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Integrating Ethno-Realistic Mathematics Education in developing three-dimensional instructional module

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Abstract

The integration of cultural and realistic contexts in mathematics learning has shown significant potential in fostering critical thinking skills; however, existing educational resources often lack a comprehensive approach that effectively combines these aspects. Addressing this gap, this research aimed to develop a novel Ethno-Realistic Mathematics Education (Ethno-RME)-based three-dimensional learning module that meets the criteria of validity, practicality, and has a positive impact on students' critical thinking skills. Utilizing a Design Research methodology, specifically a development study, the research was conducted in two stages: Preliminary Evaluation and Formative Evaluation, involving 12th-grade students from SMAN 5 Purworejo as participants. Data collection included product quality assessment sheets evaluated by material and media experts, student response sheets, and critical thinking test instruments. The results demonstrated that the module achieved high validity, with content rated "Good" (average score: 141 out of 175) and media rated "Very Good" (average score: 131 out of 135). Practicality was confirmed through student responses, with small group and field test average scores classified as "Very Good" (72.33 and 72.36, respectively). Furthermore, post-test results for critical thinking skills revealed an average score of 75.51, indicating the module's potential to enhance students' critical thinking. These findings establish the Ethno-RME-based module as a valid, practical, and impactful teaching resource, offering a culturally relevant and application-oriented alternative for mathematics education.

Keywords: critical thinking skills, design research, development, Ethno-RME, three-dimensional instructional module

Introduction

The challenges encountered in the mathematics learning process necessitate students' intellectual readiness and cognitive capabilities (Niss & Højgaard, 2019). A pivotal element in

mathematics education is the development of critical thinking skills (Basri et al., 2019). Consequently, integrating mathematics education with real-life contexts is essential to address the demands of intellectual readiness, cognitive abilities, and critical thinking development among students. The emphasis should be placed on fostering mathematical skills rather than focusing solely on the memorization of formulas (Ramdani et al., 2021; Schoenfeld, 2020).

In this regard, critical thinking skills are fundamental for a comprehensive approach to mathematics education (Arisoy & Aybek, 2021; Lestari et al., 2021). These skills encompass mental processes such as analysis, reasoning, evaluation, investigation, and decision-making (Alsaleh, 2020; Mahanal et al., 2019). They play a crucial role in enabling individuals to seek, analyze, and critically evaluate information, as well as to make informed decisions grounded in factual evidence (Dwyer et al., 2014; Jaelani et al., 2023). Therefore, mathematics education extends beyond the rote memorization of formulas, emphasizing the construction of knowledge through active engagement and the cultivation of critical thinking abilities via cognitive processes.

Despite its importance, many schools have yet to fully incorporate the development of critical thinking skills into routine learning activities (Bezanilla et al., 2019). Teaching practices often emphasize the memorization of facts, neglecting the cultivation of critical thinking abilities. This limitation frequently results in subpar critical thinking skills among students, ultimately diminishing their analytical and problem-solving capacities (Peter, 2012; Ristanto et al., 2022). Therefore, fostering a meaningful learning environment is imperative, which includes providing adequate learning resources and engaging instructional materials.

Instructional materials, as essential tools in the learning process, must be designed in accordance with established learning principles (Ovcharuk et al., 2020). This entails ensuring alignment with the curriculum content, tailoring materials to students' needs, integrating evaluation mechanisms, and fostering active student engagement (Hailikari et al., 2022; Loughlin et al., 2021). An effective strategy to enhance learning outcomes involves the use of modules—customized teaching resources designed to address specific learning objectives within particular subject areas (Bragg et al., 2021; Pratamadita & Dwiningsih, 2022). Thus, instructional materials should be developed in alignment with pedagogical principles and student-centered approaches, integrating content, evaluation, and engagement features, with further refinement through the creation of subject-specific modules.

Modules are not merely instructional packages prepared and compiled by educators; they also include various activity sheets and practice exercises designed to enhance student engagement (Pitriani & Pratama, 2021). The selection of appropriate learning modules plays a crucial role in promoting students' independent learning. These modules support teachers in managing classroom activities, improving students' learning experiences, fostering creativity, and encouraging critical thinking development (Susandi & Widyawati, 2022).

In response to these challenges, the Ethno-Realistic Mathematics Education (Ethno-RME) approach has been proposed as a potential solution. This approach emphasizes the practical application of mathematics in real-life contexts while incorporating local cultural elements to boost students' interest and motivation in learning (Prahmana, 2022; Prahmana et

al., 2023). Despite its promise, students often struggle to connect mathematical concepts to real-life scenarios, particularly in complex topics such as three-dimensional geometry (Alghadari et al., 2020; İbili et al., 2020; Prahmana et al., 2020).

The study of three-dimensional geometry is inherently connected to daily life and cultural elements (Owens, 2014). A notable example of a cultural artifact that can enhance three-dimensional geometry learning is the Sunan Geseng Mosque in Loano District. The mosque's interior and exterior designs serve as excellent models for teaching three-dimensional geometry concepts.

Discussions with mathematics teachers at SMA Negeri 5 Purworejo highlighted several challenges in teaching three-dimensional geometry. Students exhibit notably poor learning outcomes in Basic Competence 4.1, which involves determining and analyzing spatial distances, such as the distance between points, between a point and a line, and between a point and a plane. Moreover, the available teaching materials are predominantly limited to textbooks and Student Worksheets (SW), offering little variation in instructional methods. This lack of diversity in teaching approaches results in minimal student engagement, with students providing limited open feedback and facing difficulties in applying mathematical concepts to real-life situations. Additionally, teachers emphasized the critical need for teaching materials that actively engage students and promote independent learning during the instructional process.

In the field of mathematics education, three-dimensional geometry is consistently ranked as one of the most challenging topics for 12th-grade students. This is reflected in the notably low average scores in this subject at SMA Negeri 5 Purworejo, as illustrated in Figure 1.

