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The complexity of preservice teachers doing an interdisciplinary statistics project

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Abstract

In this article, 30 preservice teachers' engagement in an interdisciplinary project about the settlement of Iceland by the Vikings is investigate. A qualitative research method was applied to examine photos that documented preservice teachers' group work over three days and their responses to an open-ended questionnaire about their learning outcomes. The preservice teachers' responses to the questionnaire suggested that they failed to recognize the statistical aspects that they had engaged with, such as averages, which could be identified in photographs of their group work over three days. The photographs showed that students had participated in the different cycles of the statistical enquiry model, at least to some degree, with one major lack being variation. These results have implications for teacher educators who want to support preservice teachers to adopt interdisciplinary projects in their future classrooms.

Keywords: Interdisciplinary Education, Landscape of Investigation, Preservice Teachers, Statistical Enquiry, Variation

Introduction

In the new curriculum in Norway (Utdanningsdirektoratet, 2019), implemented in 2020, interdisciplinary projects are promoted as a pedagogical approach that teachers should adopt. Although involving students in interdisciplinary projects requires teachers to be familiar with understandings of key aspects of projects, little research has been undertaken on how preservice teachers make sense of such projects. In this paper, we explore 30 Norwegian preservice teachers' participation in a week-long, interdisciplinary project which combined learning about the Viking settlement of Iceland with statistical enquiry. The data included both the preservice teachers' responses to a questionnaire at the end of the project and our interpretation of 170 photos taken during the group work. Though many of the preservice teachers stated in the questionnaires that they had not used statistical understandings, our analysis of the photos revealed that they had engaged in many aspects of statistical enquiry. By examining the data using the four dimensions in Wild and Pfannkuch's (1999) model for empirical enquiry, we discuss these contradictions and what they might mean for teacher educators.

In designing the interdisciplinary task, we purposefully chose "Landscapes of Investigations" (Alrø & Skovsmose, 2002) as our framework. Landscapes of investigation situate learning as action, in which a scene is set that provides opportunities for the students to ask questions and become part of an inquiry process. Landscapes of investigation start with students' previous understandings and do not have a predetermined outcome. Investigating the scene requires the communication between teacher and students to be explorative and dialogic. As teacher educators, we consider that landscapes of investigation can be achieved through interdisciplinary projects, which are promoted in the new Norwegian curriculum, implemented in 2020 (Utdanningsdirektoratet, 2019). As Rodrigues et al. (2017) noted, real life problems and situations are complex, but school students do not have opportunities to reflect on this when a curriculum delivers these ideas in a fragmented way. Williams et al. (2016) defined interdisciplinarity as integrating two or more academic disciplines into a teaching unit. Combining disciplines provides opportunities to explore problems that require more than procedural calculations and so are in alignment with landscapes of investigation.

Williams et al. (2016) provided examples of interdisciplinary projects that involved mathematics, generally statistics. One case study analyzed the use of statistical ideas connected to nutrition and physical activity by students from six classes. The initial question that the students were challenged with was, "Is stopping eating the right way to lose weight?". The students then refined the question, so they could answer it meaningfully as well as statistically correct. Both students and teachers found the question important and that this type of complex problem "provides opportunities to explore important concepts from mathematics and science" (Williams et al., 2016, p. 27). In Alrø and Skovsmose's (2002) terms, the students had accepted the invitation to the landscape of investigation and in so doing changed their ideas about mathematics and how it could be learnt.

Like the studies situated in schools, the few empirical studies about teacher education have focused on different aspects of interdisciplinary projects. As well, most research about introducing preservice teachers to interdisciplinary research remains at the level of principles or suggestions on how to do it (see for example, Corlu et al., 2014; Furner & Kumar, 2007).

Most interdisciplinary projects investigated the integration of mathematics with science teaching. For example, Adams et al. (2014) described 50 preservice teachers' integration of science and mathematics with social studies through the development of place-based activities that focused on the local environment. They found that "integrating a suite of novel experiences into methods courses can provide pre-service teachers with valuable experiences that will help them conceptualize and deliver similarly situated experiences for their future students" (p. 18). This suggests that preservice teachers are more likely to integrate interdisciplinary projects into their own teaching if they have experienced them themselves. Similarly, An et al. (2016)

described a project in which preservice teachers designed and implemented an interdisciplinary project that combined music with mathematics education. Their results were encouraging as most of the participating preservice teachers indicated that music as a pedagogical strategy allowed them to go beyond traditional mathematics instruction and allowed students to understand mathematics concepts in meaningful ways. Moreover, participants expressed that contextualizing activities could help students view mathematics from diverse perspectives (p. 31).

Nevertheless, in research on interdisciplinary projects, it has been found that not all the disciplines are supported to the same degree. Saçkes et al. (2012) investigated preservice early childhood teachers' self-efficacy in integrating mathematics and science in their teaching because of an intervention completed over eight weeks. The preservice teachers worked on projects to do with bird habitats and weather recording which involved collecting data and then planning teaching for young children. At the conclusion of the project, preservice teachers considered that science was pre-eminent in the interdisciplinary units with mathematics and other disciplines took sub-ordinate roles.

Other research found that preservice teachers and their students encountered difficulties in combining a biological context with mathematics and in posing relevant problems (Hansen & Hana, 2015). Reflecting over their role as future teachers, the preservice teachers noted that it could be difficult to choose problems that students could solve with mathematical methods within their reach and be related to the mathematical topic that they were supposed to learn. However, when modifying the problems so that these requirements were met, school students risked losing ownership of them (Hansen & Hana, 2015).

Thus, preservice teachers' work on interdisciplinary projects can provide benefits, both to their understandings about mathematical concepts and for planning of their future teaching. At the same time, preservice teachers face challenges, like those of school students, about ensuring that the disciplines have equal weight, about ensuring that the mathematical concepts remained visible, and in formulating appropriate problems.

Da Ponte and Noll (2018) stated in an overview of the results of professional development work on statistical teaching, that although teachers tended to improve their content knowledge, they struggled to model statistical investigations in their classrooms. Given that our mathematics teacher education program promotes landscapes of investigation, one of the purposes of our research was to find out how preservice teachers at the end of their second year of mathematics education (third year of their study program) engaged in an interdisciplinary project connecting statistics with social studies. The analysis of the photos and questionnaires was guided by the following research question: how do preservice teachers engage in statistical enquiry when working with an open, interdisciplinary project blending social science studies and statistics?

Statistical Thinking

Following the research of Rodrigues et al. (2017) on preservice teachers' interdisciplinary projects, we adopted Wild and Pfannkuch (1999) four-dimensional framework for statistical thinking in empirical enquiry to analyze our data. The four dimensions are the investigative cycle, types of thinking, the interrogative cycle, and dispositions. In an empirical study such as ours, all four dimensions are considered simultaneously. The multi-dimensional nature of the framework provides opportunities to explore the preservice teachers' interactions with the project from different perspectives. In the following sub-sections, we describe each of the dimensions.

