

How does interactive case-based learning improve students' complex mathematical problem-solving abilities?

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Abstract

Complex Mathematical Problem Solving (CMPS) is a crucial competency that equips students to navigate uncertain future situations. To enhance this skill, there is a need for more effective instructional models. One promising approach is Interactive Case-Based Learning (ICBL), an advanced iteration of the Case-Based Learning model. ICBL engages students with intricate real-world cases, enabling them to grasp mathematical concepts and adapt to novel and unfamiliar scenarios encountered in everyday life. This study aims to evaluate students' CMPS abilities following participation in ICBL-based instruction and to assess their responses to this instructional approach. A quasi-experimental design was employed, involving an experimental group and a control group. The participants were seventh-grade students studying sequences and series. The research utilized ICBL-based teaching materials, CMPS ability tests, and student response questionnaires. The study's findings are that students instructed using the ICBL model demonstrated superior CMPS abilities compared to those instructed using traditional methods, and students responded positively to the ICBL instructional model in mathematics learning. This research underscores that the ICBL model can significantly enhance students' ability to solve complex problems. Consequently, educators should consider incorporating the ICBL model into their teaching strategies, and curricula should be adapted to support its implementation.

Keywords: complex mathematical problem-solving, interactive-case based learning model, interactive problem-solving, sequences and series, seventh-grade students

Introduction

Real-world problems often present complex scenarios that involve intricate issues requiring distinct skills and attitudes compared to well-defined problems with clear goals and known solution procedures (Jonassen, 1997). It is imperative to provide students with educational experiences that foster the development of skills and attitudes essential for tackling ambiguous and uncertain problems encountered in real-life situations (Choi et al., 2012). Kuhn and Dean (2005) demonstrate that possessing knowledge of problem-solving strategies does not necessarily equate to the ability to apply these strategies to novel and unfamiliar problem contexts. Consequently, in addition to learning general problem-solving strategies (Mayer & Wittrock, 2006), students must acquire a repertoire of cross-curricular skills applicable to addressing new and complex problems (OECD, 2010).

Contemporary research on problem-solving focuses on complex and unstructured knowledge, reflecting the nature and structure of real-life problems (Wüstenberg et al., 2014). Complex Problem Solving (CPS) is defined as the ability to address dynamically changing and non-transparent problems through behavior or cognition across multiple stages (Frensch & Funke, 1995). The CPS process generally encompasses two distinct phases, namely Knowledge Acquisition, where the problem solver explores the behavior of the system using known strategies to understand the system's state, and Goal-Oriented Knowledge Application, where the problem solver makes informed conjectures about the system's dynamics to determine appropriate interventions and their potential consequences (Leutner et al., 2005). Given its significance, CPS is increasingly recognized as a critical skill for the 21st century (Mainzer, 2005), posing challenges within the mathematics education context. It is essential to provide students with learning experiences that enhance their ability to solve complex mathematical problems (Vye et al., 2016). Beyond knowing specific strategies (declarative knowledge) and their application (procedural knowledge), students need meta-strategic knowledge relevant to unfamiliar problems, enabling them to apply suitable strategies confidently (Koh et al., 2008).

The Organization for Economic Co-operation and Development (OECD) has incorporated CPS skills into the PISA test framework (OECD, 2014). Since 2012, PISA has featured "interactive problems" characterized by non-transparency and multiple interconnected elements (Poddiakov, 2016). However, recent PISA results indicate that Indonesian students' mathematics problem-solving abilities remain below the global average, with a 2022 mathematics literacy score of 366 compared to the global average of 472, and among the lowest in ASEAN countries (OECD, 2023). This is consistent with Wahyuni et al. (2023), which identifies difficulties among Indonesian students in understanding textual problems, using mathematical terminology, connecting problems, making decisions, and handling uncertainty. These challenges highlight the need for improved approaches in developing complex problem-solving skills.

Case-Based Learning (CBL) offers a promising approach by presenting students with real-world cases that approximate real-life contexts. CBL facilitates the application of theoretical knowledge to practical situations, promotes critical thinking, encourages action planning, and fosters self-knowledge through comparative perspectives (Williams, 2005). The

integration of feedback in CBL has been shown to enhance conceptual understanding (Asfar et al., 2019), with research by Turk et al. (2019) demonstrating its effectiveness in improving learning outcomes across various fields. CBL has significantly improved exam performance, with students outperforming their peers by nearly 20% (Deshpande et al., 2019), and increased declarative knowledge scores in student groups (Jamkar et al., 2007).