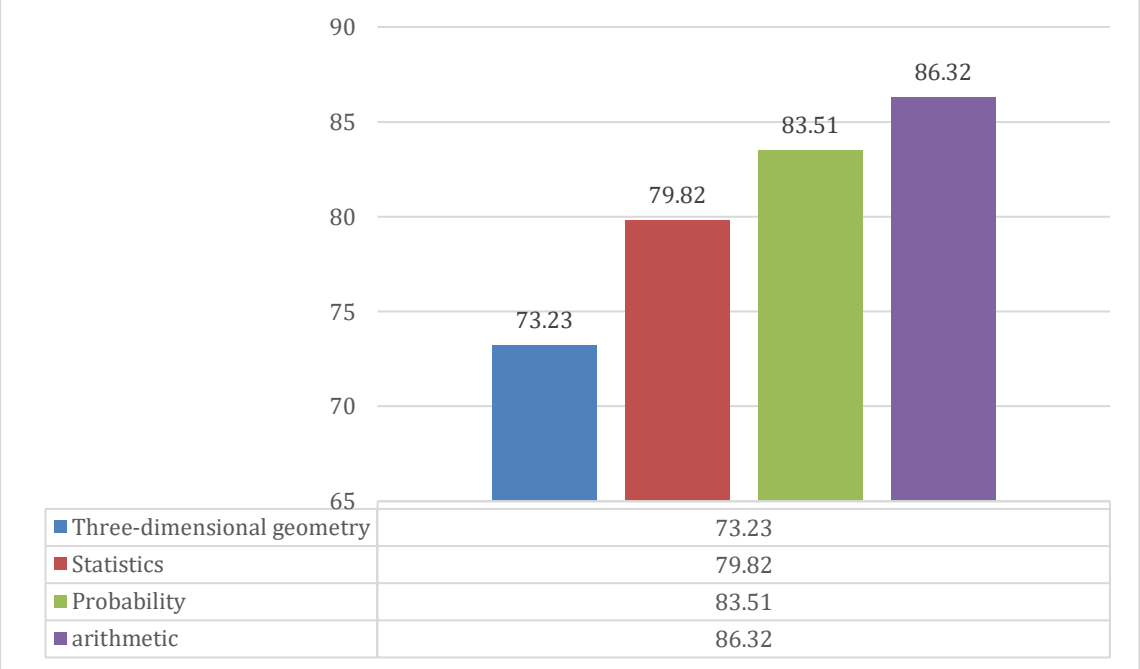

Figure 1. Average subject grades chart in SMA Negeri 5 Purworejo

Figure 1 reveals a significant disparity in students' performance across various mathematical topics at SMA Negeri 5 Purworejo. Among the topics assessed, three-dimensional geometry has the lowest average score of 73.23, compared to statistics (79.82), probability (83.51), and arithmetic (86.32). This indicates that students face greater challenges in mastering three-dimensional geometry, likely due to its abstract nature and the spatial reasoning required to comprehend its concepts. Studies have shown that difficulties in visualizing three-dimensional objects often hinder students' ability to understand and solve related problems effectively (Van Garderen & Montague, 2003). Furthermore, traditional teaching methods that focus primarily on rote learning may not sufficiently address the unique demands of this topic, thereby contributing to the lower performance. This aligns with findings by Battista (1990), who highlights the importance of developing spatial visualization skills to enhance geometry understanding.

To address this issue, integrating interactive learning tools, such as 3D visualization software like GeoGebra, can play a crucial role in enhancing students' comprehension of spatial relationships. Research by İbili et al. (2020) supports the idea that such tools significantly improve students' spatial reasoning and learning outcomes in geometry education. Additionally, the development of tailored learning modules and active learning strategies, such as problem-based learning (PBL), can increase student engagement and mastery of complex concepts. Oladayo and Diri (2024) emphasize that effective geometry instruction must focus on fostering spatial reasoning and using practical, hands-on approaches to teaching abstract topics. By implementing these strategies, educators can potentially bridge the performance gap in three-dimensional geometry and improve overall mathematical achievement.

To address these gaps, this research aims to develop a three-dimensional geometry module based on the Ethno-RME approach, with a specific focus on enhancing students' critical thinking skills. While prior studies have emphasized the lack of teaching materials and the need for diverse instructional approaches, limited research has been conducted on developing Ethno-RME-based modules explicitly designed to improve critical thinking skills in the context of three-dimensional geometry. Therefore, this research seeks to fill this gap by uniquely integrating the Ethno-RME approach with module development, addressing students' challenges in linking mathematical concepts to real-life contexts while fostering critical thinking. The anticipated outcome of this study is to provide a significant contribution to the advancement of more effective, culturally contextualized, and skill-oriented mathematics teaching strategies.

Methods

This study employed a design research methodology, with the primary goal of producing a supplementary educational product in the form of a module. The research focused on developing a valid and practical Ethno-RME-based module and evaluating its potential impact on enhancing students' critical thinking skills. The practicality of the module was assessed by its ease of use, acceptance by users, feasibility for real-world implementation, and its ability to effectively support educators and students in achieving educational objectives without

introducing unnecessary complexities or requiring additional resources.

According to Plomp (2013), development research is a branch of design research aimed at creating research-based solutions to address complex educational problems. It also seeks to deepen the understanding of interventions, as well as the design and development processes, for practical application in educational settings. Thus, in this study, design research was employed as a methodological framework to develop instructional materials in the form of modules.

The development procedure in this research consisted of two primary stages: preliminary design and formative evaluation (Tessmer, 1993). The formative evaluation process was designed to systematically refine and improve the module, as illustrated in the design flow presented in Figure 2.