The Investigative Cycle

The first dimension is the investigative cycle. The investigative cycle is also called the PPDAC model after each of the five phases that learners need to engage with in their investigations:

1. The *problem formulation (P)* which is needed for understanding the dynamics in the system, to choose the parameters that are to be taken into consideration, and to define the problem.
2. The *planning (P)* of the investigation is about how to collect data and how to manage the data.
3. The *data collection and data management (D)*, is about the storing and organizing of data in a meaningful way, including data cleaning, i.e., the removal of data which will not be used further in the investigation.
4. The *data analysis (A)* including planned and unplanned analyses and hypothesis generation.
5. In a last phase, *conclusions and communication (C)*, the analyzed data are interpreted, and the results communicated using appropriate representations.

We use the investigative cycle to structure our results. The preservice teachers' engagement with the project are described in relationship to the five phases, with information about the other three dimensions (types of thinking, the interrogative cycle, and dispositions) discussed regarding each of these phases.

Types of Thinking

The second dimension is the types of thinking that are fundamental to a statistical analysis. Wild and Pfannkuch (1999) subdivided these into general types necessary in all problem-solving activities and types of thinking required specifically for statistical analyses. The general types are:

1. Strategic thinking: what to do? How to do it? Identifying subtasks; anticipating problems; awareness of constraints (lack of knowledge, limitations of available data, preconceptions, ability; available time or money)
2. Seeking explanation
3. Modelling

4. Applying techniques: following precedents; recognition and use of archetypes, i.e., mapping a new situation on a problem that already has been solved; use of problem-solving tools

The types of thinking fundamental to statistical thinking are:

1. Recognition of the need for data; recognition of inadequacy of personal experience and anecdotal evidence.
2. Trans numeration: change of data representation to gain understanding; expressing and re-expressing data, experimenting with different representations to gain insight.
3. Consideration of variation: notice and acknowledge variation that occurs through changes in the system and/or due to data collection processes; deal with variation in an appropriate way: ignore/allow for/control variation; understand and distinguish correlation and causation; identify response variables; search for explanatory variables; probabilistic and deterministic thinking modes and their impact on the quest for causes.
4. Reasoning with statistical models: deal with variation (explained and unexplained variation; random variation (due to random sampling).
5. Synthesis of context knowledge and statistical knowledge.

For each phase of the investigative cycle, we identify which type or types of thinking were applied by the preservice teachers.

The Interrogative Cycle

From their analysis of students engaged in statistical projects, Wild and Pfannkuch (1999) identified five thinking stages that were continuously in operation, at both the macro and micro level. These thinking stages involved interrogating and reflecting on the processes of solving the different aspects of their statistical problem. The five stages are:

1. *Generate* which is to do with learners brainstorming or imagining scenarios about the different aspects of investigating the problems.
2. After generating possibilities, there is a need to *seek* information about these possibilities which could involve recalling information, including from earlier investigations.
3. Using the found information, learners then need to *interpret* it so that a variety of connections between the information gained from the seeking and the problem itself are identified. At this point, it is not about judging, but about making connections.
4. As connections are made through seeking information, learners also critique the information and ideas that they are interpreting against previous experiences (internal reference points) and other people's input (external reference points).
5. The final aspect is *judge* which is about determining the usefulness of the various ideas generated, found, interpreted, and criticized. This provides possibilities for not just for deciding whether to use or dismiss an idea as relevant, but about how to utilize the most relevant parts of an idea.

Dispositions

Wild and Pfannkuch (1999) described dispositions as personal qualities that support thinking, including in relationship to statistical problem solving. Dispositions were considered to be dependent on the person's engagement with the problem and, thus, were not innate abilities. They also considered that some dispositions such as skepticism came from experience and so were possible to learn. From interviews with statisticians and backed up by observations of students, the important dispositions that they identified, were:

1. Curiosity and awareness
2. Engagement
3. Imagination
4. Skepticism
5. Being logical
6. A propensity to seek deeper meaning
7. Openness
8. Perseverance

In our project, preservice teachers' dispositions were identified based on their answers to the open-ended questions in the questionnaire handed out at the end of the group work.

In Rodrigues et al.'s (2017) study, the disciplines of mathematics and science were integrated through incorporating technology into projects to do with healthy schools, for preservice teachers to improve their statistical understandings. Although the implementation of the project was tightly controlled by the researchers, it was considered that the preservice teachers had shown many of the aspects in Wild and Pfannkuch's (1999) four-dimensional framework of statistical thinking, such as planning their data collection and analysis, feeling the need for data collection, or displaying curiosity. Inspired by Rodrigues et al. (2017), in our research, we aimed to describe the preservice teachers' engagement with an interdisciplinary project that blended statistics with social studies. To do this, we identified the most common types of thinking and dispositions, so that we could improve how we presented such projects in our teacher education by strengthening the types of thinking and dispositions that were found less frequently.

Methods

This qualitative study investigates Norwegian preservice teachers' engagement with statistical enquiry in an interdisciplinary project during a one-week study tour to Iceland in April 2017. Thirty preservice teachers, enrolled in a teacher education program for grades 1-7, were at the end of 2 years of courses in mathematics teacher education (60 ECTS) and their third year of study. They were specializing in mathematics teaching and focused on problem solving but had not previously been engaged in an interdisciplinary project. The data collection included observation of group work, photos documenting group work, and preservice teachers' answers to a questionnaire.

The statistical enquiry task (see Appendix A) was interdisciplinary in that it required the preservice teachers to consider the social studies topic of Vikings, usually taught in Grade 5 in Norwegian schools. The task was open-ended, as are most real-world problems, and so satisfied the requirements for a landscape of investigation (Alrø & Skovsmose, 2002). Although more complex than most of the problems that the preservice teachers had been introduced to in the rest of the course, it was anticipated that they would recognize it as an opportunity to engage in a landscape of investigation and provide insights for their own teaching in mathematics classrooms.

The preservice teachers were introduced to the project by playing a board game about Icelandic settlement. From playing this game, nine randomly made working groups were formed, for the investigation, with each having 3-4 Norwegian members. The preservice teachers were sent a draft version of the task, the week before they went to Iceland. The Icelandic teacher educators adapted the task to include the creation of a story about the experiences of some settlers, which was provided to the groups of preservice teachers on the second day. They also organized two experts to give lectures about what was known about Viking settlers which provided information about what should be considered regarding the project. As well the preservice teachers visited the Viking settlement museum in Reykjavik to obtain more background into the social studies aspects of the project. The groups worked on the task for 3 days, presenting their findings at the end of the third day in a poster which they presented to others. On the final day, the Norwegian preservice teachers were invited to complete a questionnaire containing 11 open-ended questions that could inform the researchers about the preservice teachers learning outcome. The questions addressed the preservice teachers' mathematical and statistical understandings, their cultural understandings about Viking settlement voyages and the link between mathematics and culture together with the learning potential that this link might have for the preservice teachers' future students. All the data were anonymous, so the responses could not be connected to specific groups.

