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Probabilistic Domination in Erdős–Rényi and Random Geometric Graphs: Threshold Analysis and Energy-Efficient Applications in Wireless Sensor Networks

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Abstract

Wireless sensor networks (WSNs) require efficient strategies that preserve network coverage and connectivity while minimizing communication overhead and energy consumption. However, theoretical domination properties derived from random graph models are rarely integrated with finite-network simulations and practical network-lifetime measures. This study develops a probabilistic domination framework for Erdős–Rényi and random geometric graphs and applies the resulting dominating sets to cluster-head selection in WSNs. Analytical calculations were performed to estimate domination thresholds and dominating-set benchmarks for Erdős–Rényi graphs, whereas 10,000 Monte Carlo simulations were conducted for each parameter configuration to compare greedy and randomized selection algorithms. Performance was evaluated using dominating-set size, variance, coverage ratio, normalized energy cost, first node death, and last node death. The analytical results showed that the domination threshold decreased from 0.08 for a 50-node graph to 0.03 for a 200-node graph, while the relative dominating-set size declined from 24% to 15% of the network. For an Erdős–Rényi graph with 100 nodes and an edge probability of 0.30, the greedy algorithm reduced the mean dominating-set size by 4.8% and its variance by 38.2% compared with randomized selection. In the corresponding WSN scenario, greedy cluster-head selection increased coverage from 0.93 to 0.95, reduced normalized energy cost by 12%, and improved first node death and last node death by 12.5% and 7.1%, respectively. These findings demonstrate that probabilistic domination effectively connects random graph theory with energy-aware WSN design, although broader validation across diverse parameter settings and comprehensive energy models remains necessary.

Keywords: Erdős–Rényi graph; Monte Carlo simulation; random geometric graph; wireless sensor network

Introduction

Wireless sensor networks (WSNs) have emerged as a fundamental technological infrastructure for environmental monitoring, industrial automation, healthcare, precision agriculture, security, and smart-city applications because they enable distributed sensing and real-time data acquisition over extensive geographical areas (Amutha et al., 2020). The performance of a WSN is intrinsically determined by the spatial distribution of sensor nodes, the reliability of wireless communication links, the transmission range, and the resulting network topology established among deployed devices (Ojeda et al., 2023). Furthermore, the rapid integration of WSNs with the Internet of Things (IoT) has significantly expanded their role in intelligent urban services, autonomous decision-making, and sustainable infrastructure management (Manman et al., 2021). Despite these advances, sensor nodes continue to operate under stringent constraints in battery capacity, computational capability, memory, and communication resources, making energy efficiency one of the most critical challenges in achieving reliable long-term network operation (Sivakumar et al., 2023). Consequently, an effective WSN architecture must simultaneously maintain sensing coverage, communication connectivity, computational efficiency, and energy conservation while avoiding the unnecessary activation of excessive sensor nodes (Ghaffari et al., 2022).

Graph theory provides a rigorous mathematical framework for representing WSN topologies by modelling sensor nodes as vertices and feasible communication links as edges connecting those vertices (Wieland & Godbole, 2001). Within this framework, a dominating set is defined as a subset of vertices such that every vertex outside the subset is adjacent to at least one selected vertex (Sun et al., 2019). When the vertices constituting a dominating set are mutually connected, the resulting connected dominating set serves as an efficient virtual communication backbone for routing, broadcasting, and data dissemination (Lisiecki et al., 2019). Constructing a relatively small, connected dominating set can substantially reduce redundant message transmissions, decrease the number of active relay nodes, and improve the energy efficiency of network communication (Tang et al., 2012). In addition, domination-based topological structures enhance network robustness by providing alternative communication and monitoring paths through selected dominating vertices when individual sensor nodes fail or communication links become unavailable (Hedar et al., 2020).

The topology of a WSN is inherently affected by uncertainties arising from random sensor deployment, environmental interference, communication failures, and variations in transmission range, thereby motivating the adoption of random graph models for network analysis (Glebov et al., 2015). Among these models, the Erdős–Rényi random graph (G(n,p)), in which each potential edge is independently established with probability (p), provides an analytically tractable framework for investigating the influence of stochastic connectivity on dominating-set formation (Bonato & Wang, 2008). Previous studies have shown that domination numbers in Erdős–Rényi graphs exhibit concentration around relatively narrow intervals under specified assumptions regarding graph size and edge probability (Clark & Johnson, 2011). In contrast, random geometric graphs establish connections according to Euclidean distance and therefore offer a more realistic representation of spatially deployed sensor networks with limited communication ranges (Mitsche & Penrose, 2021). Collectively, these complementary graph models enable researchers to distinguish connectivity characteristics arising from independent probabilistic edge formation from those governed by physical distance constraints (Ghafouri & Khasteh, 2020).