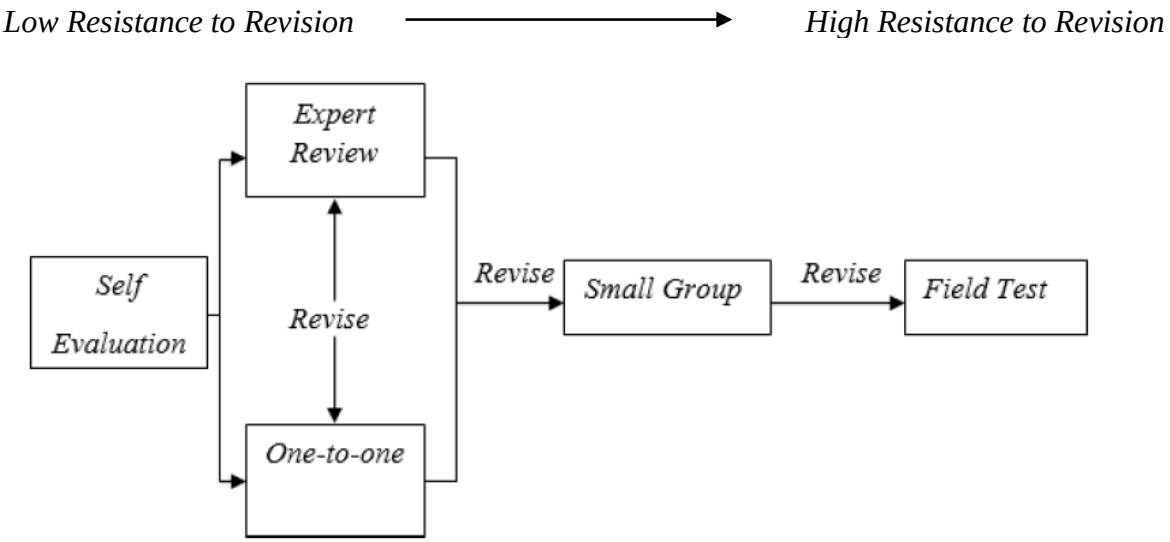

Figure 2. Formative evaluation design flow

The research was conducted during the 2023/2024 academic year and involved 12th-grade students from the MIPA 3 class at SMA Negeri 5 Purworejo as participants. The study employed multiple data collection stages to ensure the systematic development and evaluation of the Ethno-RME-based module.

In the preliminary evaluation stage, document analysis was conducted to guide the module's design. This involved integrating three-dimensional geometry content, defining learning outcomes and objectives, and developing quality assessment tools consistent with the independent curriculum and Ethno-RME principles (Prahmana, 2022; Prahmana et al., 2023).

The subsequent stages involved expert validation and iterative testing. Validation was performed through expert reviews and face-to-face sessions, which included interviews and post-tests. These activities were conducted during one-to-one interactions, small group sessions, and field tests, each designed to achieve specific research objectives. During the one-to-one phase, Prototype I of the module was evaluated for initial feedback. The small group stage focused on refining Prototype II based on collective insights. Finally, the field test aimed to assess the module's potential impact on students' critical thinking abilities.

Data analysis evaluated both the practicality and potential effectiveness of the module. Validation criteria were informed by feedback from content and media experts, analyzed using a Likert scale ranging from 1 to 5 (Tanujaya et al., 2022). The module's validity was determined by its alignment with the predetermined criteria, ensuring a thorough assessment through the post-test phase.

Results and Discussion

This study progressed through a series of structured development stages, including analysis, design, and prototyping, which encompassed self-evaluation, expert reviews, face-to-face sessions, small group evaluations, and field testing. These phases were meticulously implemented to refine and develop a three-dimensional geometry module grounded in the Ethno-RME framework. The module's primary objective is to enhance students' critical thinking skills by incorporating cultural values, creating a holistic approach to mathematics education. Through iterative refinement and rigorous testing in various educational settings, the research seeks to ensure the module's effectiveness in fostering critical thinking and conceptual understanding among students.

This study highlights the significance of methodological rigor in the development of educational modules, particularly within the Ethno-RME framework. By integrating diverse feedback from self-assessments, expert reviews, and practical trials, the research aims to optimize the design of the three-dimensional geometry module. This approach not only ensures alignment with educational standards but also embeds cultural and ethical principles, drawing from the local cultural context, exemplified by the Sunan Geseng Mosque in Loano. The iterative cycle of analysis, design modifications, and prototyping underscores the research's commitment to enhancing educational outcomes through innovative pedagogical strategies. By anchoring these strategies in local cultural values, this study contributes to the broader discourse on effective and culturally responsive practices in mathematics education.

Preliminary Stage

During this phase, rigorous analysis and design activities were conducted to ensure the development of a module that effectively addresses the identified educational challenges. A comprehensive literature review was undertaken alongside an analysis of student needs, curriculum content, and assessment requirements. These activities informed the creation of a module meticulously designed to integrate three-dimensional geometry content, Learning Outcomes (LOu), Learning Objectives (LOb), and quality assessment instruments. The module adheres to the curriculum while incorporating the principles of Ethnomathematics Realistic Education (Ethno-RME), ensuring alignment with cultural and contextual considerations. This systematic approach enhances the module's relevance and effectiveness in supporting student learning and development.

The analysis revealed several critical needs within the educational context:

1. Student Challenges in Understanding: Students often struggle to comprehend and analyze three-dimensional material, demonstrating difficulties in applying critical and logical

thinking to these concepts.
2. Lack of Contextual Approaches: Current teaching practices lack integration with local cultural perspectives, which could make learning more relatable and meaningful.
3. Critical Thinking Deficiencies: A significant gap exists in fostering students' critical thinking skills, identified as a priority for improvement.
4. Inadequate Teaching Materials: Existing resources are insufficient for enhancing students' critical thinking abilities, highlighting a need for more effective instructional tools.
5. Syllabus-Centric Teaching: Teachers often prioritize rapid syllabus completion using dense, monotonous textbooks, which diminishes student engagement and learning effectiveness.

