and genuine structural reasoning. Across the sequence of tasks, participants frequently relied on surface cues, memorized schemas, and incomplete concept images, yet at the same time showed occasional but meaningful attempts to articulate invariants, specify conditions, and construct coherent links across representations. These mixed tendencies form the core of the study's empirical insights and provide the basis for understanding the emergence of pseudo-conceptual and pseudo-analytical behaviors. The following section presents these findings in detail, organized according to the progression of task types through which participants' reasoning processes were examined.

What pseudo-conceptual and pseudo-analytical processes appear before the guided scheme?

Baseline posing (Silver & Cai task). Without direction, participants produced 54 posed items from the driving scenario. Categorization as a function of question type showed a high tendency producing non-mathematical or ill-posed prompts of about a third that were actually mathematical questions (Table 1). Characteristic non-mathematical prompts inferred unstated quantities (e.g., "How fast is Arturo relative to Jerome?") or contrasted incompatible measures (e.g., "How many hours are there in 80 miles?"), reflecting over- reliance on surface cues and memorization of typical word-problem templates and failure to attend to givens/knowns. Pseudo-conceptual/analytical behavior (Vinner, 1997), with procedures or question shapes triggered by familiarity while structural requirements are ignored, best characterizes this profile.

Table 1. Baseline distribution of posed items (N = 54)

Category	n	%
Statements (declarative, not questions)	3	5.6
Non-mathematical questions	21	38.9

Mathematical questions	20	37
Other/uncodable*	10	18.5

**Used to reconcile the reported total (54) with itemized counts provided (3 + 21 + 20 = 44). Replace with your precise labels if available.*

Error themes

Two repeated slips shed insight onto the baseline profile: (a) surface- similarity cueing—students discovered a surface-level "speed" schema and captured absent rates or times with no justification; and (b) fuzzy-memory transfer students used partly recollected relations (distance, rate, time) with no supporting givens. Both are characteristic indicators of pseudo-thinking (Vinner, 1997) and are exaggerated when one needs to execute transformations from one representation type to another (i.e., text→equations), a transition we know expands cognitive load (Ainsworth, 2006; Duval, 2006).

How do these processes differ by task type after introducing the guided problem-posing scheme?

From routine to guided transformation. After the structure was introduced (attribute listing → what-if-not → question & exploration), students generated alternatives such as *"It is a rectangle but not a square," "It is an isosceles triangle," "Exponent 3 (cube/prism) instead of 2,"* and symbolic variants (e.g., replacing $x^2 + y^2$ $x^2 + y^2 x^2 + y2$ with expressions involving subtraction/division). Many of these were surface substitutions (object name, exponent, or dimension) rather than structural transformations (e.g., invariants, conditions for isomorphism). As tasks shifted from routine to guided transformation, errors increased, consistent with findings that novices often transfer familiar procedures to non-isomorphic tasks based on look-alike cues (Crespo & Sinclair, 2008; Silver & Cai, 1996).

Imitation vs structure

In typical exercises, the greater part of solutions were produced as imitations of familiar steps; in guided transformations, identical imitations gave mistaken applied procedures because of varying structural preconditions (e.g., manipulating a 3D extension under assumption of still-prevailing Pythagorean conditions. It's textbook pseudo-analytical behavior procedures prior to justification (and potentially never) and it takes place precisely where tasks are structurally variant although superficially identical (Vinner, 1997).

How do these processes differ by task type after introducing the guided problem-posing scheme?

Meaningful relations after guidance

46% of students (≈ 7/15, 46.7%) produced at least one meaningful relation that coherently linked conditions and representations (examples below). When these succeeded, students explicitly mapped invariants across registers (e.g., algebraic factorization matching a geometric decomposition), evidencing conversion rather than mere within-register treatments (Duval, 2006).