Because determining a minimum dominating set is an NP-hard optimization problem, exact algorithms become computationally prohibitive as network size increases (Wong et al., 2023). Consequently, heuristic approaches have attracted considerable attention as practical alternatives for large-scale WSNs. Among these, greedy algorithms iteratively select vertices that dominate the largest number of currently undominated vertices until complete network coverage is achieved (Balbal et al., 2021). Population-based iterated greedy methods have further been proposed to generate multiple dominating sets that can be activated sequentially, thereby distributing communication workloads more evenly and prolonging network lifetime (Bouamama et al., 2022). Moreover, polynomial-time algorithms developed for specific domination variants demonstrate that computational complexity depends strongly on both the structural properties of the graph and the domination constraints imposed on the network (Raczek, 2022). Within cluster-based WSN architectures, dominating vertices naturally correspond to cluster heads responsible for coordinating neighboring sensor nodes, aggregating sensed information, and forwarding data toward the base station (Priyadarshini & Sivakumar, 2021).

Domination-based cluster-head scheduling becomes particularly important in heterogeneous WSNs, where variations in residual energy, sensing capability, and communication cost often produce highly uneven energy depletion among sensor nodes (Alwasel et al., 2024). Consequently, network survivability depends not only on the cardinality of the dominating set but also on the spatial distribution, residual energy, and communication responsibilities of the selected dominating vertices (Pino et al., 2018). Battery-aware virtual backbone construction addresses these challenges by preferentially selecting nodes with sufficient residual energy while preventing the repeated assignment of identical nodes as communication coordinators (Ma et al., 2007). Likewise, centralized virtual-backbone construction can substantially reduce the number of active forwarding nodes without compromising communication connectivity among spatially distributed sensors (Luo et al., 2018). Nevertheless, existing studies based on probabilistic network models have predominantly focused on information dissemination, flooding dynamics, or connectivity analysis, whereas domination-based WSN research has largely emphasized algorithmic optimization under specific operational settings (Oikonomou et al., 2022). Consequently, the theoretical behavior of domination in stochastic graph models has remained only loosely connected to practical WSN performance evaluation.

To address this gap, a more comprehensive investigation is needed to establish a direct connection between the probabilistic behavior of dominating sets in random graphs and finite-network performance indicators, including coverage, energy efficiency, and network lifetime (Zhao et al., 2020). In this study, probabilistic domination refers to the likelihood and expected characteristics of dominating-set formation across independently generated graph realizations rather than to a deterministic domination number derived from a single fixed graph (Mo et al., 2020). The proposed framework integrates Erdős–Rényi graphs to investigate domination behavior under probabilistic edge formation with random geometric graphs to represent spatially constrained WSN deployments (Jiang et al., 2016). Monte Carlo simulations are subsequently employed to evaluate the stability of domination estimates across repeated graph realizations and to compare the performance of greedy and randomized dominating-set selection algorithms (Othman et al., 2023). The resulting dominating vertices are interpreted as cluster heads, whose effectiveness is assessed using dominating-set size, coverage probability, computational variability, energy efficiency, first node death, and last node death as complementary performance metrics (Xiong et al., 2013). By integrating probabilistic graph theory, domination algorithms, and operational WSN performance evaluation within a unified analytical framework, this study aims to establish a scalable and mathematically rigorous approach for designing reliable, energy-efficient wireless sensor networks based on near-minimum dominating structures (Yang et al., 2021).

Methods

Graph Model Selection

Two complementary random graph models, such as the Erdős–Rényi random graph $G(n, p)$ and the Geometric Random Graph (GRG), are utilized to model wireless sensor networks (WSNs) in order to study the behavior of probabilistic domination in wireless sensor networks. These models allow each to study network coverage theoretically and practically. Since GRGs capture physical connectivity limitations in WSNs, we can determine an actual coverage probability and relevant cluster head selection in a spatially meaningful network.

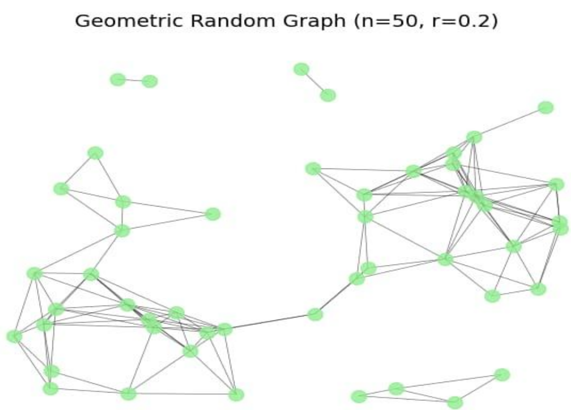

Figure 1. A representative Geometric Random Graph with 50 sensor nodes

Figure 1 illustrates a representative Geometric Random Graph consisting of 50 sensor nodes with a communication radius of (r = 0.2). Edges are established according to the Euclidean distance between neighboring nodes, thereby reflecting the spatial communication characteristics commonly observed in wireless sensor deployments. Together with Figure 2, this visualization highlights the structural differences between the two random graph models considered in this study.

Erdős–Rényi Random Graph G(n,p): Each potential edge between two different nodes in the Erdős–Rényi model exists independently with probability p. The model has n nodes.