To address these issues, researchers designed targeted interventions culminating in the development of a three-dimensional geometry module grounded in the Ethno-RME framework. The module aims to enhance critical thinking skills by integrating cultural perspectives and contextualized learning experiences. This strategic approach fosters a deeper understanding of three-dimensional geometry while addressing the critical educational needs identified. By bridging gaps in current teaching practices and resources, the module provides a robust solution to improve mathematics education quality and support meaningful student learning.

Formative Evaluation Stage

In this phase, researchers implement a structured sequence of steps to ensure the systematic refinement and validation of the module. The formative evaluation phase comprises the following critical steps:

1. Self-Evaluation: Researchers conduct an initial review of the module to identify potential weaknesses and inconsistencies. This involves a thorough examination of content accuracy, alignment with learning outcomes, and adherence to the Ethno-RME framework. Preliminary adjustments are made to improve the module’s structure and relevance.
2. Expert Review: Subject matter experts and educational practitioners evaluate the module's content, design, and alignment with curriculum objectives. Feedback from this stage focuses on ensuring the module’s validity, cultural relevance, and potential for fostering critical thinking. Suggested improvements are incorporated to enhance the module's quality.
3. One-on-One Evaluation: Individual students are engaged to evaluate Prototype I of the module. This phase allows researchers to observe how students interact with the material, identify usability issues, and gather feedback on clarity, engagement, and effectiveness. Findings from this step guide the refinement of the module into Prototype II.
4. Small Group Evaluation: A small group of students participates in testing Prototype II. This phase provides insights into group dynamics, collaborative learning outcomes, and areas where the module may require further refinement to enhance its practicality and effectiveness in real-world classroom settings.

5. Field Testing: The final version of the module is implemented in a broader classroom setting to evaluate its potential impact on students' critical thinking skills and conceptual understanding. This step assesses the module's practicality and effectiveness in achieving the intended learning outcomes within a real educational environment.

By systematically progressing through these phases, researchers ensure that the module is rigorously tested and refined, resulting in a product that is both effective and practical for enhancing students' learning experiences and critical thinking abilities.

Self-Evaluation

The development of the Ethno-RME-based three-dimensional geometry module involves a comprehensive process of design, evaluation, and refinement. Researchers meticulously review the module for content accuracy, structural coherence, and linguistic clarity, ensuring alignment with educational objectives and cultural relevance. This phase serves to identify and correct typographical errors, improve word choices, and address any conceptual deficiencies that may have emerged during the module's creation.

The evaluation process is structured into sequential stages, beginning with self-assessment, where researchers conduct an initial review of the module. This is followed by expert review, where subject matter experts assess the module's validity, relevance, and adherence to educational standards. Subsequently, one-on-one evaluation is conducted with individual students to gather feedback on usability and clarity, leading to further refinements. In the small group evaluation, a group of students tests the module in a collaborative setting, providing insights into group dynamics and its practical applicability. Finally, field testing is carried out in a real classroom environment to evaluate the module's effectiveness in enhancing students' critical thinking skills and conceptual understanding.

Each stage in the formative evaluation process contributes to the iterative refinement of the module. Feedback from each phase is systematically analyzed and incorporated, ensuring the module's educational value and instructional design are optimized. This meticulous approach not only improves the module's potential impact but also highlights the researchers' commitment to integrating cultural perspectives and fostering critical thinking within the Ethno-RME framework. Through this process, the module is designed to effectively support meaningful and culturally contextualized mathematics education.

Expert Reviews

During this phase, an Expert Review was conducted to evaluate the validity of Prototype I against predetermined criteria. The validation process involved both face-to-face meetings and email correspondence. Specifically, a face-to-face validation session took place on Tuesday, February 27, 2024, with Expert 1, a Mathematics Education lecturer from Ahmad Dahlan University. The review panel consisted of two subject matter experts, both university lecturers with doctoral qualifications from different universities, and one instrument expert, also a doctoral degree holder, who validated the developed product.

Feedback and suggestions were systematically collected using a product quality assessment sheet. Inputs were provided by subject matter experts, media experts, students, and

teachers, as well as through a critical thinking assessment instrument. These insights were used to evaluate the module's content accuracy, pedagogical alignment, and design quality.

The findings from the evaluation highlighted the need for revisions to the assessment sheet. After incorporating the suggested modifications, the assessment sheet and the module were deemed "suitable for use," ensuring they met the required educational standards and objectives. This validation process underscores the module's alignment with quality criteria and its potential for effective implementation in enhancing students' critical thinking skills.

The validation process of Prototype I of the module was carried out following revisions to the product quality assessment sheet. This phase involved evaluations by two subject matter experts specializing in Ethno-RME (Expert 1) and mathematics (Expert 2), as well as by a media expert. The results of the evaluation by subject matter experts are presented in Table 1.

Table 1. Calculation results from material experts

No	Validator	Score	Descriptive Interpretation
1	Material Expert 1	141	Good
2	Material Expert 2	141	Good
	Total Score	282	
	Average Score	141	Good

The average score of 141 out of 175 from both validators indicates that the Ethno-RME-based module demonstrates good adherence to the evaluation criteria and is considered valid. Detailed comments and suggestions for further refinement are provided in Table 2.