The task was modified from one used in Trinick and Meaney's (2017) research on Māori preservice teachers' engagement with a statistical enquiry model around the settlement of New Zealand. The task description included statistical hints, such as identifying relevant variables, such as size, surface area, weight, age group mix, travel time, and supplies, which might affect the modelling of the situation. The stages of Wild and Pfannkuch's (1999) PPDAC model were also included in the description.

This article focuses on the work of the 30 Norwegian preservice teachers, which was documented through 170 photos of the group work and their individual answers to a questionnaire. It was decided not to audio-record the group work so as not to interfere with the developing work relationships between the Norwegian and Icelandic preservice teachers. We acknowledge that the photos and questionnaire responses provide insights into only some aspects of the project process. Nevertheless, the photos provide relevant details about the progression of each group's work on the project, while the questionnaire responses provide insights into the preservice teachers' views of interdisciplinary projects.

The photos were taken of the preservice teachers' preliminary handwritten notes, computer screens from their internet searches, models of Viking ships, pages of books used as data sources, and the posters created by each group. Approximately 19 photos from each group' progress were taken over the 3 days by one of the authors, who visited each group regularly, photographing what was happening at the time and talking to the preservice teachers about what they were doing. At times, this included challenging the preservice teachers to consider specific aspects of the interdisciplinary, such as the calories that rowers might need for the journey and how this could be provided, when they seemed to be making unrealistic decisions.

The photos of each group's work were arranged in chronological order. Then, the photos were classified according to which of the stages of the PPDAC model were identifiable, problem formulation, planning, data collection and management, data analysis and conclusions (Wild & Pfannkuch, 1999). In the following sections, we describe how we determined how a photo was classified as one stage or another.

Analysis

The PPDAC model provides the structure for the presentation of the results. In the following subsections, how these were identified in the photos is described first. Then, how the photos and the responses to the questionnaire were analyzed, based on Wild and Pfannkuch's (1999) questions, are described to provide information about the other dimensions of the framework.

Problem Formulation (First Phase)

In the investigative cycle, preservice teachers first needed to formulate and recognize statistically relevant problems. This was considered important for two reasons: to support their own problem solving (Lowrie, 2002); and for their future teaching (Hansen & Hana, 2015). Strategic thinking, from the second dimension about types of thinking (Wild & Pfannkuch, 1999), is important for identifying a problem and the kind of data needed to solve it. From the interrogative cycle, defining the problem was connected to generating possibilities and the dispositions of curiosity and awareness, with imagination being important for this process.

The preservice teachers showed their engagement in defining a problem in similar ways, such as writing down ideas and checking information on the internet. Figure 1 provides an example from one group. The preservice teachers were gathered around handwritten notes, which began with the name of the main character of their story, Ketilbjorn. However, the details of the story seemed not to be settled because questions about the plot in their notes included, "Why did they leave?" and "How long did it take?". These questions refer to both the potential narrative and aspects of time and distance that could be part of a statistical enquiry. The notes also included a list of items to be taken on the ship. Several points had question marks, such as the words for gender, age, livestock, eat and drink, suggesting that they were not yet finalized. On a separate sheet, the group had begun calculating the weight and dimensions of the boat, but there was no evidence that the collection of data had started, nor that a systematic plan for data collection had been made. Therefore, we considered this photo showed that the group was working on the first phase in the cycle, determining the problem and identifying relevant parameters.

