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Epistemological obstacle on the topic of prism: a phenomenological study

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Abstract

Mathematics learning is often hindered by epistemological obstacles that affect students' conceptual understanding. In geometry, students frequently struggle to identify and analyze fundamental properties of prisms, despite instructional efforts to improve comprehension. Prior research has primarily focused on procedural fluency rather than the cognitive barriers students face in interpreting mathematical definitions. Addressing this gap, this study investigates the epistemological obstacles eighth-grade students encounter in understanding prisms. Specifically, it examines students' ability to determine whether a geometric figure qualifies as a prism based on definitional characteristics and to analyze the relationship between two figures with equal volumes. This qualitative study employs a phenomenological approach, involving six purposively selected eighth-grade students. Data were collected through written tests and semi-structured interviews, then analyzed in three stages: identifying core ideas from student responses, categorizing these ideas into conceptual groupings, and thematizing the categorized data into key discussion themes. Findings reveal that students struggle to identify prisms due to difficulties recognizing defining characteristics and determining bases, resulting from didactic transposition issues such as oversimplified definitions, misinterpretation of concepts, and curricular limitations. At the C4 level of Bloom's Taxonomy, students also struggle to analyze mathematical statements due to reliance on teacher-provided examples. This study contributes to mathematics education by highlighting cognitive barriers in geometric reasoning. The findings emphasize the need for instructional strategies that enhance conceptual clarity and adaptive problem-solving, ultimately fostering deeper geometric understanding.

Keywords: Bloom's Taxonomy, Epistemological Obstacle, Phenomenological Methodology, Prism

Introduction

Mathematical competence necessitates the development of conceptual knowledge, procedural knowledge, and procedural flexibility among students (Rittle-Johnson, 2017). Conceptual knowledge pertains to the comprehension of overarching abstract principles that serve as a foundation for an in-depth understanding of mathematical concepts (Rittle-Johnson, 2019). In contrast, procedural knowledge involves familiarity with the sequential steps or operations required to achieve a specific objective, which is progressively refined through experience in problem-solving. Both forms of knowledge contribute to the enhancement of procedural flexibility, which refers to the ability to apply diverse strategies adaptively in solving mathematical problems based on the given context (Rittle-Johnson et al., 2015).

A robust understanding of mathematics is a critical determinant of academic success (Rittle-Johnson, 2017). However, acquiring mathematical proficiency is often challenging due to various obstacles encountered in the learning process. These difficulties may stem from multiple inhibiting factors, encompassing both didactic and pedagogical dimensions (Yuliani, 2016). Common challenges include the misalignment of didactic situations with the intrinsic nature of mathematics and its instructional methodologies, discrepancies between the applied learning approaches and their underlying philosophical foundations, and conceptual mismatches between educators, students, and established scientific frameworks. Furthermore, difficulties in knowledge transposition—both from didactic and pedagogical perspectives along with suboptimal didactic design, significantly influence the effectiveness of mathematics instruction (Suryadi, 2019a).

A fundamental challenge in mathematics learning is the discrepancy between students' prior knowledge and the new concepts they are expected to acquire (Suryadi, 2019b). Suryadi (2019a) identified various learning obstacles encountered by students in mathematics education, categorized as ontogenic, didactic, and epistemological obstacles. Ontogenic obstacles pertain to students' cognitive readiness and mental maturity in assimilating new mathematical knowledge. These challenges arise when a misalignment exists between students' developmental readiness and the cognitive demands or complexity of the didactic environment in which they learn (Hendriyanto et al., 2024).

Beyond ontogenic constraints, didactic obstacles emerge from the instructional system, encompassing curriculum sequencing and the methods used to present mathematical concepts in the classroom (Fauzi & Suryadi, 2020). The structuring of instructional content, both in terms of conceptual relationships and the progression of cognitive processes, plays a crucial role in shaping students' learning experiences. Furthermore, the depth of content presentation whether excessively detailed or insufficiently comprehensive must be carefully evaluated to mitigate didactic obstacles (Brousseau, 1976).