However, Maer & Hendrayani (2022) argue that CBL is more suitable for structured, modular problems, and that students require complex problem-solving skills for effective learning. Thus, the cases used in CBL should encompass not only well-structured problems but also complex problems. Effective complex problem-solving necessitates a blend of task-specific knowledge and abstract thinking skills (Goode & Beckmann, 2010). To address this, the Interactive Case-Based Learning (ICBL) model has been developed by adapting the CPS process model (Frensch & Funke, 1995; Grünig & Kühn, 2017; Chevallier, 2016) and integrating it with CBL learning syntax (Williams, 2005). This study aims to assess students' CMPS skills following ICBL-based instruction and to evaluate their responses to this learning approach.

Interactive Case-Based Learning Model

Case-Based Learning (CBL) is a pedagogical approach closely related to Problem-Based Learning (PBL) (Williams, 2005). While PBL emphasizes that problems drive the learning process, CBL requires students to apply prior knowledge to address complex cases (Garvey et al., 2000). Despite numerous studies demonstrating the effectiveness of PBL in enhancing students' problem-solving skills, a systematic review by Koh et al. (2008) challenges its long-term efficacy. Kitchener (1983) criticized PBL as ineffective, arguing that it disregards or contradicts human cognitive architecture and cognitive load principles. However, this critique has been countered by Hmelo-Silver (2004) and Schmidt et al. (2007), who provided cognitive science principles and empirical evidence supporting the foundational theories of PBL. This debate highlights the need to refine the implementation of PBL, and by extension, CBL, within educational practices.

Research into instructional design for unstructured problem-solving is limited, particularly regarding how to support both students and educators in designing effective instruction (Choi & Lee, 2009). To address this gap, researchers have developed the CBL model, which presents complex cases as a means to enhance learning. CBL is deemed more efficient for acquiring content knowledge (Kirschner et al., 2006) and is effective in fostering students' abilities to tackle unstructured problems (Williams, 2005). The selection of cases in CBL involves identifying problems relevant to students and those they may encounter in the future (Bridges & Hallinger, 1999).

In problem-solving, the nature of the problem influences the approach taken. Problems can be categorized into two types: choice problems and design problems. Choice problems have predefined solution options, whereas design problems require decomposition into smaller sub-problems, which are addressed sequentially or concurrently. Design problems can be solved in three ways, namely Series, where the problem is broken down into sub-problems solved

sequentially, Parallel, where independent sub-problems are solved concurrently, with the overall solution derived from coordinating these solutions, and Combined Series-Parallel, where the problem is tackled using both sequential and concurrent approaches.

Barrows (1986) proposed a taxonomy for PBL, classifying it based on two variables: self-directedness and problem structuredness. This taxonomy was later illustrated by Hung (2011), and a modified version of this PBL taxonomy is presented in Figure 1.

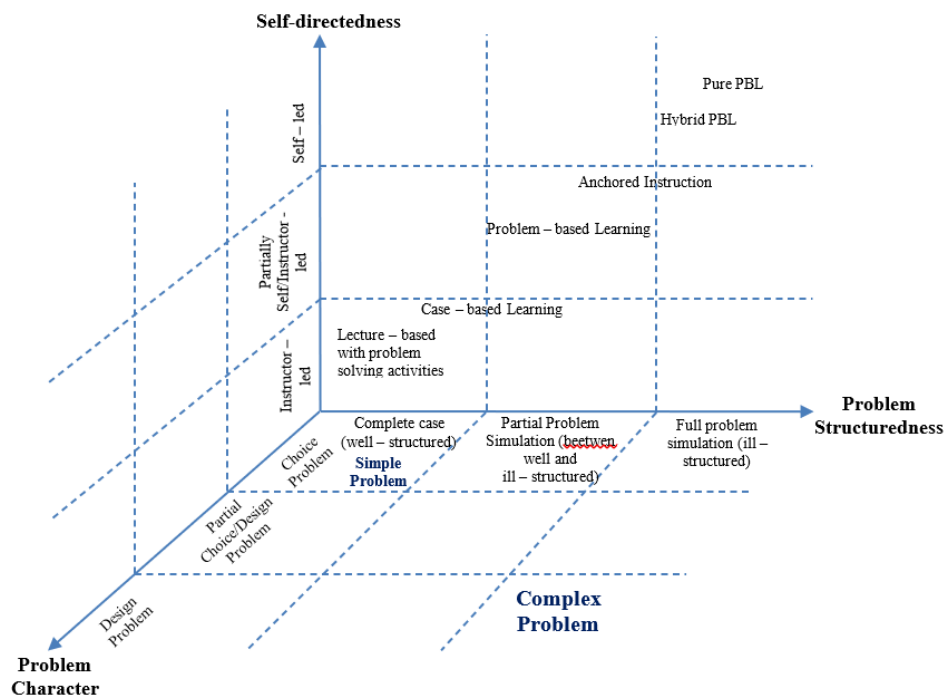


Figure 1. Modification of PBL Taxonomy

Figure 1 illustrates that problems addressed in all Problem-Based Learning (PBL) models encompass not only poorly structured problems but also complex problems. These complex problems are characterized not merely by choice issues but also by design issues. In the PISA assessment, the term "interactive problem solving" is employed to describe the resolution of complex problems. Consequently, the term "interactive" is applied to complex cases in the development of case-based learning models. This has led to the creation of the Interactive Case-Based Learning (ICBL) model.