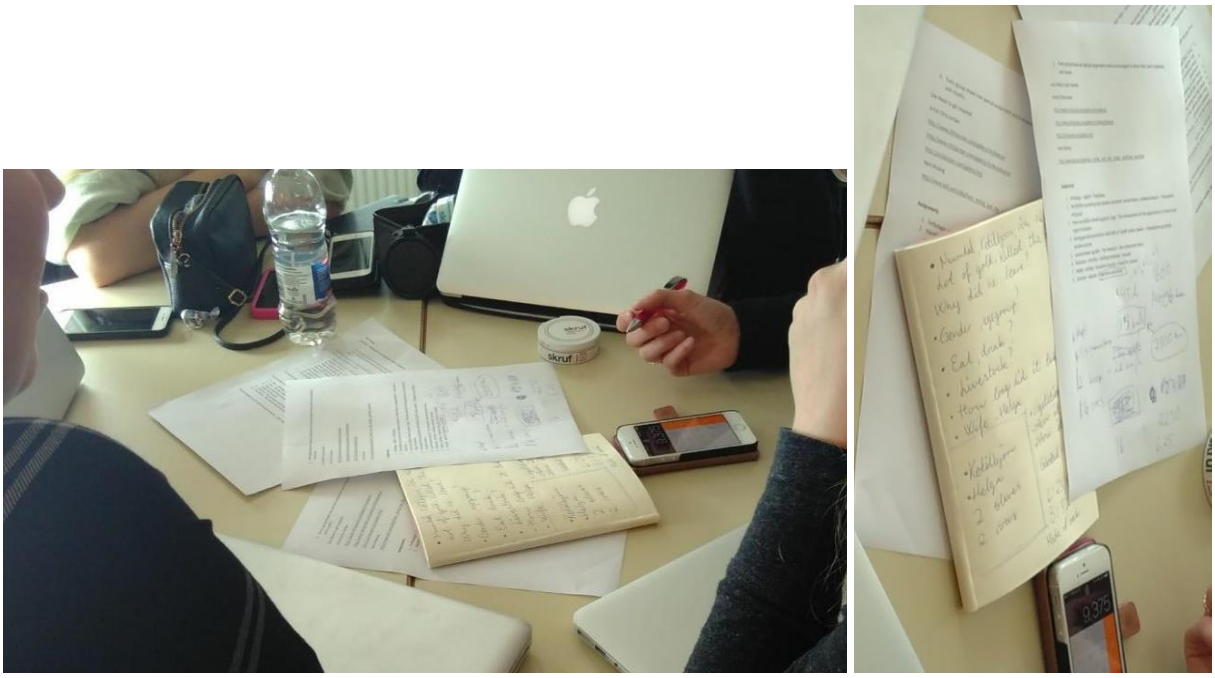

Figure 1. Defining the Problem

In contrast, the responses to the questionnaires indicated that many preservice teachers considered that it was difficult to define a statistical problem in their project. Several wrote that they learnt a lot about cultural and historic aspects, but felt they needed more instructions about how to include statistics. Preservice Teacher 13's (S13) comment was typical, "The task could have been a little clearer in which mathematical aspects should emerge. I have not completely seen the mathematical part." In contrast, the history and culture of the Vikings often fired their imagination and curiosity. For example, in responding to a question about what they had learnt about culture and history when doing the investigation, S14 wrote, "How people travelled, what they had to take into account, what could happen, what was wise to bring along and how they lived long ago", while also stating they wanted more information about the statistical aspect of the project. Although the preservice teachers could enact their dispositions to become involved in what they considered to be the social studies aspects of the investigation, seen in the photos of this first stage, many struggled to recognize these also included statistical thinking. The weights and distances evident in these photos from early in the investigation process seemed to only be considered as measurements to be calculated, not as statistical data that could be operated on to determine different possible outcomes. This suggests that some preservice teachers lacked the statistical imagination, one of the dispositions identified by Wild and Pfannkuch (1999), needed to generate a wider range of possibilities for investigation, connected to the interrogative cycle.

Nevertheless, in the questionnaire responses, some preservice teachers did state that they viewed the project as a statistical investigation, "one can use other people's statistics to calculate different situations and challenges (e.g., how much food you need)" (S30). The preservice teachers were also generally positive about how interdisciplinary project could support students' motivation to learn mathematics. S22's comments were typical, "Because there is a lot of mathematics in everyday life, to make mathematics fun/exciting and problem solving". This suggests that if the preservice teachers used statistical imagination, they could recognize that they were engaging in a statistical enquiry and formulate problems that could be solved using statistical thinking.

From Problem Formulation to Data Collection – Moving Back and Forth Between First and Second Phase

In the second stage of the PPDAC model, the preservice teachers were expected to seek relevant information. Seeking information is also important in the interrogative cycle and is about the recognition of the need for data, one of the types of thinking for statistics, and a disposition to want to seek deeper meaning (Wild & Pfannkuch, 1999). In analyzing the photos and the responses to the questionnaires, it seemed that the preservice teachers were aware that their identification of relevant information affected their problem posing, but saw their searching as being about social studies, not statistics.

It was difficult to identify photos that clearly showed the preservice teachers involved just in the planning stage of the investigative cycle as they seemed to combine it with data gathering. Hansen and Hana (2017) pointed out that problem posing within modelling often requires several attempts until a problem is found that is mathematizable and solvable with mathematical methods that are within reach of the learner. The photos suggested that the preservice teachers worked simultaneously in several stages of the investigative cycle and that the collected information might have influenced their final problem formulation. For example, the preservice teachers in Figure 2 were studying a map to determine the length of the journey. However, as there is no evidence of storing this data, it could also be about planning the data collection.

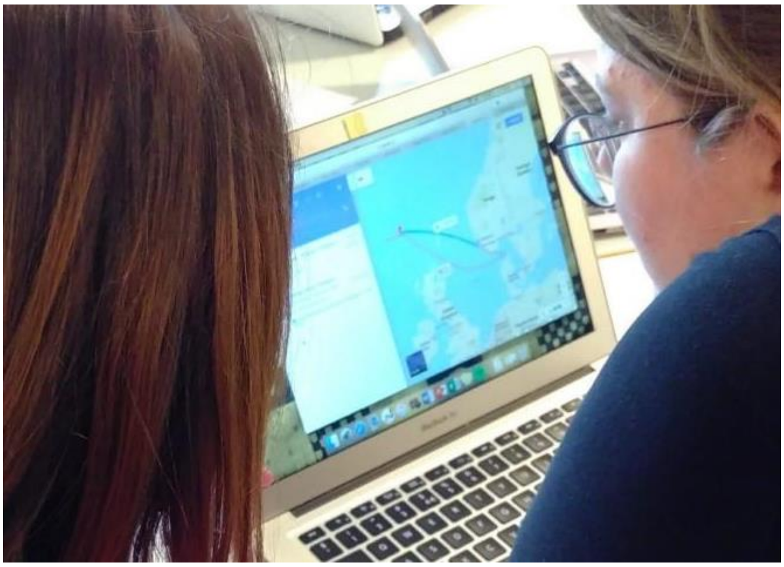

Figure 2. Travel Routes to Iceland

Photos of other groups from the first day showed similar combinations of problem formulation, planning and data gathering. For example, in Figure 3, there is a copy of the task, along with handwritten notes that included a list of ship measurements and items, both of people and things, to be taken on board. These items were young people, slaves, animals, seeds, hay, and items that could be obtained during the journey, such as fish. The open book suggests that the group were identifying relevant information and what new information they might need. The measurements of the ship in the handwritten notes were those of the Gokstad ship, a burial Viking ship found in south-east Norway. The recording of this information suggests that the data collection had already begun.

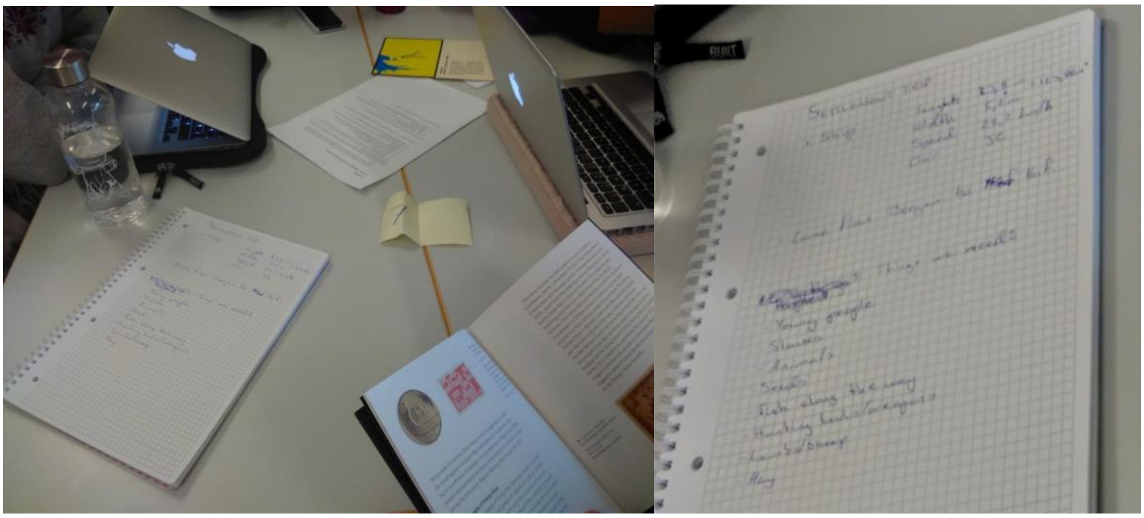

Figure 3. Ship Dimensions and Sources of Data

In these stages of the enquiry cycle, we anticipated that the preservice teachers would incorporate uncertainty in their planning and data gathering. This is because aspects of the investigation, such as the time the journey took, would have a range of answers, requiring the preservice teachers to consider probability in everyday settings. This aligns with Wild and Pfannkuch's (1999) statement that modern statistical thinking is about decision making under uncertainty.

