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Monte Carlo method at the 24 game and its application for mathematics education

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Abstract

Students often find mathematics a challenging subject and turn it into a scourge for them. Game-based learning, such as "24-card game", help engage students in a self-paced and fun learning process and thus may overcome students' math anxiety and promote mental math skills. This research aims to examine how the 24-card game works using the Monte Carlo method and the possibility to overcome students' mathematics anxiety. The meta-analysis method was used to explain Monte Carlo's simulation to solve the solution for all possible combinations of cards in the game and respectively assign difficulty levels. The student's proficiency level was evaluated based on the divergence value in the number of guesses required to solve the dealt combination at 87% to show full proficiency. The evaluation could also show the math difficulty of advanced operations, such as fractions and grouping games. This game is more efficient in developing students' mental math skills compared to a conventional and rigidly structured classroom lecture.

Keywords: 24-Card Game, Math Anxiety, Mathematics, Monte Carlo Simulation

Introduction

24-card game is an arithmetic game aiming to find a way to manipulate four integers resulting in the final evaluation of 24. All four integers must be included in the evaluation. The game has been widely played in China since the 1960s, called *ershisi*, with 52-deck playing cards. It is also known in Java (Indonesia) as *patlikuran*.

The more well-known variant uses only 1-9 or 1-10 face values of the card. The patented worldwide variant was popularized by Robert Sun (First in Math, Suntex) and gained its peak popularity in the late '80s to early '90s in the United States and South Africa through organized tournaments in primary and secondary schools (Tong et al., 2014). The game deals with four cards randomly for each hand. Figure 1 shows an example of a hand in the game.

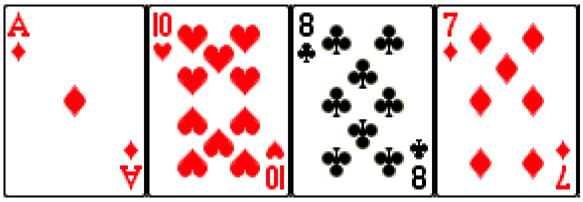


Figure 1. The combination of a card player

The players try to find the arithmetic combination of the cards making 24. The first one to find the combination scores the hand.

$$1 \;\; 10 \;\; 8 \;\; 7 \quad \text{solves as } 8 \times (10 - 7) \times 1 \; or \; 10 + 8 + 6 - 1.$$

This combination skill can stimulate the player's mental math and mental calculation skills. The calculation procedures could help children recognize and operate with numbers (Gusty, 2005). Furthermore, mental math skill necessitates a basic comprehension of mathematics. In other words, This game activity will foster the skills, allowing players (children and teenagers) to overcome their aversion to learning mathematics.

Math anxiety

Math anxiety is a negative emotional reaction (stress or anxiety) when dealing with mathematical problem-solving. Math anxiety during childhood, specifically, has long-term adverse consequences (Supekar et al., 2015). It was found that math anxiety has a detrimental impact on math performance regardless of whether the anxiety is related to the math itself or to the situational or social experience of doing math (Wu et al., 2012). Generally, perceived pressure influences both strategic and emotional aspects of math execution, leading to suboptimal strategy selection (Caviola et al., 2017).

Math anxiety not only hinders children in arithmetic development but is also associated with altered brain structure in areas related to fear to process (Kucian et al., 2018). Math anxiety is related to both individual (cognitive, affective/physiological, motivational) and environmental (social/contextual) factors (Chang et al., 2016). Math anxiety also increases activity in regions in the brain associated with visceral threat detection, leading to the experience of pain. Interestingly, this relation was not seen in math performance, suggesting that it is not that math itself hurts; rather, the anticipation of math is painful (Lyons et al., 2012) Prior studies revealed that sustained exposure to mathematical stimuli could reduce math anxiety (Supekar et al., 2015) and self-stimulating neural circuits involved in cognitive control (Chang et al., 2016). Games might encourage children to engage with and enjoy math more. However, children may be unwilling to persist if they interpret struggles as a product of their inability rather than a natural part of learning (Ramirez et al., 2018). Thus, supervision is required to effectively encourage individuals to reappraise their math anxiety reaction and embrace that struggling during learning can be helpful for brain development.