Table 2. Comments and suggestions from subject matter experts

No	Comments and Suggestions
1	Re-examine operational verbs in indicators and align them with Bloom's Taxonomy and critical thinking stages.
2	The material depth is appropriate for high school level competencies.
3	Definitions and concepts are accurate.
4	Data and facts presented are accurate.
5	Examples are relevant to three-dimensional material but require greater emphasis on Ethnomathematics concepts.
6	Clarify terminology in diagrams (e.g., the term ABCD.EFGH on page 6) to improve comprehension.
7	Add a glossary to define terms for better student understanding.
8	Illustrations align with content but need clearer adaptation to the Ethnomathematics context.
9	Images utilize mosque contexts effectively.
10	Improve the definition of lines using Euclidean geometry references.
11	Ensure logical progression in drawing material conclusions.
12	Replace images with those more familiar to students' daily lives.
13	Simplify sentences to enhance problem comprehension.

These insights are instrumental in refining the module's content, making it more accessible and aligned with Ethno-RME principles. Furthermore, the media expert evaluation scores are detailed in Table 3.

Table 3. Media Expert Evaluation Scores

No	Validator	Score	Criteria
1	Expert 3	131	Very Good
	Total Score	131	Very Good

With a score of 131 out of 135, the module's visual and media components received a "Very Good" rating. It means that the RME-based module developed is considered valid. Furthermore, based on feedback and recommendations from expert reviews, significant revisions were implemented to improve various components of the module. In particular, the introduction of Prototype I was strategically revised in response to expert suggestions, focusing on enhancing layout and visual presentation. Comments and suggestions from media experts are listed in Table 4.

Table 4. Media expert comments and suggestions

No	Comments and Suggestions
1	The back cover design is missing.
2	Consider adding images of study objects (e.g., a cube) in the empty photo section to clarify its role as a mathematics module, not a history book.
3	The placement of elements is very good; no parts are cut off or hanging.
4	The font color in the Table of Contents lacks sufficient contrast with the black background, making it unclear.
5	Adjust the size of the girl in the image to ensure proportionality with the boy, and consider using a different color for the "Example" text instead of red.
6	Ensure consistency in page design throughout the module.

The revisions addressed these recommendations to enhance the module's professional presentation and visual clarity:

1. Back Cover Design: A back cover was added, incorporating thematic elements that align with the Ethno-RME framework.
2. Use of Visual Aids: Study object images, such as geometric shapes, were included in appropriate sections to reinforce the module's mathematical focus.
3. Improved Contrast: The font color in the Table of Contents was adjusted to ensure better visibility and readability.
4. Proportionality in Illustrations: Adjustments were made to the size of figures in images, ensuring proportional representation and aesthetic balance.
5. Consistency in Design: Page layouts were standardized, creating a uniform and cohesive design throughout the module.

In addition to these visual and structural enhancements, further refinements were made to align the module's content with educational standards. For instance, operational verbs in the learning indicators were revised in line with Bloom's Taxonomy and critical thinking stages. This ensures that the learning objectives are clear, measurable, and effectively aligned with the goal of fostering critical thinking skills.

These improvements reflect a commitment to integrating expert feedback and paying meticulous attention to detail. The enhanced visual presentation, structural coherence, and alignment with pedagogical principles ensure that the module is not only more engaging but also more effective in supporting its educational objectives. By optimizing the module's clarity and professional quality, these refinements strengthen its utility in an educational context, further supporting the principles of Ethno-RME.

Ultimately, the revised module is better equipped to create a positive first impression, facilitate clear identification of key elements, and align with cultural and contextual learning goals while fostering students' critical thinking abilities. It is also stated that the validation process highlighted that the Ethno-RME-based module is valid and suitable for use, with both material and media experts providing valuable recommendations for further improvement. These suggestions aim to enhance the module's educational effectiveness, cultural relevance, and alignment with Ethno-RME principles, ensuring it supports the development of students' critical thinking skills and understanding of three-dimensional geometry.

After a thorough review by experts, significant enhancements were made to the module, particularly addressing the depiction of the rectangular prism-shaped open-top ablution pool. The researchers revised the illustrations to ensure greater alignment with the overall visual appeal of the module's pages. In addition, consistency was established in the representation of student imagery throughout the module to maintain a cohesive design.

Furthermore, the 'questions' section, which initially lacked integration with the ethnomathematical context, was revised to incorporate elements of ethnomathematics, specifically contextualizing the questions within the framework of a mosque. This modification, emphasized within the updated presentation, marks a notable improvement in the contextual relevance and coherence of the module's content.

These changes reflect a commitment to addressing feedback aimed at improving the clarity, readability, and consistency of the teaching materials. The addition of a glossary further enhances the module's comprehensibility by clarifying previously challenging terms. These refinements underscore the dedication to fostering an optimal user experience and increasing the module's effectiveness, particularly in promoting Ethno-RME and boosting student engagement.

Moreover, revisions to the visual elements were implemented to improve aesthetic cohesion and adherence to educational standards. By adapting imagery to a building-related context, the module achieves a more unified visual identity while enhancing readability and thematic consistency. These updates further reinforce the module's objective of aligning with the principles of Ethno-RME and ensuring a more engaging learning experience for students.

Lastly, updates to the cover design, incorporating a building-based theme, contribute to the module's overall aesthetic coherence and professional presentation. This adjustment highlights the commitment to delivering educational materials that align with academic standards while fostering improved student interaction and comprehension. These enhancements collectively emphasize the module's role in advancing the principles of Ethno-RME and creating a more effective learning tool.

One-to-One

At this stage, Prototype II was provided to three students from class XII MIPA 3 at SMA Negeri 5 Purworejo, representing different cognitive abilities. These students, identified as S, A, and N, had previously participated in the testing of Prototype I. The primary objective of this phase was to identify challenges and gather student responses regarding the module's usability. Feedback and suggestions from these students, summarized in Table 5, played a crucial role in further refining the module to enhance its clarity and engagement. These adjustments aimed to better align the module with the diverse cognitive abilities of students within the framework of Ethno-RME.