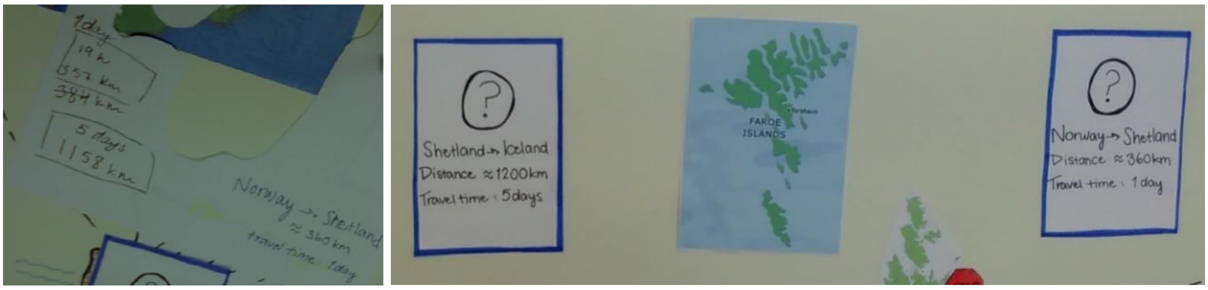

Figure 4. Travel Time Calculations and Presentation in the Poster

Instead, many groups seemed to reduce the uncertainty very quickly. Figure 4 shows that one group chose a rate for travelling for their calculations, but there is no information about how, for example, the suggested 19 hours of daily rowing time was chosen. Although Vikings travelled to Iceland only during summer when they had almost 24-hour daylight, the lack of reality in having Vikings row at close to Olympic-rowing-record speeds for 19 hours was not raised in the notes. Even if the preservice teachers actually assumed that the Vikings sailed to Iceland, the average distance is significantly more than the average travel distance of 120 km/day that has been suggested for the Viking cargo ships, knarr, used in settlement journeys (see https://en.wikipedia.org/wiki/Viking_ships). In the notes in Figure 4, the travel time to the Shetland Islands was approximated using a rounding up from 357 to 360 km but did not include reflection on what might have influenced the travel time. A slower travel rate for the trip from the Shetland Island to Iceland was included, although with no indication about why this was the case. It may be that the spoken discussions, which we did not capture, may have provided more insights into this.

If uncertainty seemed to be raised, it was to do with probability issues. Some preservice teachers noted in their responses to the questionnaire that they worked with probability at a general level. S9's response about learning probability in the project was fairly typical, "We have used probability in that we have calculated various challenges that could have arisen if we had sailed from Norway to Iceland". However, we did not find any traces in the photos of the preservice teachers defining variables, that included variation, in the problem formulation or planning of the investigation, suggesting that this was not part of their thinking about the project.

This is reinforced by the fact that many preservice teachers wanted more support to recognize the probability possibilities in the project, specifically how to use the probability calculations they had worked on in their teacher education class. In answering a question about what they would like to learn more about in relationship to statistics and probability, S23 wrote, "We could get specific topics where we should use probability and other topics, so we used it more clearly". The preservice teachers did not seem to recognize the need for them to have dispositions to do with curiosity, awareness, and openness (Wild & Pfannkuch, 1999). This could be because their statistical imaginations were not stimulated by this project. Their need to have the connections to statistical and probability aspects identified clearly suggests that they did not equate this task with entering a landscape of investigation (Alrø & Skovsmose, 2002). Yet, when they did see relationship to probability, they saw this as being linked to school students being creative. Having acknowledged that they had calculated the probabilities of different events happening, S30 responded to the questionnaire question about how an interdisciplinary project could be used with school students by stating, "How to get students to figure out what they need to work out independently. How to teach them to be creative in their mathematics." This indicates that they considered being creative in mathematics to be important, but perhaps struggled to know how to actually implement it in their own projects and perhaps also in their future work with school students.

Data Collection, Data Management, and Data Cleaning (Third Phase)

Data collection, management and cleaning relies on statistical thinking, particularly in relationship to variation and an openness to the investigation. The gathering of relevant data is expected to involve interpreting, criticizing, and judging the data according to the interrogative cycle. However, the preservice teachers mostly seemed to want to determine one correct answer, rather than considering different possibilities that could affect the outcome. Focusing on calculating the correct answer seems more closely related to the exercise paradigm, than to the landscape of investigation (Alrø & Skovsmose, 2002).

Unlike the New Zealand preservice teachers who surveyed relatives in a similar project (Trinick & Meaney, 2017), the Norwegian preservice teachers predominantly relied on the internet to gain information, as well as the lectures on Viking settlement. In many of the photos, preservice teachers can be seen using their mobile phones and computers, often on webpages with information about Vikings and Viking ships. Other sources which were not related to Vikings were consulted, as well. For example, one photo of a website showed horse trailers and their payloads, perhaps used to estimate the weight of a horse.

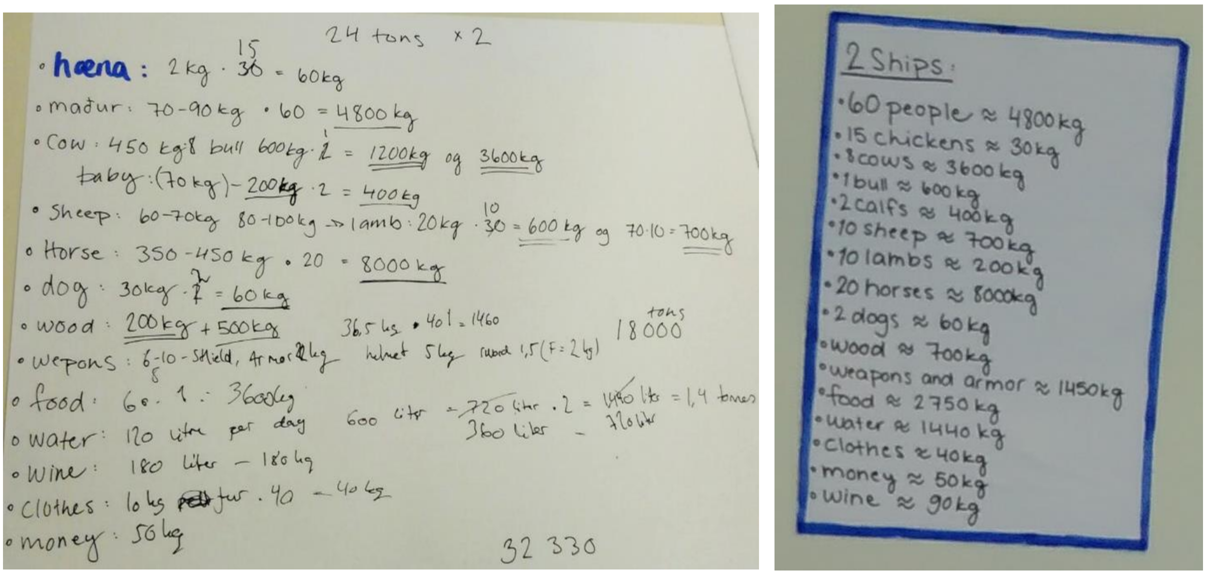

Figure 5. Calculations and Data Tidying along with the Poster Version of the Amounts