In contrast, epistemological obstacles arise when students' understanding of a mathematical concept remains confined to specific contexts, limiting their ability to generalize and apply knowledge to unfamiliar situations. These obstacles become evident when students can correctly respond to problems presented in familiar formats, such as those modeled by instructors or textbooks, yet struggle to solve analogous problems in different representations or contexts. This limitation indicates a difficulty in transferring knowledge across diverse mathematical settings, ultimately hindering the development of higher-order mathematical thinking (Hendriyanto, 2024). Errors in problem-solving frequently serve as indicators of epistemological obstacles (Brousseau, 2002; Cornu, 1991), often resulting from poorly structured prior knowledge (Brousseau, 2002). Furthermore, research by Moru (2009) highlights that deficiencies in conceptual understanding represent a primary manifestation of epistemological obstacles.

Effectively addressing obstacles in mathematics learning necessitates a comprehensive understanding of students' prior knowledge and their cognitive models of mathematical concepts (Suryadi, 2019a). Educators must be able to identify specific learning difficulties encountered by each student and employ the most appropriate instructional strategies to facilitate overcoming these challenges (Yuen et al., 2003). Numerous studies have documented various learning barriers in mathematics, particularly within the domain of geometry.

Kandaga et al (2022) examined epistemological obstacles at different levels of geometric reasoning based on the van Hiele model. Similarly, Sunariah and Mulyana (2020) identified students' difficulties in recognizing contextual problems involving geometric transformations. Their study revealed a limited understanding of the properties of geometric transformations namely, rotation, dilation, translation, and reflection along with a tendency to misapply previously learned concepts. Furthermore, students predominantly relied on procedural knowledge rather than conceptual comprehension when solving problems related to geometric transformations.

Epistemological obstacles in understanding the volume of pyramids were explored by Mujahidah and Rosjanuardi (2024), who identified key difficulties such as inadequate comprehension of square root simplification, area calculations, and unit conversions. Moreover, students often memorized the pyramid volume formula without grasping its underlying conceptual basis, leading to misinterpretations. Safitri et al. (2023) further highlighted that such obstacles arise from contextual limitations, as students are not accustomed to integrating multiple mathematical concepts. Meanwhile, Angraini and Prahmana (2019) identified conceptual errors in geometry, including formula misapplication, misidentification of geometric properties, and difficulties in translating word problems into mathematical representations. Additionally, Aziiza and Juandi (2021) reported epistemological obstacles in students' understanding of prism surface area, noting that students frequently relied on rote memorization of definitions and formulas rather than developing a comprehensive conceptual understanding.

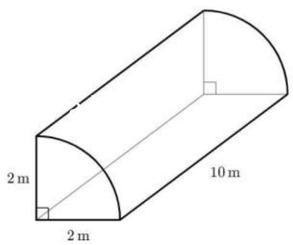
These findings underscore the significance of epistemological obstacles as a major challenge in geometry learning. Consequently, recognizing and addressing these learning barriers is essential for designing more effective instructional approaches. The present study aims to identify epistemological obstacles that students encounter in understanding the concept of prisms. Specifically, students are asked to determine whether a given geometric figure qualifies as a prism based on its defining characteristics. Additionally, they are required to analyze the relationships between two geometric figures with equivalent volumes. A key novelty of this study lies in its emphasis on students' analytical reasoning in applying previously taught prism concepts a focus that has been largely overlooked in prior research. This investigation is particularly relevant, as analytical reasoning skills, particularly those classified as C4 and above in Bloom's Taxonomy, are crucial for success in both contemporary and future educational contexts.