Table 5. Student feedback and suggestions

No	Student Comments and Suggestions
1	The module is very easy to learn.
2	Good, there is a column for exercises.
3	The cover is too crowded.

During this phase, student feedback primarily emphasized the need for more visually appealing illustrations. However, the findings from this stage did not produce significant breakthroughs, prompting the researchers to advance to the next phase, which involved small group testing sessions. These subsequent sessions aimed to gather broader insights to ensure the module’s design and content were both engaging and effective in addressing diverse learning needs.

Small Group

The module's refinement progressed to small group testing after integrating feedback from one-on-one sessions and expert reviews. This phase involved six students selected based on their cognitive diversity to assess the module's readability and gather additional feedback prior to field testing. During this stage, the students exhibited a strong understanding of the module's instructions and successfully completed the activities, demonstrating its practicality and effectiveness.

To further evaluate the module, the researchers distributed a student response questionnaire. The results, as illustrated in Figure 3, revealed an average score of 72.33, categorized as "Very Good." This outcome reflects the module's high level of practicality and positive reception among students. These findings affirm the module's alignment with the principles of Ethno-RME, emphasizing its role in enhancing the learning experience and promoting student engagement.

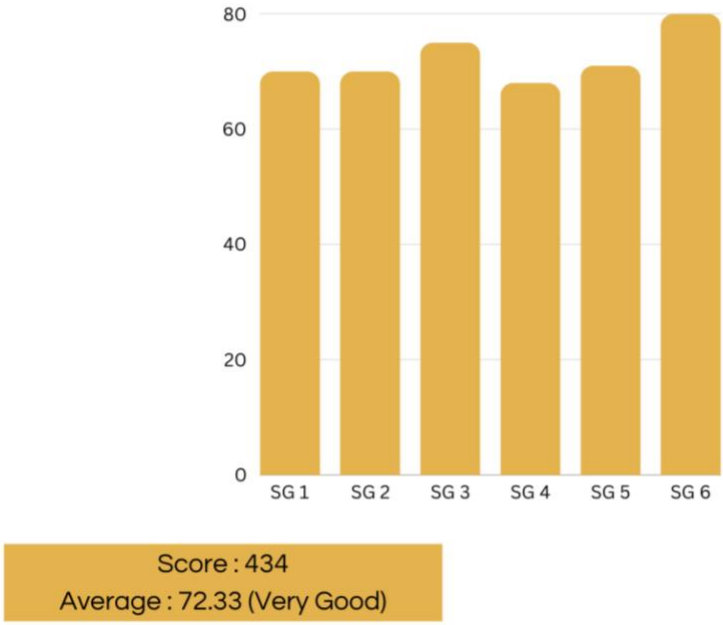

Figure 3. Results of small group student questionnaire responses

The feedback and suggestions gathered during the small group testing phase were utilized to refine Prototype II before advancing to the Field Test phase. Student comments and suggestions are summarized in Table 6.

Table 6. Comments and suggestions from small group

No	Comments and Suggestions
1	The module is good, ma'am, with many questions about everyday life.
2	The module's presentation is attractive, making me more interested in studying.
3	The module allows me to learn independently.

The feedback from students indicated that no major revisions were necessary for the module. Following enhancements made during the one-on-one and small group phases, Prototype II was further refined, culminating in the development of Prototype III.

Prototype III was subjected to validity and practicality testing. Quantitative evaluations from questionnaires rated the material validity as "Good" and media validity as "Very Good." Similarly, the module's practicality was rated as "Very Good." These results affirmed the quality and effectiveness of Prototype III, which was subsequently tested in the Field Test phase to ensure its suitability for broader implementation.

Field Tests

At this stage, Prototype III was tested on 28 students from class XII MIPA 3 at SMA Negeri 5 Purworejo, who served as the research subjects. The testing process was divided into two phases. In the first phase, the researchers observed how teachers implemented the module in their teaching practices. During this phase, the researcher acted as an observer, assisting the teacher in clarifying sentences or concepts in the module that students found challenging.

Following the teaching phase, a student survey was conducted to evaluate the practicality of the module and gather suggestions for further improvements. The findings from this survey are presented in Figure 4.

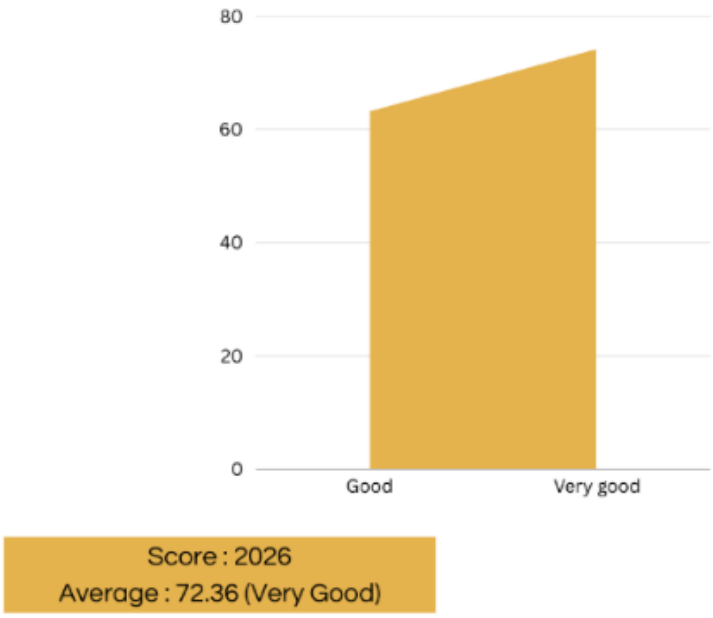

Figure 4. Results of the student response survey field test

The results indicate that the practicality of the module during its use in the field test was rated as "Very Good". Based on their experience with the Ethno-RME-based Three-Dimensional module, students provided comments and suggestions, summarized in Table 7.