Figure 5 is a typical example from the photos of data management and data cleaning in that it shows a list of items to be taken on the ship together with estimates of their weights, both from the work-in-progress and then from the final poster. In the work-in-progress, several numbers were crossed out and there are some different values, suggesting some consideration of variation. The first two lines include the sorts of common changes to the collected data seen in other photos:

$$\text{Hæna (egg-laying chicken):}\ \ 2\text{kg} \bullet \overset{15}{3\emptyset} = 60\text{kg}$$

In this line, 30 is changed to 15 although the change in combined weight to 30 kg is only evident in the poster presentation. The preservice teachers probably made the change either because 30 hens could have been too many for the allocated space on the ship or there was a need to reduce the overall weight of items on the ship.

In the next line, the preservice teachers seemed to have used 80 kg as the average of 70 and 90 kg in their calculation. The preservice teachers could identify the average variation in how much a human (at this time) weighed, but only by simplifying it down to one figure:

$$\text{Madur (human): } 70 - 90\text{kg} \cdot 60 = 4800\text{kg}$$

This can be seen in the final poster, where this variation was absent. On the same poster, the suggested supply of water was stated to be 1.5 m^3. This amount of water is related to the 60 embarking humans, who would need 2 liters of water per day. The water needs of the other living creatures on board the ship seemed to have been forgotten. Varying the number of animals would have substantially affected the weight of the cargo, not just because of the animals' own weight, but also because of their food and water needs on the voyage.

In our data, there is no evidence to the preservice teachers considering the impact of averages or a range of amounts in their calculations. In the photos, most groups used a specific value for each item. This could have been because only an average value appeared in the internet information. In the responses to the questionnaire, the preservice teachers wrote about organizing of the data as a form of data management, for example S23 wrote, "Systematized the information we found, made a list of the different things to bring on the ship". However, they did not discuss using a critical approach as required in a landscape of investigation (Alrø & Skovsmose, 2002). Although Wild and Pfannkuch (1999) highlighted variation as a central concept in statistical thinking, if variation in the information was recognized, its management did not include considering how it could have affected the outcome of the investigation. For the Vikings who settled Iceland, variations in what was brought would have affected whether or not they had enough food for the journey and could survive the first growing season, as well as whether the boat would have been stable during the journey. These considerations were discussed in the two lectures provided when in Iceland and so it was surprising for us, as teacher educators, that the preservice teachers did not raise these issues. This suggests there is a need discuss with the preservice teachers the critical thinking involved in statistical enquiries.

In the interrogative cycle, it is expected that learners would take a critical stance towards the data, but there was little evidence in the photos or the questionnaire responses that this occurred. It may be that finding the weight of humans and animals or the speed of ships in an historic context was outside of the preservice teachers' previous experiences. Wild and Pfannkuch (1999) highlight that all modelling is a simplification of the reality. The use of modern values might have been such a simplification, as precise historic data were not easily available. Wild and Pfannkuch (1999) also described the problem-solving strategy of mapping a new problem onto a problem that had previously been solved, known as problem archetypes, as being particularly efficient. In later discussions with the preservice teachers, there was some agreement that their previous experiences with average amounts may have been a problem archetype, which worked against them considering the impact of variation on their statistical enquiry. This may have led to the preservice teachers not considering the need to incorporate a critical stance into their data management.

Perhaps unsurprisingly, in the responses to the question about the advantage for school students in doing interdisciplinary projects, the preservice teachers made few references that could be linked to learning about data management. Sometimes there was mention of using other methods, but this seemed mostly to do with the calculations. S12 stated "engage students and such that they can see the benefit of being able to do math (calculate)", either associating doing mathematics in general with doing calculations or not being able to associate more complex mathematics to this particular task. As the task on its own did not support preservice teachers to understand how variation could affect their investigation, we, as teacher educators, need to highlight more clearly the importance of preservice teachers having a critical stance both regarding the kind of data that they decide to use but also about how their own previous learning may affect their expectations in tasks.

Data Analysis (Fourth Phase)

Data analysis deals with data exploration and forming hypotheses. Wild and Pfannkuch (1999) coined the expression trans numeration to describe how a change in data representation could give insights into the problem being investigated. Although there were several examples of information being presented in new ways, which could potentially be examples of trans numeration, it is unclear if these changes led to new insights.

In the photos, it seemed that often a transformation in representation was merely a change in medium. Several groups conveyed information with tables, graphs, models, and manipulatives. In Figure 6, one group showed the number of Icelandic hens in Iceland at their peak, 30-40 000 hens, compared to the present time, 3-4000 hens, with a representation built of yellow and red unifix-cubes. Although this presentation made it easy to interpret the data, it was not clear if this transformation produced new statistical insights. In the questionnaires, several preservice teachers mentioned that data representation was the statistical topic that they learned most about, for example they mentioned "learned to make tables" (S3), "used concrete materials to present statistics" (S8) for representing the data. Descriptive statistics traditionally receives a lot of attention in Norwegian school (see for example, Grønmo & Onstad, 2009), therefore the preservice teachers may have had previous experiences with various techniques to represent data and so felt comfortable representing information in these ways.