Methods

This study employs a qualitative research design using a phenomenological approach. Phenomenology is commonly defined as the study of phenomena as they manifest in human experience, focusing on how individuals interpret and comprehend these phenomena and the meanings they hold in subjective experiences (Neubauer et al., 2019). This approach aims to describe and interpret the essence of lived experiences, recognizing their significance within the fields of pedagogy, psychology, and sociology (Fuster Guillen, 2019). By analyzing participants' accounts and subjective perceptions of objects and events, phenomenological research provides an in-depth exploration of individuals' lived experiences. A distinguishing feature of phenomenological analysis is the active involvement of the researcher in the interpretive process (Tuffour, 2017). In this study, the phenomenological approach was employed to investigate epistemological obstacles encountered by students in learning mathematics. Specifically, the research analyzed students' errors in solving mathematical problems related to prisms and examined their learning experiences through teacher interviews.

The study involved six eighth-grade students from a junior high school, selected through purposive sampling based on teacher recommendations. The selection criteria prioritized students with strong communication skills, enabling them to articulate their mathematical reasoning effectively. Data collection was conducted using written assessments and semi-structured interviews. The written assessment focused on students' conceptual understanding of prisms, with two key tasks: determining whether a given geometric figure qualifies as a prism and providing justification and evaluating the truth of statements regarding the surface area and volume of prisms, as presented in Figure 1.

1. Does the figure below represent a prism? Provide a justification for your response.



2. Do two prisms with the same volume necessarily have the same surface area? Explain your reasoning.

Figure 1. Test questions

Following the written assessment, all six participants took part in semi-structured interviews designed to validate their test responses and gain insight into their experiences in learning about prisms in the classroom. The collected data were analyzed qualitatively through a structured thematic analysis. First, the researcher identified key concepts from students' written responses and interview transcripts. Second, these key concepts were categorized into broader themes. Finally, the themes were synthesized to highlight the main learning obstacles encountered by students. The thematic analysis resulted in two primary discussion themes: difficulties in identifying the characteristics of prisms and challenges in verifying the accuracy of statements related to the volume and surface area of prisms.

Results and Discussion

This section presents an analysis of students' responses, obtained through written assessments and interviews, regarding two problem-based questions on prism geometry.

Case 1: Learning Obstacles in Identifying Prism Characteristics

Subjects 1 (S1) and 2 (S2) identified the given shape as a prism, justifying their answers by stating that it possessed identical bases and a top, specifically a quarter-circle, with upright rectangular lateral faces. Additionally, both students identified the prism's height as 10 cm. However, they overlooked the presence of a curved lateral face, which contradicts the defining properties of a prism. Their understanding of a prism was based on an incomplete definition: "Having the same base and top sides and having upright sides that are rectangular." This definition does not explicitly require that the base and top sides be polygons, leading to the misclassification of a shape with circular or quarter-circular bases as a prism.

In an interview, the teacher was asked whether they had explicitly stated that prism bases must be polygons. The teacher explained that instruction on prisms typically follows a structured sequence: defining prisms, naming different types, and identifying prism elements. Although the definition provided in class does not specify that the bases must be polygons, the examples used consistently featured polygonal bases, such as triangular and pentagonal prisms. Consequently, the naming convention of prisms depends on the polygonal shape of their bases. The absence of a "circular prism" in common terminology highlights students' difficulty in applying a complete definition of prisms. Their reasoning also disregarded the criterion that the lateral faces of a prism must be rectangular, further contributing to their misconceptions.

Conversely, subjects S3, S4, S5, and S6 correctly identified that the given shape was not a prism, but their justifications varied. S3 and S6 provided accurate explanations: S3 noted that one of the lateral faces was curved, and S6 identified the quarter-circle base as an issue. Meanwhile, S4 and S5 arrived at the correct conclusion but for incorrect reasons. S4 claimed that the shape lacked a top, arguing that the rectangular portion was the base and that the opposite side was curved, making it impossible for the shape to have a "lid." Similarly, S5 reasoned that the base and top were not identical, incorrectly perceiving the rectangular side as the base and the curved side as the top. These responses highlight limitations in S4 and S5's ability to correctly determine base and top sides, often associating the base with the bottommost surface of a given diagram. The summarized responses of all subjects are presented in Table 1.