24-card game for mathematics education

The game requires quick mental arithmetic as pen, notes, or calculators are not allowed in most games. It can be used to introduce and instil mathematical concepts, e.g., commutative properties, mathematical operations priority orders, and combination/permutation, in exciting and engaging ways. The game encourages computation skills, problem-solving, number sense, critical thinking, and pattern sensing, mainly targeted in tweens/early teen school children.

As far as mathematics education is concerned, students are required to understand the concept of numbers. Moreover, playing is one of the teaching methods that help students reach the classroom's objective. Students will be motivated and challenged through the activities; they will also be able to formulate their way to find the formula.

Games can raise students' interest, lower boredom, facilitate a better understanding of the math concept, and improve their focus, which in turn help students acquire new knowledge easier (Debrenti & Laszlo, 2020). The development of mental computation skills can stimulate a higher level of mathematical thinking skills (Reys, 1985), aligning with the Indonesian government's objective to emphasize the Higher Order Thinking Skills (HOTs) into mathematics education.

A study by Sulistiawati and Wijaya (2019) revealed that mathematical games like the 24-card game could stimulate students' number sense in an arithmetic operation. In their work, students could get the correct answer by doing mental computation during the game. However, when it came up with communication on their mathematical ideas, they struggled to understand the meaning of the equation. In the second meeting of their game, students were accustomed to the game, and they could provide different strategies, indicating that their number sense abilities had improved. When the students can explore the relationships among those numbers and mix operations, they can see and understand how mathematics relates to the various situations (Greeno, 1991).

Additionally, Debrenti and Laszlo's (2020) work convinced us that card games are more efficient in developing students' mental math skills than traditional ones. The result showed that the students in the games group could perform mental computation in a shorter time than those who were not.

24-card game variants

Several variants of the game are developed to add complexities and challenges (Aviezri, 2004). Other mathematical operations, such as square root, logarithmic, exponentiation, and the fastest solving sets of hand, may be added to the rule.

Another number, other than 24, can be set as the target number. However, these variants still use 2 and 3, such as 18, 36, or 48 as the target number. It proves that 24 is not the most common; the game needs to be hard enough to create a competitive environment among players. Only about 80% of cards combination is solvable.

Some other variants use algebraic variables incorporated as one or two of the cards, of which the final solution also needs to solve the value of x and y. Fractions are also used in some variants.

Existing solvers

Several solvers have been developed to automatically solve the hands possible in the game (retrieved from https://www.4nums.com/solutions/; https://web.archive.org/web/20080919110911/http://reijnhoudt.nl/24game/appendixa.html). The algorithms iteratively go through all the combinations from the dealt hand and the operators (plus, minus, multiplication, division, bracket); however, given the small number of card combinations and operators, iterating through all possible combinations should not take too long for most computers. However, such an iterative method does not mimic how the human player solves the problem.

Such exhaustive-search iteration also does not specifically assign the difficulty level of each hand. Instead, some other metrics are used to evaluate the difficulty of solving a hand.

1. Number of possible solutions from the dealt hand
2. Mean of the duration to solve the game from actual gameplay by actual players
3. Whether any employed arithmetic operation results in an intermediate non-integer number

These metrics are then adjusted to the probability of dealt cards combination, as shown in the following equation.

$$C_n = \frac{n!}{(n-k)!\ k!}$$

For:
n = number of cards values
k = number dealt cards for a hand
10 cards: 210 combinations
13 cards: 715 combinations

To fill the gap from other solvers, this research proposes a Monte Carlo solver to find a solution for all potential card combinations in the game and assigns a difficulty level accordingly.