The ICBL model is designed to facilitate effective interaction between students as individual problem solvers and the situational conditions of the task when confronted with complex problems. Such interaction necessitates the utilization of cognitive, emotional, and social resources, alongside knowledge (Frensch & Funke, 1995).

Teachers play a critical role in supporting the teaching and learning process by incorporating activities that explore students' foundational skills and promote conceptual understanding before they engage with complex problems (Edo & Tasik, 2022). Thus, the ICBL model is defined as an instructional approach that involves cases characterized by complexity,

realism, and relevance to the subject matter. It requires students to actively seek and utilize pertinent information to solve these cases based on their prior knowledge and experiences.

Figure 2 presents the ICBL learning design framework, which is based on learning design theory as proposed by Reigeluth (1999). This framework adapts the steps of complex problem solving outlined by Chevallier (2016), Frensch and Funke (1995), and Grünig and Kühn (2017), and serves as the foundation for modifying the CBL learning syntax according to Williams (2005).

Complex Problem Solving Process Model							
Model Chevallier (2016)	Step 1 Framing the problem (the what)		Step 2 Diagnosing it (the why)		Step 3 Finding solutions (the how)		Step 4 Implementing the solution (they do)
Model French & Funke (1995)	Step 1 Goal-elaboration	Step 2 Hypothesis-Formation	Step 3 Prognosis	Step 4 Planning and Decision Making	Step 5 Monitoring	Step 6 Self-Reflection	
Model Grünig, R. & Kühn, R. (2017)	Step 1 Verifying the discovered decision problem	Step 2 Problem analysis	Step 3 Developing solution options	Step 4 Determining the decision criteria	Step 5 Determining environmental scenarios	Step 6 Determining the consequences of the option	Step 7 Overall evaluation of the options and decision
Adaptation Model	Step 1 Understand the problem situation and determine the goal to be achieved		Step 2 Identify the core problem based on previous suspected causes of the problem		Step 3 Plan solution actions based on the possible impacts that will occur and make decisions for resolution		Step 4 Reflect on the process and make conclusions.
Interactive Case-Based Learning (ICBL) Model							
Problem Solving Activity by ICBL	Problem interpretation	Consider the problem from different perspectives formulate conjectures on the causes of the problem and identify the root cause of the problem.		Explore potential solutions and decide on the best solution based on the likely impact and monitor its effects.		Conduct an overall evaluation of the problem-solving process and conclude.	
ICBL Learning Syntax based on Complex Problem Solving	Stage 1 Determine objectives based on problem overview	Stage 2 Diagnosing the problem		Stage 3 Creating solutions		Stage 4 Reflecting on the results	
Learning Environment Design Model (Adaptation of French & Funke, 1995)	Interactive Case	Expert Perspective		Theory and Literature		Opinions on results	

Figure 2. ICBL Learning Design Framework

The Interactive Case-Based Learning (ICBL) model is characterized by several key elements:

1. Case

A case is deemed complex when it encompasses at least two attributes of a complex problem. Effective cases should include a compelling idea, focus on controversial or novel topics, foster empathy with the central character, and be relevant to the audience. Additionally, cases should have pedagogical utility, present a significant decision-making scenario, and be concise.

2. Study Questions

The study questions, presented at the conclusion of each complex case, are designed to enhance understanding by prompting students to establish goals, diagnose the underlying issues, and determine solutions for the case.

3. Group Discussion

Addressing complex cases necessitates collaborative group discussions where students engage actively with the case's central concepts. The teacher facilitates these discussions to help students derive meaning and deepen their understanding.

4. Follow-Up

To further their understanding, students are encouraged to seek additional information beyond the case study, stimulated by class discussions. The aim of ICBL is to ensure mastery of content, promote collaboration, and develop problem-solving skills for complex scenarios.

The learning steps of the ICBL model are derived from the Case-Based Learning (CBL) framework established by Williams (2005). Table 1 provides a detailed outline of the ICBL learning steps.