Table 7. Student responses to the module in field tests

No	Comments and Suggestions
1	Using this module makes me understand better.
2	Because it uses local culture, I find it more enjoyable.
3	The lessons with this module are very enjoyable, ma'am.
4	The sentences and paragraphs in this module are clear and easy to understand.
5	The module's appearance is very good, but it needs more pictures.

The feedback emphasized positive aspects such as the module's clarity, cultural relevance, and engaging presentation. However, some students suggested adding more illustrations to enhance its visual appeal. Overall, the responses reflect strong acceptance of the module, affirming its effectiveness in supporting learning objectives consistent with the principles of Ethno-RME.

During the third meeting, a test was administered with two questions targeting six critical thinking ability indicators. The results of this test are depicted in Figure 5, with the following percentage achievements for each indicator, such as Focus: 78.57%, Reason: 83.04%, Inference: 84.82%, Situation: 75.60%, Clarity: 88.39%, and Overview: 87.50%.

The results indicate that the Ethno-RME-based module positively influenced students' critical thinking abilities. By incorporating real-life problems rooted in local cultural contexts, the module enabled students to better understand three-dimensional concepts, thereby fostering critical reasoning skills and increasing engagement in the learning process. These findings confirm the module's potential to effectively support learning outcomes and demonstrate its alignment with Ethno-RME principles. The positive reception and measurable improvements in critical thinking validate the module as a valuable tool for enhancing educational quality.

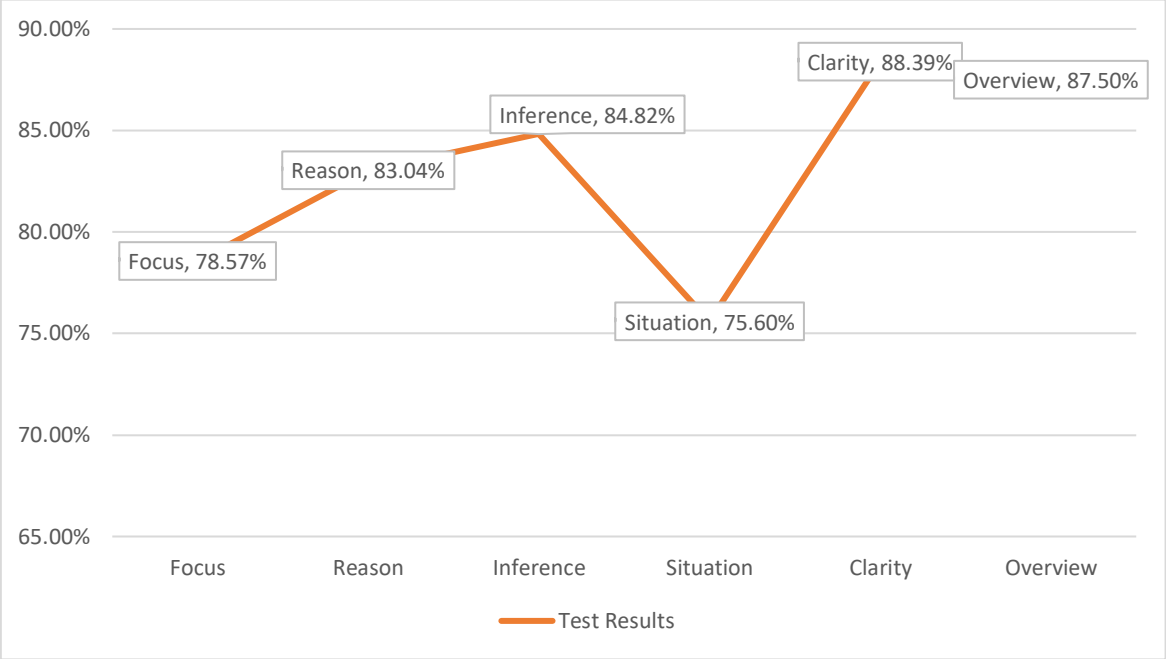

Figure 5. Results of the student critical reasoning ability test

The positive potential impact of the Ethno-RME approach module on students' critical thinking skills is clear, as students found it easier to understand three-dimensional concepts because the learning involved everyday problems related to local culture, especially in three-dimensional material. The research results indicate that the developed module meets the criteria for module feasibility, both in terms of content and media. Language, content, presentation, and graphics suitability are requirements that must be met during module development. The lack of practice and students' unfamiliarity with critical thinking-based learning processes led to weak critical thinking skills (Lestari et al., 2023). Ethnomathematics emerges as an engaging method to enhance students' mathematical understanding (Rosa & Orey, 2021). The developed module, with its Ethno-RME approach, successfully meets the requirements for use by students to improve their critical thinking skills.

The module development process included two stages: the initial evaluation stage and the formative evaluation stage. Based on the evaluation results from each phase of developing the Ethno-RME-based three-dimensional material module, the module was found to be valid, practical, and potentially influential on students' critical thinking skills at SMA Negeri 5 Purworejo, class XII MIPA 3. Curriculum analysis, material, and student characteristics were conducted in the first stage, the initial evaluation stage, to determine learning needs.

Mathematics learning requires specific skills to understand, study, and solve problems (Al-Mutawah et al., 2019).

The process included conducting open interviews with mathematics teachers and the curriculum deputy head at SMA Negeri 5 Purworejo. The interviews revealed that the school uses the 2013 curriculum and students need a module that connects with everyday life and contains cultural values, as SMA Negeri 5 Purworejo is a Cultural Support School in the Loano area. This served as the basis for continuing the research to the design phase of the learning device, referred to as Prototype I.