Monte Carlo solver on the 24-card game (10 cards and 13 cards variant)

Monte Carlo search algorithm (and its variant) has been implemented to find metaheuristic solutions for various games, such as Scrabble (Zook et al., 2019), Go (Gelly et al., 2012), Do Di Zhu (Whitehouse et al., 2011), *7 Wonders* (Robilliard et al., 2014), Scopone (Di Palma, 2018), Skat (Kupferschmid et al., 2007), *Magic: The Gathering* (Cowling et al., 2012). The Monte Carlo search algorithm works by random sampling selection across the possible states and narrowing down the solution based on fitness (score) evaluation. The algorithm can further be improved to fit the context better by several techniques, such as score-biased sampling (Zook et al., 2019), adaptive subset sampling (Vrugt et al., 2009), rule-turn optimization (Cowling et al., 2012), and other techniques.

The Monte Carlo solver for the 24-card game works by uniform-randomized search across the operands the dealt card faces. The difficulty of the hand is evaluated linearly based on how many random guesses/iterations are needed to solve a particular hand. The solver works by employing random swap operations between the cards while assigning different sets of operators for each guess. The difficulty value is evaluated based on the number of guesses required to solve the dealt hand and the probability of the dealt hand appearing from the 52-cards deck.

Monte Carlo solver has yet to incorporate several common human-player strategies, such as avoiding division and getting a factor of 24 from the set of cards (2, 3, 4, 6, 8, 12), so that the final operation of the card is the multiplication resulting in 24.

The following flowchart explains how the Monte Carlo solver works (Figure 2). The solver will stop after the guess increment counter exceed an arbitrarily set value (n = 4000), assigning that the dealt hand is impossible to solve. The solver is implemented on C programming language, available from https://github.com/jendralhxr/patlikuran.