Table 1. ICBL Model Learning Steps

No	Learning Steps	Description
1	Dividing students heterogeneously into groups	
2	Presents interactive cases (complex)	In the early stages of learning, it is possible that students are given simpler cases before being given complex problems. This is done so that students do not have difficulties in carrying out the learning process.
3	Determine objectives based on an overview of the complex problem	(1) Reviewing the Case a. Identify the problem in the case b. Looking at expert perspectives on the case (if required) (2) Objective Identification a. Construct questions about the case that you want to know about b. Determine the various objectives that may be associated with the solution
4	Diagnosing the Problem	(1) Formulate conjectures on the causes of the problem

No	Learning Steps	Description
		(2) Identifying the core case (root cause) (3) Select key objectives based on the results of goal identification (4) Seek information from various sources related to key objectives (5) Determine the solution relationship between objectives. The relationship can be parallel, sequential, or a combined parallel-sequential relationship
5	Creating solutions	(1) Plan the solution action to be taken (2) Identify possible impacts that will occur when the solution is implemented (3) Determine or make a decision on the solution to be taken based on how much effect it will have. (4) Performing settlement based on the solution taken (5) Monitoring the effects of the implemented solution
6	Reflecting on learning outcomes	(1) Reflecting on the problem solving process that has been done (2) Conclude the learning process
7	Presentation. Groups present the results obtained based on the agreed-upon outcomes.	
8	Improvement: correcting answers that are not correct	

Methods

This study employs quantitative methods to generalize students' Complex Mathematical Problem Solving (CMPS) abilities, which can be measured and observed through learning with the Interactive Case-Based Learning (ICBL) model and a carefully developed theoretical framework (Siswono, 2018). A quasi-experimental design was utilized, incorporating both an experimental group and a control group. The study employed a randomized two-group post-test-only design, involving two randomly selected classes: the control class using a conventional model and the experimental class using the ICBL model. The average post-test scores of these two groups were compared. This design was chosen for its efficiency and to mitigate response shift bias (Geldhof et al., 2018).

To ensure the validity of the research, triangulation was conducted through the following stages: (1) Preparation: This involved interviews with mathematics teachers to understand student characteristics and the learning process, analyzing core and basic competencies, determining material and question grids, and preparing lesson plans based on ICBL learning stages; (2) Implementation: This stage involved selecting the control and experimental classes and administering post-test questions after the completion of instruction for both groups; and (3) Final Analysis: The CMPS abilities of students in both classes were described based on post-test scores to assess the impact of the ICBL model on students' CMPS abilities.

The study population comprised all 10th-grade students at a Madrasah Aliyah in South Jakarta during the odd semester of the 2023/2024 academic year. Cluster random sampling was employed to select participants, as this technique is cost-effective and suitable for random

selection at multiple stages (Parrott, 2014). Classes from level X were randomly chosen to form the control and experimental groups. Class X-1 was designated as the experimental group using the ICBL model, while Class X-4 served as the control group utilizing the conventional model, with each class consisting of 30 students.

The learning process spanned six sessions, including five face-to-face meetings and one post-test. Data collection involved administering CMPS ability tests in the form of post-test scores for both groups. These tests were conducted after all row and sequence materials had been covered. Additionally, a questionnaire assessed students' responses to the ICBL model, employing a Likert scale with 17 positive statements. The research instruments included Students Worksheets or in Indonesia it's called LKPD aligned with the ICBL model, CMPS ability tests featuring descriptive questions on row and sequence material, and student response questionnaires evaluating the ICBL learning process. The CMPS ability indicators used in this study are adapted from Chevallier (2016) and contextualized for mathematics. Finally, Table 2 presents the CMPS indicators developed for this study.

Table 2. Indicators of CMPS Ability

No.	Indicator	Description
1	Framing the problem	Identify the math problem and explain the purpose of the problem in parts.
2	Diagnosing the problem	Determine the information needed and possible causes of the problem then determine the core of the problem based on several possibilities related to the math problem.
3	Finding a solution	Identify alternative ways to solve problems and explain the possible effects of these alternatives in relation to mathematical problems.
4	Implementing the solution	Determine the appropriate solution to be implemented in solving the problem and then evaluate the implemented solution in relation to the mate problem.

The test instrument utilized in this study has undergone thorough validation and reliability testing. Additionally, a scoring rubric was developed for each item to accurately measure students' Complex Mathematical Problem Solving (CMPS) abilities in the context of row and sequence material. The validity test revealed that, out of the six questions, one question was deemed invalid due to a Sig. (2-tailed) value of 0.244, which exceeds the threshold of 0.05. This question was excluded based on its inadequate representation of the CMPS indicators.

The reliability test indicated a reliability coefficient of 0.579, categorizing the reliability of the questions as medium. Subsequent data analysis involved hypothesis testing to determine whether the CMPS abilities of students using the ICBL model were superior to those of students using the conventional model. Prior to hypothesis testing, prerequisite analyses were conducted, including normality and homogeneity tests, to ensure the data met the necessary assumptions.

Results and Discussion

Results

The results of the data analysis revealed that, in the experimental class, scores ranged from a minimum of 25 to a maximum of 80. In contrast, the control class exhibited a range from 15 to 70. The average score for the experimental class was 40.17, compared to 39.67 for the control class. These findings suggest that students who received instruction through the ICBL model demonstrated superior CMPS abilities compared to those who experienced conventional instruction. The average scores for each CMPS ability indicator are illustrated in [Figure 3](#).