Table 1. Summary of student answers

Subject	Prism		Reason	
	Yes	No	True	False
S1	√			√
S2	√			√
S3		√	√	
S4		√		√
S5		√		√
S6		√	√	

As shown in Table 1, S1 and S2 incorrectly classified the given shape as a prism, while the remaining four subjects correctly identified that it was not. Among them, only S3 and S6 provided valid justifications, while S4 and S5 demonstrated epistemological learning obstacles. This included failure to identify both key properties of a prism ignoring the presence of a curved lateral face and failing to recognize that the base must be a polygon and incorrectly determining the base and top by assuming that the base is always the lowest surface in a given diagram.

Ricardo & Schwartzman (1995) defines a prism as "a polyhedron with two parallel, congruent polygon bases, and lateral faces that are parallelograms." Similarly, Unlu & Horzum (2018) emphasize the essential properties of prisms as follows: (1) two parallel bases, (2) identical bases, (3) polygonal bases, and (4) lateral faces that are parallelograms. These criteria align with Euclid's (Fitzpatrick, 2008) classical definition of a prism as a solid contained by planes, where opposite faces are equal, similar, and parallel, and the remaining faces are parallelograms. The teacher's definition "a geometric shape with identical base and top sides and vertical rectangular faces" does not explicitly require that the base and top be polygons. This exemplifies didactic transposition, a process in which knowledge is adapted for instructional purposes, sometimes altering its original meaning (Chevallard & Bosch, 2020).

Didactic transposition may lead to conceptual errors when teachers do not fully grasp the transformations involved. Common sources of error include:

1. Oversimplification – In adapting mathematical concepts for teaching, educators may inadvertently obscure essential properties, resulting in misconceptions among students (Balacheff et al., 2002).
2. Inaccurate Interpretation – Educators may misinterpret adapted concepts, leading to instructional inconsistencies (Artigue, 2009).
3. Curricular Limitations – Educational policies and curricula influence the way concepts are introduced, sometimes omitting critical elements (Chevallard, 2007; Chevallard & Bosch, 2020).

For a didactic transposition to be effective, it must preserve the scientific integrity of mathematical concepts while ensuring that they remain comprehensible to students at their respective cognitive development levels. Several key principles contribute to the success of this process. The first principle is a deep understanding of the original mathematical concept. According to Balacheff et al (2002), educators must possess a thorough comprehension of the subject matter before presenting it to students to prevent misinterpretations and conceptual errors. A study by Unlu and Horzum (2018) revealed that none of the Mathematics Teacher Candidates (MTCs) surveyed could accurately articulate the definition of a prism, indicating a significant gap in their ability to correctly define the concept and identify its essential attributes as outlined in existing literature (Bozkurt & Koç, 2012). This finding underscores the critical importance of subject-matter expertise among mathematics educators, as emphasized by Adler et al (2014). Effective mathematics instruction necessitates a profound conceptual understanding (Türnüklü, 2005), which minimizes the risk of teachers misrepresenting key ideas and students developing misconceptions.

The second principle involves avoiding excessive simplification of concepts. Chevallard and Bosch (2020), as well as Chevallard (2007), warn that if educators oversimplify mathematical content during the didactic transposition process, essential conceptual properties may be distorted, leading to student misunderstandings. The third principle is ensuring alignment with the curriculum while maintaining scientific accuracy. Artigue (2009) highlights the need for curricula to strike a balance between mathematical rigor and accessibility, ensuring that concepts are both comprehensible and scientifically valid.

The fourth principle pertains to employing appropriate didactic strategies. In the context of this study, geometric concepts are best understood through the examination of a diverse set of examples. Research suggests that conceptual understanding is reinforced when students can accurately identify the defining characteristics of a concept (Fried, 2006; Ulusoy, 2019). The process of concept formation begins with distinguishing between examples and non-examples (Levenson et al., 2011; Ulusoy, 2019). Vinner (1991) and Ulusoy (2019) define a concept image as the mental representations that students associate with a given mathematical term. However, when instruction is limited to providing examples without non-examples, students' conceptual frameworks remain incomplete. Interviews with teachers at the research site revealed that instructional practices predominantly involved presenting examples of prisms, with little emphasis on non-examples. This pedagogical approach likely contributed to students' difficulties in correctly identifying prism properties, as they lacked experience in distinguishing between valid and invalid representations of the concept.