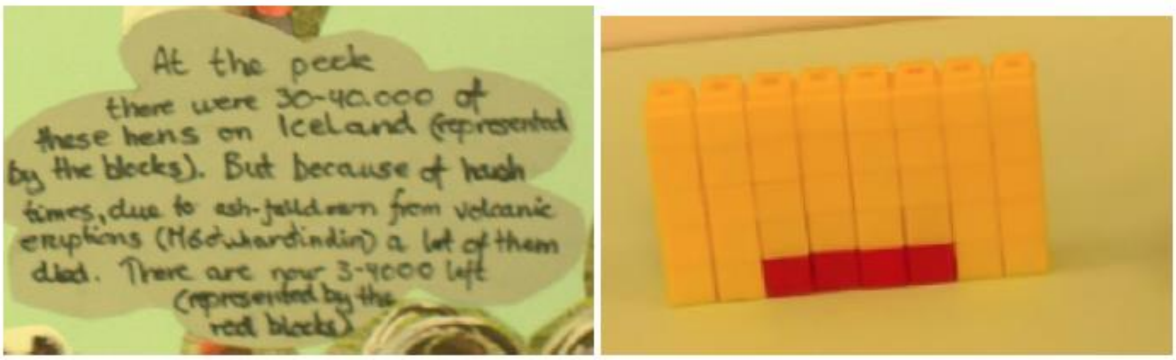

Figure 6. Icelandic Hen Representations

In some cases, the data analysis was based on previous experiences from science and geometry, rather than statistical considerations. Several preservice teachers mentioned they used physics to determine speed of the Viking ships. In one photo, a group showed an estimation of the payload of a ship:

$$20200 \quad - \quad 8855\ \text{kg payload}\ 11{,}345\ \text{kg}$$

It is not clear how the number 20200 was calculated, but other photos from this group included a sketch of a boat and a cube alongside the expression 1000 liters. It may be that 20200 represented the amount of water displaced by a boat, similar in size to the Gokstad ship. Previous experiences from science education may have supported the preservice teachers to use the data about the size of the Gokstad ship to determine the payload of their ship to settle Iceland.

If the preservice teachers considered that students should focus on calculating one correct answer, then gaining insights from trans numeration would not be seen as supporting this focus. To change this view, we, as teacher educators, need to provide experiences with trans numeration so that the preservice teachers are engaged in explicit reflections about what this contributes to doing interdisciplinary projects involving statistics.

Conclusions and Communication (Fifth Phase)

The last stage of the PPDAC model is about communicating the results, including considering how they and the new ideas in them should be interpreted. This stage can involve a range of thinking types and dispositions. All nine groups presented their results in posters as requested in the task description, with all but one group including a story about a group of settlers. One group only included a graph about the growth of population in Iceland after the settlements which they had copied from a book. The other groups provided conclusions about the journey, such as length of journey, travel time, people in the settler group, amount of food, livestock and other items like money, weapons, and clothes. The results were presented in the form of: texts about a group of settlers (7 groups); tables or lists of items to be taken on the ships (5 groups); paper models of a Viking ship (4 groups); time line of events (2 groups); graph of population growth (1 group); map showing size of settlements in Viking times compared to the situation today (1 group); and representation of amounts by using manipulatives (2 groups).

In most posters, only the final list of items to be taken on board were provided. A few posters showed some information about the calculations (see for example, Figure 7). The calculations showed evidence that this group of preservice students were using the disposition of being logical, within the confines of the problem that they had set themselves. Having seen the problem as one of getting the correct answer, it was logical for them to provide information about how they obtained that answer as a justification for their results. This group of preservice teachers also used average speed to estimate the time it would take to travel to Iceland. They made a model of a ship which was divided into strips (see Figure 7), with each strip indicated the space one adult would have onboard.

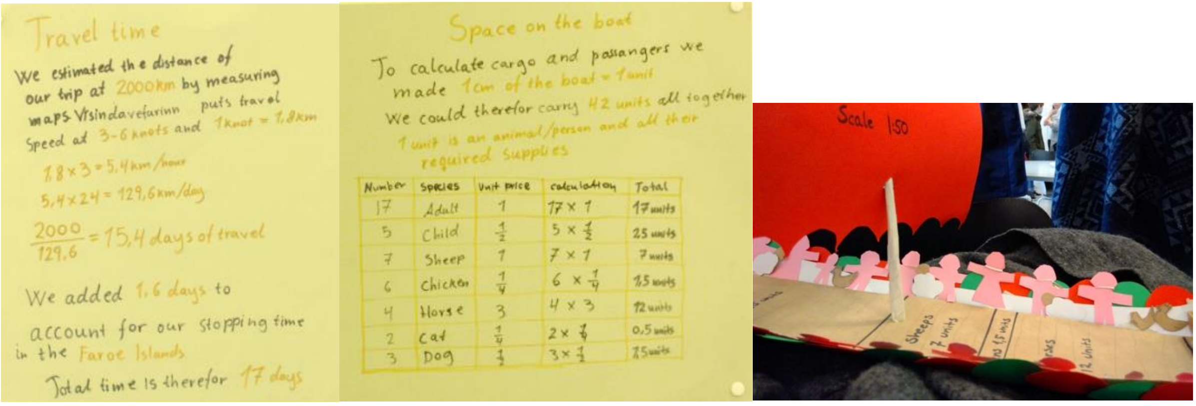

Figure 7. Use of Averages

In the responses to the questionnaire, some preservice teachers wrote that they were focused on ensuring that their calculations were accurate, “It would also have been good to have a little more time to work on the project, so that the calculations become a bit more detailed and concrete” (S27). It seemed that the preservice teachers considered that justifications of the calculations were sufficient, and they did not need to reflect on the appropriateness to the task requirements. This suggests that the final stage of the interrogative cycle, judge, did not move beyond determining whether the calculations were correct or not. As well, although they had the disposition to be logical regarding those calculations, they did not show a disposition towards “a propensity to seek deeper meaning” (Wild & Pfannkuch, 1999). As teacher educators, we need to highlight the importance, in statistical enquiries, of reflecting on how the outcomes of an investigation answer the original question. This, of course, links back to having the skills to pose interesting mathematical or statistical questions.

Discussion of the Results from the PPDAC Model

From our analysis of the photos and the questionnaire results, we have shown that the preservice teachers were engaged with some aspects of the four-dimensional framework for statistical thinking in empirical enquiry (Wild & Pfannkuch, 1999). However, the preservice teachers’ interpretation of the task was more in alignment with the exercise paradigm, than a landscape of investigation (Alrø & Skovsmose, 2003) and was about social studies and history, rather than statistics. It is, therefore, important to determine what might have caused the difficulties many preservice teachers had in establishing connections between disciplines. Although Saçkes et al. (2012) and Hansen and Hana (2015) found similar issues with preservice teachers focusing on other discipline knowledge and struggling to formulate their own problems, our results provide more nuanced insights which can support teacher educators to better support preservice teachers to engage in landscapes of investigation and recognize statistical aspects in real life or even historical settings.

The preservice teachers described, in their questionnaire responses, that the main purpose of interdisciplinary projects was to motivate students to learn mathematics. However, by making explicit the statistical questions that the students had to answer, as many preservice teachers requested that they themselves needed, then this motivation was likely to be lost as the tasks would be more in alignment with the exercise paradigm. Also, by focusing only on motivational aspects, the potential of using an interdisciplinary approach to provide opportunities to students for a deeper conceptual understanding of mathematics and statistics is neglected.