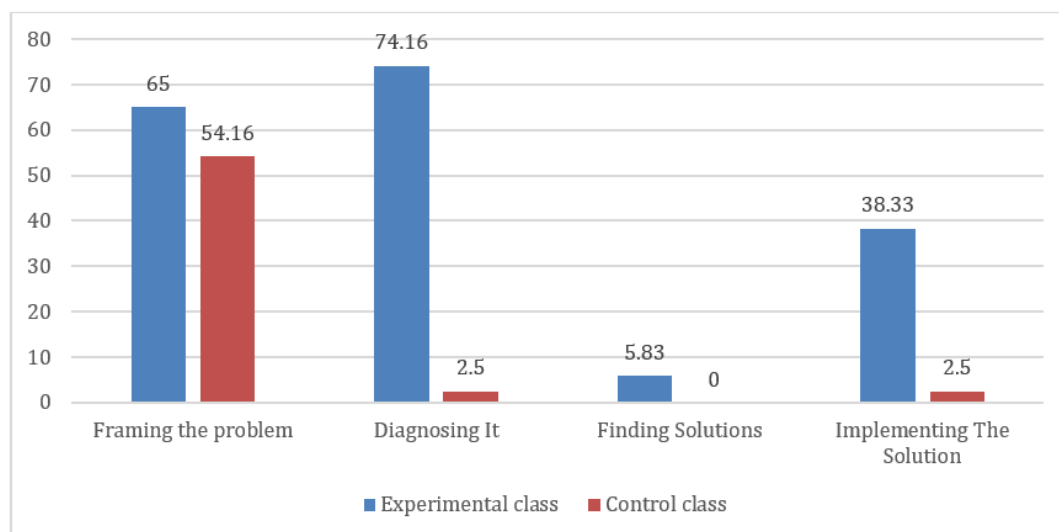


Figure 3. Average CMPS Ability of Students

[Figure 3](#) illustrates that students in the ICBL model learning group outperform their peers in the conventional class across all CMPS ability indicators. Notably, the average score for the problem diagnosis indicator shows a significant disparity: students in the experimental class achieved an average score of 74.16, compared to just 2.5 in the control class. This substantial difference indicates that students in the experimental class demonstrate markedly superior problem diagnosis skills.

For the solution-finding indicator, none of the students in the control class were able to provide answers, whereas the average score for students in the experimental class was 5.83. This result reflects a generally low ability among students to find solutions in both groups, although those in the experimental class performed slightly better.

To find out whether the average difference in the CMPS ability of students in the experimental class and control class is significantly different or not, it is necessary to conduct a statistical test. The results of the normality test using SPSS obtained the Sig. value of the experimental class is $0.048 < 0.05$, then H_0 is rejected, meaning that the sample comes from a population that is not normally distributed. It is also known that the Sig. value of the control class is $0.001 < 0.05$, then H_0 is rejected and the sample comes from a population that is not normally distributed so hypothesis testing uses the Mann-Whitney Test (U-Test). The results of

hypothesis testing can be seen in Table 3 that the Asymp. Sig (2-tailed) is $\frac{0,00}{2} = 0,00 < 0.05$, then H_0 is rejected and H_1 is accepted, so that the CMPS ability of students using the ICBL model is higher than the CMPS ability of students using the conventional model. Table 3 shows the results of hypothesis testing using SPSS software.

Table 3. Hypothesis Test Results of CMPS Ability

Test Statistics	
	Value
Mann-Whitney U	42.000
Wilcoxon W	507.000
Z	-6.088
Asymp. Sig. (2-tailed)	.000

a. Grouping Variable: Kelas

The learning process using the ICBL model is implemented according to the steps outlined in Table 1. Initially, students are introduced to a simple case before progressing to a complex (interactive) case. This approach aims to minimize potential difficulties that students may encounter during the learning process. While analyzing the complex case, students are encouraged to discuss various aspects that contribute to the solution. They are tasked with mapping out both the core solution and the intermediate solutions that lead to it, and are guided to identify various simpler objectives to facilitate the discovery of the core solution.

During the first meeting, students faced challenges in identifying and solving the complex cases, despite prior exposure to simpler cases. This difficulty is attributed to students' unfamiliarity with the learning approach. However, by the second and subsequent meetings, students began to acclimate to the process of handling complex mathematical problems. As a result, all group members became actively involved in discussions and collaboratively worked towards solving the cases. Figure 4 illustrates students engaging in discussions within their groups while working on complex cases related to the material of rows and series.