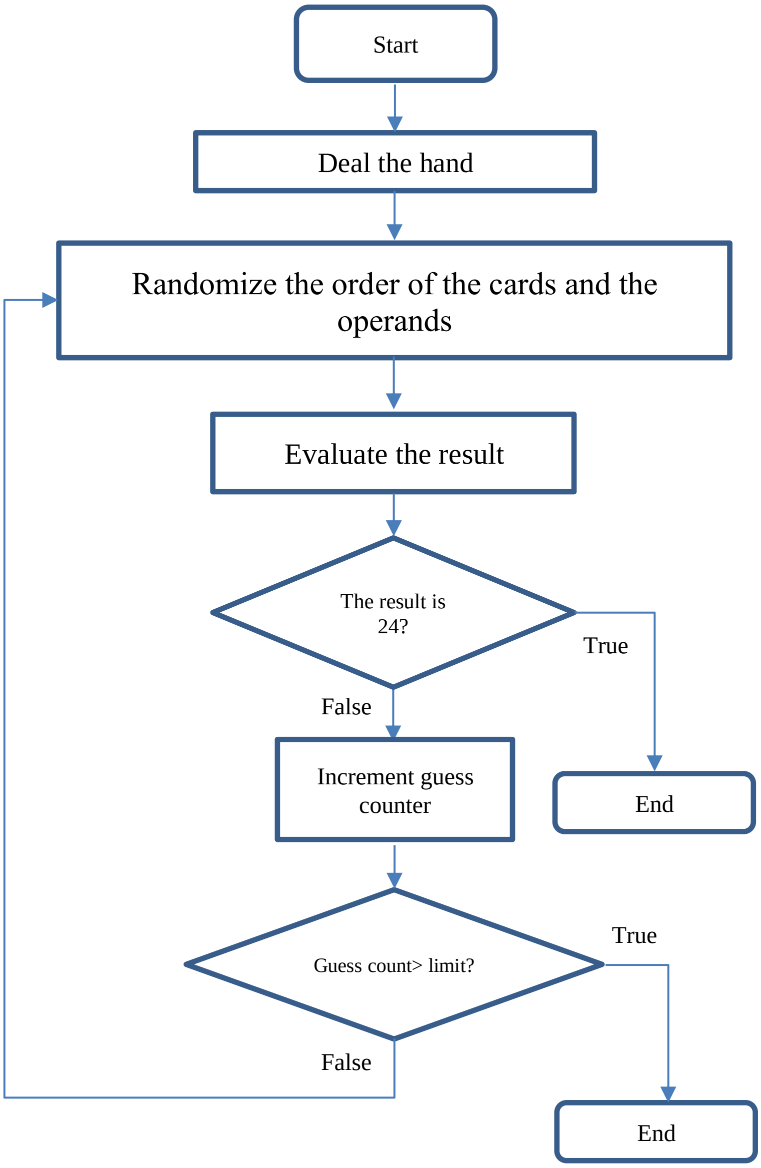


Figure 2. Flowchart of 24 game Monte Carlo Solver

Methods

This study employed a meta-analysis to integrate the findings by combining two or more published primary investigations (Ahn et al., 2012; Schmidt, 2017; Glass, 2015). This research looked at preliminary studies that questioned the effect of a 24-card game on pupils' mental math skills. The next section explains the result by utilizing Monte Carlo simulation to solve all potential card combinations in the game and give each difficulty level. The steps are described as follows.

1. Preparation Stage
   a. Data Management was sourced from Google Scholar, JSTOR, eric, and journal research websites.
   b. The research keywords used were math anxiety, 24-card game; mathematics; Monte Carlo simulation.

2. Implementation Stage
   a. Collecting data through literature sources November 1 - December 1, 2021
   b. Using the Monte Carlo method to find all possible combinations of cards

3. Data Analysis Stage
   a. Examining the correlation among the difficulty level, combination, difficulty rank and number of guesses
   b. Examining the correlation among the number of card values, the number of hands and solvable hands, and the ratio of solvable hands.

Results and Discussion

Monte Carlo refers to a mathematical procedure to compute findings using a game of chance, probability theory, and repeated random sampling. Monte Carlo's general approach to problem-solving is to set up an experiment with a random element. This experiment is repeated several times, yielding an approximated solution to the problem.

Monte Carlo's main advantage is its simplicity. Sampling and evaluation are simple procedures once a random variable has been identified. Another significant benefit of Monte Carlo methods is that they apply to various issues. Monte Carlo is often the sole viable answer for many complex issues (Ščigulinský, 2012). The results in Table 1 and Table 2 show the difficulty levels and skill ceiling.

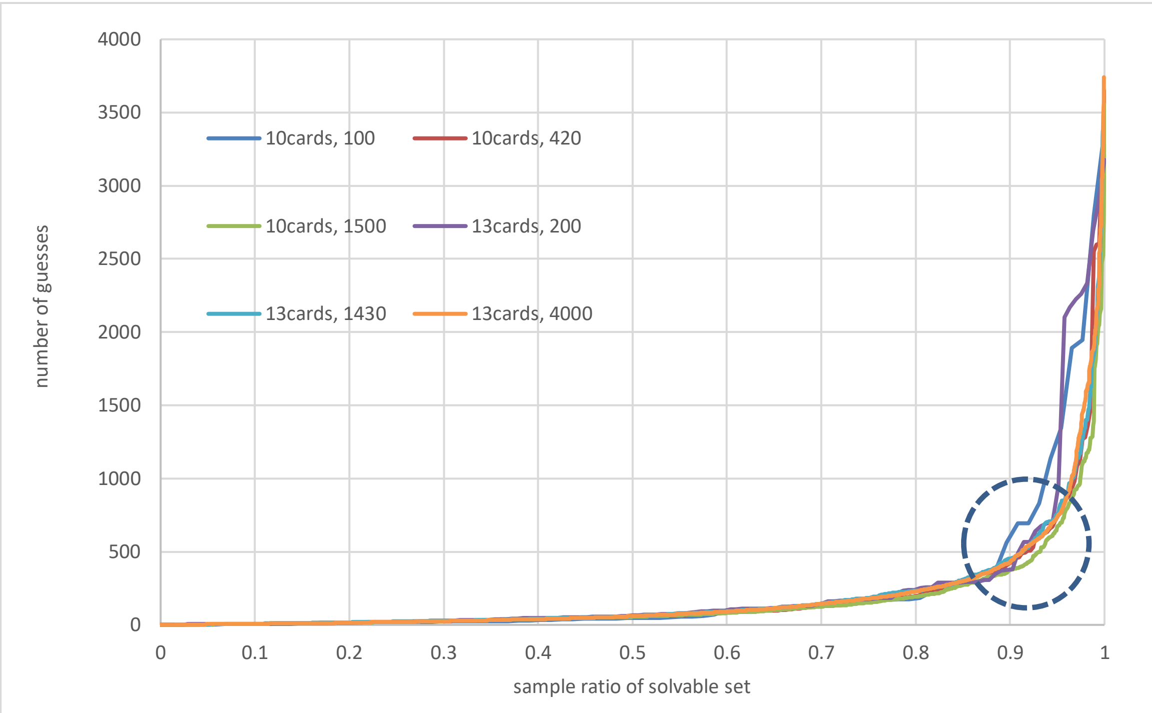
Table 1. Solvable hands at difficulty levels in 10-card variant with Monte Carlo Solver from 100 plays