Case 2: Learning Obstacles in Analyzing the Validity of Statements Related to the Volume and Surface Area of a Prism

In this section, participants were required to evaluate the validity of the statement: “If two prisms have the same volume, then they must also have the same surface area.” This question falls under the category of analytical reasoning and is classified at Level C4 in Bloom’s Taxonomy. To effectively address this problem, students needed to deconstruct the given information and explore multiple possible conditions that could arise from the statement. Three primary scenarios can be considered: (1) two prisms with identical dimensions (e.g., two cubes with side lengths of 5 cm) that yield both the same volume and surface area, (2) two prisms of the same type (e.g., rectangular prisms) but with different dimensions, leading to the same volume but different surface areas, and (3) two distinct types of prisms (e.g., one being a cube and the other a rectangular prism) that share the same volume but exhibit different surface areas.

The responses revealed significant difficulties among participants in providing a well-reasoned justification for their answers. Two participants (S1 and S2) correctly identified the statement as false, while four participants (S3, S4, S5, and S6) incorrectly considered it to be true. However, five out of six participants struggled to articulate their reasoning clearly. For instance, S6, when asked to justify their response, simply stated, “I don’t know.” Similarly, although S1 correctly identified the statement as false, they were unable to explain their reasoning further. None of the participants attempted to visualize the problem by sketching the two prisms, assigning arbitrary side lengths, or calculating whether the same volume necessarily leads to the same surface area. Instead, most students relied on verbal reasoning, without employing any form of visual representation. This suggests that visualization plays a crucial role in problem-solving, as verbal explanations alone may be insufficient for conceptual understanding.

Among the responses, S2 demonstrated a partial understanding of the concept. S2 stated that “if a prism has different base and height values that produce the same numerical result, the surface area may differ.” This response was interpreted by the researchers to mean that two prisms with different dimensions can share the same volume while having varying surface areas. When asked for further clarification, S2 provided an analogy: “For example, 5 and 10 give the same result as 11 and 4. However, when extended to surface area, the values may differ.” While this analogy suggested some level of comprehension, it lacked mathematical precision. The explanation implied that different sets of dimensions could yield the same volume, but S2 was unable to extend this reasoning to explicitly demonstrate how surface area is affected. Since prisms typically have three key dimensions, an appropriate explanation would require a more comprehensive numerical justification.

The participants’ struggles in evaluating the validity of the given statement and their difficulty in articulating a clear rationale indicate a lack of familiarity with higher-order cognitive tasks, such as those classified at Level C4 and above in Bloom’s Taxonomy. These tasks require analytical reasoning and problem-solving skills beyond routine computations. Interviews with teachers confirmed that students are predominantly exposed to exercises that closely resemble worked examples rather than non-routine, conceptual problems. This finding aligns with previous research indicating that an overreliance on example-based learning can impede students' ability to transfer their knowledge to novel contexts (Hendriyanto et al., 2024). Sierpińska (1987) and Modestou and Gagatsis (2007) similarly observed that students who rely excessively on worked examples often struggle with knowledge application in unfamiliar situations.

Furthermore, epistemological obstacles were evident in students' difficulty in analyzing the given statement. These obstacles arise when students encounter limitations in applying their mathematical knowledge to new and complex problems (Supandi et al., 2021). According to Hendriyanto et al. (2024), students who predominantly engage with routine exercises of uniform difficulty may develop cognitive barriers that hinder their ability to adapt their understanding to novel scenarios. Such challenges often stem from an inadequate grasp of fundamental mathematical principles, as noted in studies on epistemological barriers in mathematics education (Moru, 2009).