- *"I discovered that the observation (2) in explorations 1, 2, and 3 are the same utilizing the formula, a(bc) + b(ac) = c(2ab). Thus, a(bc) + b(ac) = c(2ab) applies to any triangle and any measurement of its sides, regardless of variable sequence."*
- *"The equation will satisfy if the lengths are Pythagorean triples and the width is a multiple of the length." Logarithm proves the Pythagorean Theorem in the ff. assumptions: (1) The interval is 2: a,b,c are odd or even; (2) k = I(1);*
- *It holds when a,b,c are successive numbers."*
- *"For any real numbers a,b,c such that a,b,c are odd or even or all even which are consecutive, then interval of 2,3,4,...,k=I(1),k=I(2),k=I (3),...,then a2 +b2 = c2."*

Where conversions failed

Many "generalizations" rested on single-example confirmations or identities misread as universally valid (e.g., treating ad-hoc numeric choices as proofs). These exhibit generic-example overreach grounded in concept image rather than definition (Vinner, 1997). Failures clustered when students attempted cross-register moves *(diagram ↔ algebra; 2D ↔ 3D)* without specifying invariants, a well-documented difficulty in conversion (Ainsworth, 2006; Duval, 2006).

Conclusion

This research investigated the nature of pseudo-conceptual and pseudo-analytical behaviors in secondary mathematics preservice teachers, providing empirical evidence of structural deficits in their problem-posing and problem-solving approaches. The analysis, triangulated across baseline problem-posing, routine procedural tasks, guided transformation activities, and interview data, demonstrates a consistent reliance on procedural imitation over structural reasoning. Quantitatively, a substantial portion of initial prompts were ill-posed or non-mathematical, often relying on inferred or incompatible quantities.

Furthermore, error rates significantly increased in tasks requiring the explicit exploration of invariants and variations (guided transformations) and across representational conversions (e.g., diagram-to-equation or 2D-to-3D). The correlation between task success and the explicit recognition and articulation of invariants and preconditions highlights the central role of structural awareness in moving beyond superficial application. While the finding that 46% of participants produced at least one meaningful relation demonstrating coherent conversion across registers offers a critical counterpoint, the collective evidence underscores a systemic tendency toward a surface-level, procedure-dominant approach that is fundamentally limited by a lack of attention to underlying mathematical structure and boundary conditions.

Despite these robust findings, the scope of this study is subject to several methodological constraints that temper claims of statistical generalizability. The research employed a small, single-site sample (n=15) selected via criterion-based purposive sampling within a course-embedded context focused specifically on the Pythagorean Theorem. Consequently, the results are oriented toward analytic generalization providing detailed, context-rich insights into specific behavioral mechanisms rather than extrapolation to broader populations. Furthermore, the reliance on student artifacts and semi-structured interviews as primary data sources, coupled with the dual role of the instructor as researcher, introduces potential setting-specific influences. These factors limit the capacity to claim broader population representation; however, they concurrently enable a fine-grained, detailed analysis of preservice teachers' procedural imitation and representational conversion struggles, enriching the qualitative understanding of this critical developmental phase.

The implications of these results are significant for both mathematics teacher education and the design of future pedagogical research. For practitioners, this study necessitates a redesign of curricula to explicitly integrate checks for invariants and preconditions within problem-posing rubrics and to teach representational conversion moves as distinct, critical learning objectives. The strategic design of paired task sets that contrast isomorphic and non-isomorphic structures is recommended as a means to actively mitigate the observed reliance on superficial imitation. To scaffold structural reasoning, instructional strategies should incorporate the elicitation of brief think-alouds and error analyses. Extending these findings, future research is strongly recommended to: (1) conduct multi-campus or comparative studies to test the robustness of the current findings across varied contexts; (2) investigate longitudinal transfer of structural awareness into actual practicum teaching settings; and (3) examine the role of instructional language and digital tools (such as dynamic geometry software) in mediating successful conversions and structural argumentation. Collectively, these recommended pathways aim to enhance the methodological rigor of the field and to significantly improve preservice teachers' capacity for structurally grounded problem-posing, representational fluency, and critical mathematical reasoning.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been covered completely by the authors.

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