The second stage is the formative evaluation stage. Validation instruments included validation of material expert assessment sheets, media expert assessment sheets, student response sheets, and post-test validation sheets, which were used to assess the potential impact of the module on students' critical thinking skills. After validating and revising the instruments, Prototype I underwent expert review and one-to-one testing. The results showed that the developed module was considered valid in terms of content and media. In the expert review phase, material experts rated it as "Good," while media experts rated it as "Very Good." In the one-to-one phase, it was rated as "Very Good."

Subsequent revisions based on feedback and suggestions led to Prototype II. After Prototype II was deemed valid, it proceeded to the small group phase to test its practicality. In the small group phase, class XII MIPA 2 was used with 6 students of varying cognitive abilities (high, medium, and low). A student response questionnaire was used to assess the practicality criteria after students in the small group understood the material and instructions provided in the module steps. Feedback and suggestions from the small group were used to revise Prototype II into Prototype III, which was valid and practical with a "Very Good" rating.

Prototype III was tested in the field test phase after being validated and deemed practical. The field test phase was divided into two parts: first, the teaching and learning activities using the Ethno-RME approach module, and second, the administration of tests to assess the potential impact of the module on students' critical thinking skills. The classroom learning process during the field test phase was carried out using the Ethno-RME approach, which included:

1. Determining the Ethnomathematics Context: Students were given contextual problems related to the Al Imam Loano Mosque. They understood the problems and then expressed their knowledge by writing down the information they knew and the questions posed in the problems.
2. Exploring and Processing Information in the Ethnomathematics Context: Students were asked to explore the information they obtained and then process that information.
3. Finding Mathematics in the Ethnomathematics Context: Students were asked to create mathematical models to solve the given problems.
4. Applying the Self-Development Model: Students were asked to use the model or ideas they had created to find solutions to the problems based on the provided questions.
5. Conducting Critical Reflection as Assessment: This means students were involved in explaining, correcting, discussing, and asking questions to the teacher and other students about the given problems.

The integration of Realistic Mathematics Education (RME) with cultural elements has been recognized as an effective approach to enhancing students' mathematical abilities, particularly critical thinking skills. Research indicates that cultural elements play a vital role in shaping positive attitudes toward mathematics learning by making the material more engaging and relatable. For instance, studies by Palunisa (2013) and Nashrullah et al. (2023) demonstrate the effectiveness of this approach, while Hubert (2014), Bishop (1988), and Orozco-Guzmán et al. (2020) emphasize how cultural integration fosters a more enjoyable and meaningful learning experience.

In the second phase of the field test, a post-test was administered to 28 students from class XII MIPA 3 to assess the potential impact of the Ethno-RME-based module on critical thinking skills. The test focused on six critical thinking indicators, revealing high levels of achievement across all categories. Specifically, the focus indicator reached 78.57%, reason 83.04%, inference 84.82%, situation 75.60%, clarity 88.39%, and overview 87.50%. These results highlight the module's positive effect on students' ability to engage with and understand three-dimensional material. The findings are consistent with the research of Arnellis et al. (2020), which concludes that the Ethno-RME approach effectively enhances mathematical critical thinking skills while benefiting students across varying cognitive levels, including high, medium, and low performers.

The Ethno-RME approach has garnered substantial support from prior research, highlighting its dual significance in education and cultural preservation. Nuraina et al. (2021) demonstrated that Ethno-RME enhances students' critical thinking skills while simultaneously serving as a medium to preserve cultural heritage through mathematics education. Similarly, Widada et al. (2019) reported that students who engaged with an Ethnomathematics-based RME model exhibited superior critical thinking abilities compared to those taught through conventional instructional methods.

Further advancements in this field include the development and validation of Ethno-RME-based e-modules, which have shown notable effectiveness. For instance, Alghiffari et al. (2024) reported the positive impact of such e-modules on eighth-grade students' problem-solving skills. Pujiastuti et al. (2025) explored the alignment of Ethno-RME with the Pancasila Student Profile and found significant improvements in students' critical reasoning skills. Additionally, Nurnaningsih et al. (2024) successfully integrated Ethno-RME into the MathCityMap application via the Mangkujo Math Trail, which effectively enhanced students' understanding of linear equation systems by connecting abstract mathematical concepts to real-world contexts.

In summary, the Ethno-RME approach offers a comprehensive framework for fostering critical thinking while embedding cultural elements into the learning process. By harmonizing cultural relevance with mathematical instruction, this approach not only bolsters cognitive development but also strengthens students' appreciation of their cultural heritage, establishing its value as a modern educational tool.

Conclusion

This study successfully developed a three-dimensional geometry module based on Ethno-RME aimed at enhancing the critical thinking skills of senior high school students. The results confirmed both the validity and practicality of the module in promoting critical thinking within

the context of three-dimensional geometry. Furthermore, the findings suggest that the module has the potential to significantly improve students' critical thinking abilities, making it a valuable tool for fostering higher-order thinking skills in mathematics education.

Despite its promising outcomes, this research has certain limitations. The study was conducted with a relatively small sample size, which may limit the generalizability of the findings to broader student populations. Additionally, the focus was restricted to three-dimensional geometry, leaving the potential application of the Ethno-RME approach to other mathematics topics unexplored. The cultural diversity of the participants was also limited, which could affect the adaptability of the module to varying cultural contexts. These constraints highlight the need for further refinement and broader testing of the module.

Future research should focus on expanding the sample size and implementing the module across diverse educational and cultural settings to enhance its applicability and robustness. Researchers are also encouraged to explore the integration of Ethno-RME principles into other areas of mathematics to broaden its impact on critical thinking skills. Moreover, collaboration between educators and researchers should be strengthened to ensure effective implementation and adaptation of the modules. Expanding the question types and diversifying the content within the modules will also be vital to address varied learning needs and further improve the educational outcomes. This study provides a solid foundation for advancing Ethno-RME-based mathematics education and paves the way for more comprehensive and inclusive educational practices.

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Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been covered completely by the authors.

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