Difficulty Level [1]	Combination	Solution	Difficulty Rank [2]	Number of Guesses
1	1135	(5+1) × (3+1)	336	87.65
2	2344	(3+2) ×4+4 4×4×3/2 (4-2) ×4×3	813	25.38
3	5822	(8+5) ×2-2	1254	342.19
4[3]	1346	6/ (1-3/4)	1361	1233.51*

Table 2. Skill Ceiling

Number of Card Values	Number of Hands	Solvable Hands	The Ratio of Solvable Hands
	100	87	0.870
	420	344	0.819
10	1500	1303	0.869
	200	164	0.820
	1430	1132	0.792
13	4000	3201	0.800

Besides, Figure 3 shows the number of guesses produced from the Monte Carlo solution.



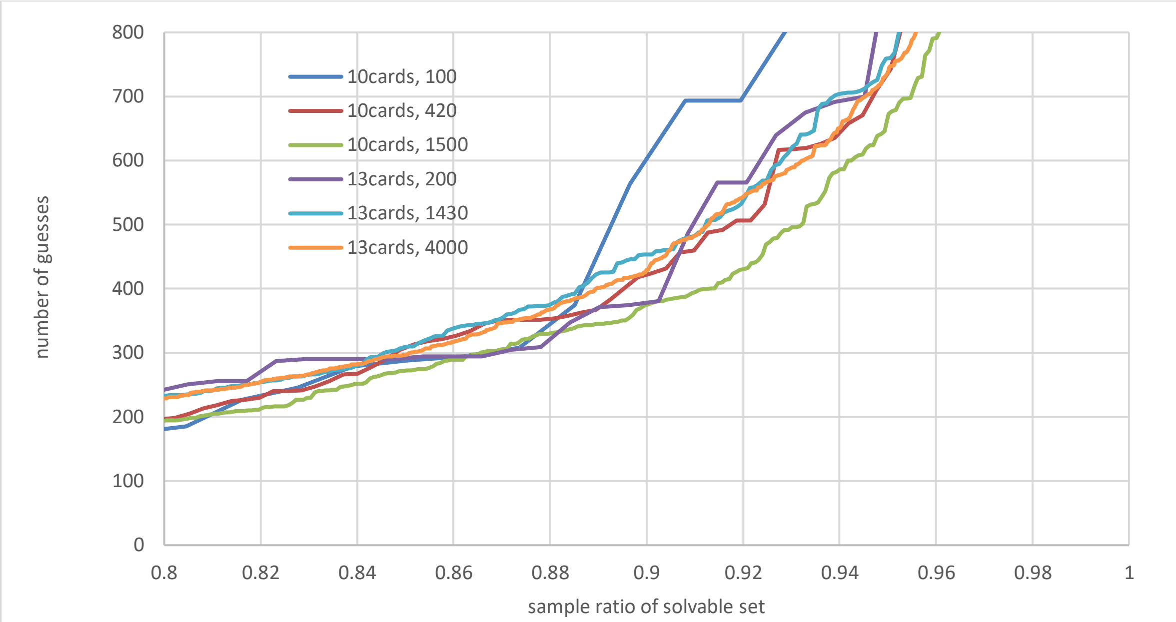


Figure 3. Number of guesses to solve different number sets of hands*

*(10cards: 100, 420, 1600; 13 cards: 200, 1430, 4000).
Note the divergence at ~0.87.