Figure 4. Student's situation when solving a complex case

Discussion

Based on the results of the statistical analysis, students using the ICBL model demonstrated better complex mathematical problem-solving (CMPS) abilities compared to those using the conventional model. This is supported by the average results of each CMPS ability indicator, as depicted in Figure 4. These findings suggest that the ICBL model effectively enhances students' capacity to solve complex mathematical problems. For instance, in problem diagnosis, students displayed the ability to identify and address key questions leading to solutions. This skill is crucial for developing overall problem-solving abilities, as it aids students in comprehending mathematical concepts and applying this knowledge to unfamiliar situations (Geeganage et al., 2016; Krulik & Rudnick, 1988).

Complex Mathematical Problem Solving (CMPS) Ability

The first indicator of complex mathematical problem-solving (CMPS) is students' ability to frame mathematical problems. This skill involves identifying, defining, and clearly formulating the problem. Effective framing enables students to view a situation or challenge from multiple perspectives, thereby grasping the essence of the mathematical problem. Without this ability, students struggle with planning and executing problem-solving strategies. Complications arise when problems involve lengthy sentences and extensive information, which can confuse students and obscure the problem's objective. This confusion can lead to uncertainty about the appropriate course of action, increasing the likelihood of errors. The following example illustrates a problem used to assess students' problem-solving abilities as shown in Figure 5.

A contractor wants to build a housing estate consisting of 50 houses. The houses measure $6m \times 8m$ and use a pyramid roof with a roof angle of 30° and a roof overstep of $1m$. The roof will use concrete tiles and each m^2 of the roof requires 10 tiles. Then the contractor collaborated with a roof tile company "Ratu Atap". With the demand from the contractor, the company finally increased productivity and labour, so that every month it is always more than twice as much. As a result, the company increased its production in the third month by 8,000 roof tiles. In what month was the company able to fulfil the demand for roof tiles from the contractor?

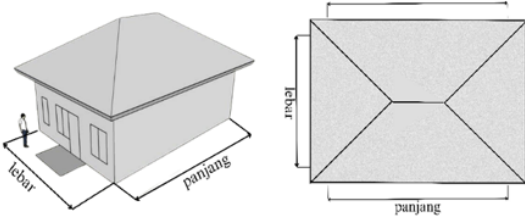


Figure 5. The given problem to assess students' problem-solving abilities

This problem is considered complex due to its characteristics of politeness and transparency. Politeness is evident as the problem involves multiple objectives, including calculating the triangular area of a pyramid roof, determining the area of a trapezoid (a component of the roof), estimating the number of roof tiles required for one house, and computing the total production of roof tiles for the fourth month. Transparency issues arise because the problem does not provide the height of the triangular roof, which is necessary for

calculating its area, nor does it provide the height of the trapezoid, which is needed for determining its area.

Among all CMPS indicators, the greatest average difference is observed in the problem diagnosis indicator, with a difference of 71.66. This significant difference is attributed to the ICBL model's effectiveness in enhancing students' diagnostic skills. Through the ICBL approach, students are guided to identify the core problem and establish key objectives for solving the case. Conversely, in the control class, such diagnostic activities did not develop as effectively. Figure 6 displays an example of a student's response from the experimental class demonstrating their ability to diagnose cases.