One potential strategy to address these obstacles is the development of students' metacognitive skills, which involve self-awareness and self-regulation of cognitive processes (Schraw & Dennison, 1994). By engaging in metacognitive activities such as self-monitoring and reflective thinking, learners can critically assess their understanding, identify misconceptions, and adjust their problem-solving strategies accordingly (Vaughan, 2022). Research suggests that fostering metacognitive awareness enables students to move beyond rigid, absolutist conceptions of knowledge toward a more flexible and evaluative approach to learning (Double, 2025). In addition to improving conceptual understanding, metacognitive practices promote the acquisition of higher order thinking skills, including analysis, evaluation, and synthesis (Hamzah et al., 2022). Integrating reflective questioning, goal-setting, and strategic thinking into the learning process can thus support students in becoming more independent and adaptable problem-solvers (Schraw, 1998).

Moreover, teachers must recognize epistemological obstacles and their role in shaping students' comprehension of mathematical concepts during the didactic transposition process (Robutti, 2020). Effective didactic transposition requires instructional strategies that directly address students' preconceptions and facilitate conceptual restructuring (Suryadi, 2019a). Designing learning tasks that challenge existing misconceptions, providing varied problem-solving contexts, and incorporating diverse instructional approaches can enhance students' conceptual understanding (Prihandhika et al., 2022; Jamilah et al., 2021). By acknowledging and addressing these epistemological challenges, educators can create more effective learning environments that foster deeper mathematical reasoning and critical thinking.

Conclusion

The findings of this study reveal that students face epistemological obstacles in identifying prisms, primarily due to their inability to recognize the two fundamental characteristics of a prism as defined in mathematical theory. Specifically, students struggle to understand that a prism must have two parallel polygonal bases and that a curved vertical surface is not a characteristic of a prism. This misconception arises from the didactic transposition process, where the original mathematical concept is adapted for instructional purposes but loses essential elements in the process. The study identifies three key factors contributing to this issue: oversimplification of definitions, misinterpretation of concepts, and curricular limitations that do not accurately reflect the fundamental properties of prisms. To address these epistemological obstacles, it is crucial to ensure that didactic transposition retains conceptual accuracy while being accessible to students at their cognitive development level. This can be achieved by applying specific principles, including a deep understanding of the original mathematical structure, the avoidance of excessive simplification, the use of appropriate didactic strategies, and the integration of concepts into the curriculum without compromising their scientific integrity. Additionally, the study highlights that students face challenges in solving non-routine problems due to their reliance on repetitive exercises that closely mirror the teacher's examples. To overcome this limitation, fostering metacognitive awareness and refining didactic transposition techniques that acknowledge epistemological obstacles can empower students to take a more active role in their learning process.

Despite its contributions, this study has certain limitations that must be acknowledged. The research primarily focuses on epistemological obstacles and does not examine ontogenic and didactical obstacles, which are equally significant in understanding students' difficulties in learning the prism concept. Ontogenic obstacles, which stem from students' cognitive development and prior knowledge, play a crucial role in their ability to grasp abstract mathematical ideas. Similarly, didactical obstacles, which arise from instructional methods and learning environments, can influence students' misconceptions and problem-solving abilities. The absence of an analysis of these two types of obstacles limits the comprehensiveness of the study's findings. Furthermore, the study does not explore how different instructional approaches, such as inquiry-based learning or technology-assisted teaching, might mitigate these learning challenges. Addressing these limitations in future research would provide a more holistic understanding of the factors affecting students' comprehension of geometric concepts. Finally, future research should expand the scope of investigation to include ontogenic and didactical obstacles in order to develop a more comprehensive didactic design for teaching the prism concept. A multidimensional analysis incorporating these factors would offer deeper insights into the cognitive and instructional barriers that hinder students' understanding. Additionally, further studies could explore the effectiveness of various pedagogical approaches, such as problem-based learning, the use of digital manipulatives, and collaborative learning strategies, in addressing students' misconceptions about geometric figures. Longitudinal studies tracking students' progress over time could also provide valuable data on how conceptual understanding evolves with different teaching interventions. By broadening the research focus, future studies can contribute to the development of more effective instructional strategies that enhance students' geometric reasoning and problem-solving skills.

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Conflicts of Interest

There is no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been covered completely by the authors.

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