Difficulty levels

Difficulty level (Dempsey et al., 2002) is designated difficulty level from First in Math official game. Difficulty rank is obtained from 4nums rank (Papadopoulos & Robert, 2018) from 1362 solvable hands, based on the mean of the time required to solve the hand in an online game. Difficulty level 4 is proposed by Triplet (2011) as the extension of First in Math's difficulty level, which includes only hands with addition/subtraction with fractions.

Monte Carlo solver requires more guesses to solve a hand with the increasing difficulty level. Hand 2344 (difficulty level 2) requires fewer guesses than 1135 as it has more available solutions. The solver failed to solve the hand 1361 on 4 out of 100 occasions; the hand has subtraction operation in fractional terms, which remarkably increase the difficulty for the human player.

Skill ceiling

Table 2 describes the number of solvable hands for 10-card and 13-card variants from randomly assigned hands. Figure 3 explains how many guesses are required to solve a particular hand. The 13-card variant of the game does not introduce any additional difficulty to the solver despite intuitively being harder to be solved by human player given the additional prime number of 11 and 13, although, to some extent, these two cards can help to achieve 12 factors from with evaluation of 11+1 or 13-1, strategically.

The figure is subject to slight variation due to the pseudo-randomized nature of the Monte Carlo solver. Yet, the figures show that for both 10-card and 13-card variants of the game, approximately 80% of hands are solvable, and 360 guesses are required to solve the majority of all solvable hands (87%). This denotes that if a player can solve 87% of the time, such player is already proficient enough. Also, it is noted that a different number of hands dealt had no apparent effect on the number of guesses required in the whole simulation, citing the consistency of the solver.

To evaluate the "skill" limit, we use the Monte Carlo solver for random four-card combinations: variance 1-10 or 1-13(JQK) a total of 100, 420, or 1500 simulated times. From the results: ~85% of all card variations can be completed without a significant difference in difficulty level; i.e. the more to the right means, the more difficult the combination.

An example:
1 1 2 10 = easy
3 3 8 8 = hard

As the divergence starts at that 85% (about 360 guesses), this value can be used as a benchmark for the level of proficiency. The time limit for each hand can perhaps be defined based on the time to complete 360 guesses, adjusted accordingly. The curve similarity 100, 420, 1500 simulation states that the solver algorithm is linear and stable. The similarity of the 10 and 13 card variance curves means that the algorithm fails to show the level of difficulty experienced by real (human) players.

This research on games demonstrated that this 24-card game had produced students' mental math level proficiency based on the difficulty level and skill ceiling. This game allows students to see the number of guesses to enhance their math skills (Istikomah & Wahyuni, 2018). Furthermore, it was proposed that while choosing the type of mathematical games to use, teachers should think about establishing clear goals and regulations, having flexible learner control, and picking relevant assignments at an appropriate level of difficulty for students. Employing Monte Carlo stimulates the teacher to improve student's proficiency level, which relates to their mental maths skills and overcome their anxiety towards mathematics. For mathematics education, this paper expands the traditional 24-card game and lets elementary and middle school students improve their knowledge work in numbers (Bilgin, 2021; Gok, 2020).

Conclusion

Application of Monte Carlo method at the 24-card game for mathematics education is in numbers operation, probability, arithmetic, and divide-and-conquer algorithm. Approximately 85% of all card variations can be completed without a significant difference in difficulty level; the more to the right means, the more difficult the combination. Through the game, students' impressions of mathematics will change and decrease the level of math anxiety. In further research, solver for 24 Games can be modelled with AI or Neural Networks to better represent the cognitive strategy model of human players.

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Conflicts of Interest

The authors declare no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies, have been completed by the authors.

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