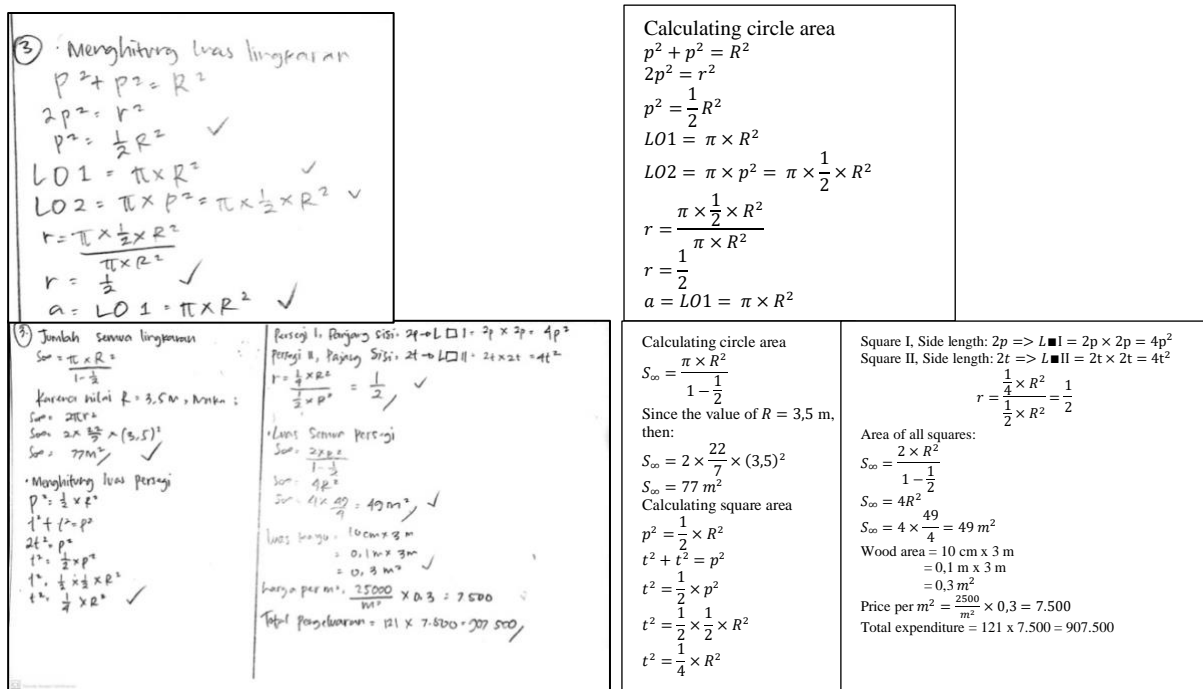


Figure 6. Student Answers on the Indicator of Diagnosing Cases

At the stage of diagnosing cases, which is the initial phase of the learning process, students often encounter difficulties due to the requirement for deep knowledge and analytical skills. Diagnosing cases is crucial for problem-solving as it involves clinical reasoning and navigating various pathways to reach the correct solution (Doleck et al., 2016).

The second indicator of CMPS ability is the capacity to diagnose problems. This ability is vital for students as it supports their understanding of mathematical concepts and contributes to their success in mathematics learning. It is equally important for teachers, who need to diagnose students' abilities thoroughly. However, this aspect often lacks serious attention from teachers. Teachers frequently do not conduct in-depth diagnoses of students' difficulties in learning mathematics (Wijaya et al., 2019). Effective diagnosis of skills involved in problem-solving helps students recognize and address their weaknesses (Tambychik & Meerah, 2010). Teachers need insights into cognitive processes to guide attention to critical aspects of the

problem (Trismen, 1988). Students' difficulties in problem-solving can stem from individual traits, attitudes, and their relationship with the tasks provided by the teacher (Sirant, 2022).

The third indicator of CMPS capability is the ability to find solutions. This involves planning solution actions and considering the potential impacts of those actions. The average score for experimental class students on this indicator was very low at only 5.83, though it was still better than the control class where no students were able to answer this question. In the experimental class, students were exposed to complex math cases and trained on how to solve them, with activities conducted in heterogeneous groups. Providing problem-solving experiences is crucial, as it encourages students to share their learning experiences and collaboratively identify challenges and solutions (Moleko & Mosimege, 2020). This collaborative approach allows students to internalize knowledge with relevance and meaning (Anaya et al., 2020). Problem-solving activities not only help students apply their understanding but also foster deep understanding and improve mathematical competence (Putri et al., 2023). In contrast, students in the control class were given practice questions that did not promote a deeper understanding or the ability to solve complex problems. Insufficient math skills can hinder various stages of the problem-solving process (Tambychik & Meerah, 2010), and successful problem-solving relies on both conceptual understanding and procedural knowledge (Geary, 2004).

The fourth indicator of CMPS is implementing the solution. Here, the experimental class students had an average score of 38.33, while the control class students had an average score of 2.5. This indicates that students in the experimental class performed better in this area. Implementing solutions involves applying the correct solution and evaluating its effectiveness. This process typically requires students to verify their results and assess the problem-solving process (Kamariah et al., 2023). While students generally do not struggle with applying solutions, evaluating the problem-solving process remains a challenge, as noted by Moreno et al. (2021), who found that students rarely interpret solutions in relation to the actual situation.

Students' Response to Learning Process Using ICBL Model

The student response questionnaire was administered upon completion of the learning process using the Interactive Case-Based Learning (ICBL) model. The results of this questionnaire, which assess students' feedback on the ICBL model, are summarized in Table 4.

Table 4. Results of student response questionnaire to the ICBL model

No	Statement	Percentage (%)
1	I feel satisfied following math learning with the ICBL model	78
2	ICBL learning model is more useful for math learning	78
3	Following math learning using the ICBL model has increased my motivation to learn.	81
4	ICBL learning model can eliminate boredom during the math learning process in class	78
5	Learning by using the ICBL learning model makes me understand math material better	76

6	The application of the ICBL learning model encourages me to find new ideas	78
7	Learning by using the ICBL learning model trains me to be able to express my opinion.	78
8	Learning by using the learning model trains me to be able to solve complex problems	77
9	ICBL learning model can improve complex problem solving skills	77
10	Learning by using the learning model trains me to think critically	78
11	Learning by using the ICBL learning model makes me more active in learning	79
12	Learning by using the ICBL learning model makes me actively discuss in study groups in class	79
13	Learning by using the ICBL learning model makes me challenged to solve cases	77
14	Learning by using the ICBL learning model can share knowledge with friends	83
15	Learning by using the learning model trains me to create new things	76
16	Learning by using the ICBL learning model makes the material easier to remember	78
17	Learning by using the ICBL learning model makes math material look useful in everyday life	79
Average		78