Thus, an area that teacher educators should focus on was engaging the preservice teachers in identifying relevant problems. Although these preservice teachers had experiences of this in their two years of mathematics teacher education, it seemed that this scenario was more open and the links to social studies and history provided ways to view the project, which minimized a focus on the statistics. Although many in their questionnaire responses could see the value of interdisciplinary projects for their students, they may find it difficult to support them to formulate appropriate problems that required statistical enquiry skills. This suggests that teacher educators need to discuss with preservice teachers how they would work with school students to develop relevant problems to investigate, which could support them to understand the difficulties that they themselves had in formulating such questions.

Experiences with previous statistics tasks seemed to have affected the preservice teachers' views on variation in the data. Variation was either not considered at all or only in superficial ways. Although Wild and Pfannkuch (1999) identified variation as being fundamental for understanding statistics, earlier research has shown that teachers often struggle with how it affects statistical problems (Sánchez & Garcia, 2008) and, thus, were likely to struggle supporting their own students to make sense of it in real-world contexts (Pfannkuch, 2008). Similarly, Rodrigues et al. (2017) found that the preservice teachers involved in their inter-disciplinary project struggled with utilizing understandings about the mean. Our research elaborates on this earlier research by indicating that the lack of utilizing variation seemed to be connected to views about interpreting, criticizing, or judging the data, which are important components of the interrogative cycle (Wild & Pfannkuch, 1999). To support the preservice teachers' understandings about how variation in the data, away from the mean, might affect their outcomes, teacher educators would need to highlight the value of interpreting, criticizing, and judging the data, beyond a focus on identifying the right calculations.

There would also be a need to raise discussions about the kinds of dispositions needed for statistical investigations (Wild & Pfannkuch, 1999). Although the preservice teachers showed some dispositions, they did not seem curious or imaginative about how statistics could be utilized in their investigation. Yet the preservice teachers used these dispositions regarding the non-statistical aspects about their chosen main character, their family and journey. Hansen and Hana (2015) stated that mathematical modelling problems that include other subjects can sometimes lose their link to mathematics. However, our results suggest that this potential loss was connected to preservice teachers' views of statistics as something which did not need them to be curious or imaginative. Thus, as teacher educators, we need to provide more opportunities for preservice teachers to experience curiosity and imagination in statistical investigations, which include reflective discussions about why such dispositions are important in interdisciplinary projects.

Conclusion

By identifying how different aspects of the four-dimensional framework (Wild & Pfannkuch, 1999) were connected, we have been able to extend earlier projects which highlighted similar issues in interdisciplinary projects, such as only one subject being valued or being valued more highly than the other subject. For example, it seemed that by reducing the challenge to do with statistical aspects, such as working with variation, most of the groups of the preservice teachers were able to miss recognizing the aspects of Wild and Pfannkuch's (1999) four-dimensional framework of statistical thinking that they had in fact engaged with. This contributed to them only seeing the project as being about social studies and history.

The new Norwegian curriculum, introduced in 2020, requires teachers to implement interdisciplinary projects (Utdanningsdirektoratet, 2019) and previous research has outlined the advantages of such an approach for students learning mathematics (see for example, Adams et al., 2014; An et al., 2016; Corlu et al., 2014). This approach is also in alignment with Alrø and Skovsmose's (2002) landscapes of investigation, which has been part of the mathematics teacher education program for many years at our institution. However, as the results of this research show, implementing interdisciplinary projects is challenging, including at the teacher education level. Although some of the preservice teachers accepted the invitation to participate in a landscape of investigation, that this interdisciplinary project offered, many indicated that they wanted a less-open task, where the statistics component was clearly outlined.

Our analysis of the responses to the questionnaires and the photos of the group work gives insights into how different aspects of Wild and Pfannkuch's (1999) four-dimensional framework of statistical thinking need to work together if a landscape of investigation offer is to be taken up in an interdisciplinary project. The preservice teachers' dispositions and types of thinking seemed to have affected their possibilities for identifying potential statistical problems. The preservice teachers recognized the need to find relevant information for conducting their investigation, but they did not interpret, criticize, or judge that information regarding the context which might have alerted them to the relevance of variation in their projects. It seemed that many different issues coincided and reinforced each other to limit the possibilities for entering a landscape of investigation.

These insights are important for teacher educators to provide appropriate interdisciplinary tasks that support preservice teachers to recognize possibilities for statistical investigation. In mathematics teacher education courses, this would include discussing the complexities involved in statistical investigations, particularly about future work with school students. Teacher educators also need to provide preservice teachers with experiences and reflective discussions about designing and implementing interdisciplinary projects which invite students into landscapes of investigation. As Wild and Pfannkuch (1999) four-dimensional framework for statistical thinking in empirical enquiry provided insights to us as teacher educators, it may well also provide insights to preservice teachers regarding their future teaching.

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Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been covered completely by the authors.

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Appendix A: The Task

A. Your group has settled in Iceland in one of the settlements from the settler game. Make a story about your travel and experience. Have the following questions in mind when writing the story.

The following questions might be helpful in determining what variables will affect how you model the situation and calculate the results:

1. What is the size of a Viking ship? How much surface area is there for carrying people and material to Iceland?
2. How much weight can a Viking ship carry?
3. Where do you think the Vikings came from and why?
4. Gender, age group mix of the people needed to set up a successful settlement?
5. What sort of foods/drink did they bring? For the journey? For the first few months of settlement?
6. What livestock did they bring and what food would the livestock need? On board? For the first few months of settlement?
7. How long did the journey take? Quickest? Shortest? On average?

You will be expected to work on this project with your group and then present your results on 6 April. In presenting your statistical investigation you should include a discussion of:

1. Plan – planning the procedures used to carry out the study
2. Data – the data collection process
3. Apply probability principles, i.e., Identify the various variables and probabilities and how they might be presented
4. Analysis – the summaries and analyses of the data to answer the questions posed
5. Conclusion – the conclusions about what has been learned

This presentation can be done using PowerPoint, video clips etc.

B. Every group draws one special assignment and is encouraged to show their work creatively and visually.

Assignments

1. Ferðadagar – dagleið – Travel days
2. Mannfjöldinn og skipting hans (þrælar/ambáttir, karlar/konur, strákar/stelpur) – Population and groups
3. Þróun mannfjölda á Íslandi og genin í dag – The development of the population in Iceland and origin of Icelanders.
4. Hvernig gæti aldurspýramídinn hafa litið út? Gefið nokkur dæmi. – Population pyramids, possible scenarios.

5. Landnámshænur og búfé – The livestock – the settlement hens
6. Húsakostur – efniviður – Building materials - houses
7. Þéttbýli – dreifing – Population density – travel in Iceland
8. Gróðurfar – veðurfar - Vegetation and klima