The results indicate a favourable student response to the learning process utilizing the ICBL (Inquiry-based Case-based Learning) model, with an average approval rating of 78%. This finding supports the hypothesis that students' Computational Mathematical Problem Solving (CMPS) abilities are enhanced when using the ICBL model compared to the conventional model. A closer examination reveals that the highest approval rate was for the statement, "The application of the ICBL learning model encourages me to generate new ideas," which received an 82% positive response. Conversely, the statement, "Learning with the ICBL model facilitates knowledge sharing with peers," received the lowest approval rate at 67%. These results align with Lopes and Jorge's (2000) research, which suggests that case-based learning fosters discovery by integrating rules and cases, constructing explanations for each case, and considering diverse perspectives to identify similarities.

Several factors contribute to the superior CMPS abilities observed in students using the ICBL model compared to those engaged in conventional methods. Firstly, the interactive nature of the cases presented in the ICBL model promotes active student engagement in solving mathematical problems. The complex problems embedded in these cases are designed to enhance higher-order mathematical thinking skills (Hong & Kim, 2016). Student-centered learning remains a fundamental objective of effective educational practices (Cattaneo, 2017). Secondly, the procedural stages of the ICBL model facilitate the development of CMPS skills. For instance, during the identification stage of mathematical cases, students are trained to decompose problems into smaller, more manageable sub-problems and objectives, thereby honing their ability to pinpoint the core issue of the given case.

Why Develop the ICBL Model?

The ICBL (Inquiry-based Case-based Learning) model represents an advanced modification of the CBL (Case-based Learning) model, necessitating further development. Two primary reasons underscore the need for this development: (1) The advancement of the ICBL model aligns with the fundamental objective of mathematics education, which is to equip students with problem-solving skills essential for addressing real-world challenges. This capability is a critical component of the school mathematics curriculum, as it prepares students to tackle societal problems (Erlina & Purnomo, 2020; Purnomo et al., 2022; Sintema & Mosimege, 2023). Problem-solving activities should be integrated into classroom instruction to enable teachers to assess students' complex cognitive processes (Kadir, 2023); (2) The current trend in curriculum development is towards incorporating external assessments, such as those used in PISA (Programme for International Student Assessment), to enhance the learning process. Many students struggle with solving problems at levels 4 to 6, which are characterized by their complexity (Poddiakov, 2016). Consequently, the mathematics curriculum must evolve to include complex, real-world problems to better prepare students for practical problem-solving tasks, as evidenced by curricula in places like Singapore (Kaur, 2014). Nonetheless, most schools lack effective intervention strategies aimed at improving real-world problem-solving skills (Amukune et al., 2022).

Several limitations are associated with implementing the ICBL model in educational settings: (1) The resolution of complex problems may require more time, which may not always fit within the constraints of a rigorous curriculum; (2) Teachers might need to provide additional guidance and support, potentially limiting students' independence in their learning process; and (3) Managing complex problems may necessitate more sophisticated classroom management strategies, including organizing group assignments and facilitating effective discussions. To address these limitations, the following strategies can be employed: (1) Assist students in decomposing complex problems into smaller, more manageable components; (2) Promote collaborative learning by encouraging students to work in small groups, enabling them to support and learn from each other; and (3) Utilize educational applications or software to aid students in understanding challenging concepts.

Conclusion

The findings of this study indicate that students' Computational Mathematical Problem Solving (CMPS) abilities were significantly enhanced when utilizing the ICBL (Inquiry-based Case-based Learning) model compared to those using the conventional model. Additionally, students exhibited positive responses toward learning mathematics with the ICBL model. Based on these results, several recommendations are proposed. Firstly, educators should focus on enhancing the classroom learning process by emphasizing the development of students' complex problem-solving skills, incorporating the ICBL model. This approach aligns with the primary goal of the school mathematics curriculum, which aims to cultivate students' ability to solve complex mathematical problems, a skill crucial for success in international assessments such as PISA

(Programme for International Student Assessment). Secondly, given that the ICBL model is an innovative modification of the CBL (Case-based Learning) model, further research is needed to assess its impact on other mathematical competencies, including mathematical literacy, reflective thinking, and lateral thinking. Additionally, exploring the application of the ICBL model in fields beyond mathematics could provide valuable insights into its broader efficacy and potential benefits.

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Conflicts of Interest

The authors declare that there are no conflicts of interest associated with the publication of this manuscript. Additionally, all ethical considerations have been meticulously addressed and resolved by the authors. This includes, but is not limited to, issues related to plagiarism, intellectual property infringement, data falsification, duplicate publication, and redundancy.

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