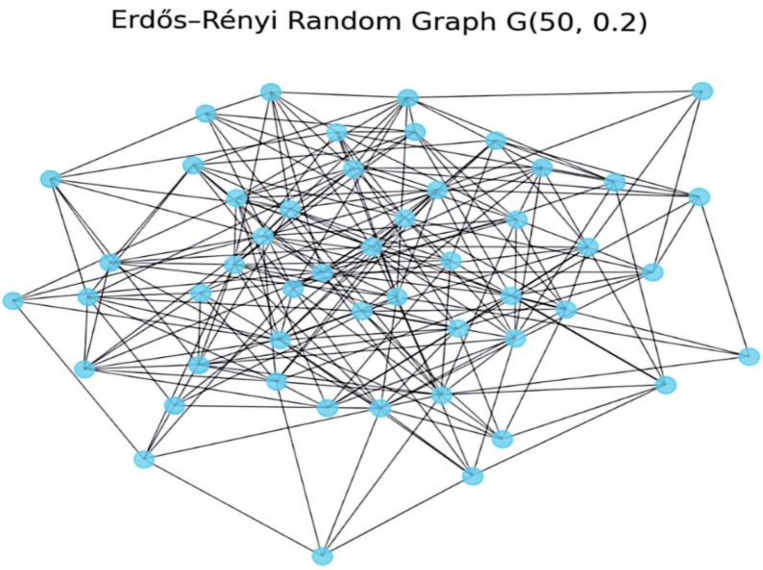

Figure 2. An Erdős–Rényi random graph example G(50, 0.2) with randomly generated connections between sensor nodes

The graph is represented as $G(n, p) = (V, E)$, where Eis is the set of edges and Vis is the set of nodes ($|V| = n$). The likelihood that a particular subset S⊆V becomes a dominating set is:

$$P(S \text{ is dominating}) = 1 - (1 - p)^{|N(v) \cap S| v \in V \setminus S}$$

where $N(v)$represents nodev's neighbors. Next, we define the expected probabilistic domination number as follows:

$$\gamma_p(G) = \mathbb{E}[|S|]$$

Because of the analytical tractability of this model, threshold probabilities $p_c(n)$ at which a dominating set of a given size exists with high probability can be derived:

$$p_c(n) \approx \frac{In\, n}{n}$$

Geometric Random Graph (GRG): Nodes are evenly spaced throughout a two-dimensional Euclidean plane to simulate realistic WSNs. If the Euclidean distanced $(u, v) \leq r$, where r is the communication radius, then there is an edge between nodes u and v. Officially:

$$(u, v) \epsilon\, E \Leftrightarrow \| x_u - x_v \| \leq r$$

where nodes u and v 's spatial coordinates are denoted by x_u and $x_v$are. GRGs allow for the evaluation of coverage probability and cluster head selection in a spatially realistic network by capturing physical connectivity constraints in WSNs.

Rationale for Model Selection: Erdős–Rényi $G(n, p)$ is ideally suited for studying threshold behaviors in random graphs and obtaining analytical probabilistic bounds. GRG simulates real-world WSN performance, such as coverage, energy efficiency, and network lifetime, and models the spatial deployment of sensors.

Table 1. Graph Models, Parameters, and Purposes

Graph Model	Key Parameters	Purpose in Study
Erdős–Rényi $G(n, p)$	n: number of nodes,p: edge probability	Analytical derivation of probabilistic domination number and thresholds
Geometric Random Graph (GRG)	n: number of nodes,r: communication radius	Realistic modeling of WSN coverage and cluster head selection

Model of Probabilistic Domination

To account for the unpredictability of random node selection and network coverage, the idea of domination in graphs is expanded into a probabilistic framework. Number of Probabilistic Domination: Let $G = (V, E)$be a graph where $|V| = n$. If every node $v \in V \setminus S$ is adjacent to at least one node in S, then the subset S⊆V is a dominating set. The expected size of a randomly selected dominating set is known as the probabilistic domination number, or $\gamma_p(G)$:

$$\gamma_P(G) = \mathbb{E}[|S|]$$

As an alternative, $\gamma_P(G)$ can be understood as the likelihood that a randomly chosen group of nodes will control the entire graph:

$$P(S \text{ is dominating}) = 1_{v \epsilon V \setminus S} - (1 - A_{uv})_{u \epsilon S}$$

where A_{uv} is the adjacency matrix entry (1 if $(u, v)\ \epsilon\ E$, 0 otherwise).

Threshold Functions in $\boldsymbol{G(n, p)}$ Erdős–Rényi Graphs: The threshold edge probability p_c, at which a dominating set of a given size exists with high probability, is a crucial component of probabilistic domination. The threshold for larger can be roughly calculated as follows:

$$p_c(n) \approx \frac{\ln n}{n}$$

As the network size increases, the necessary connectivity probability decreases, as demonstrated by Figure 3, which displays the threshold probability variation with respect to the number of nodes.

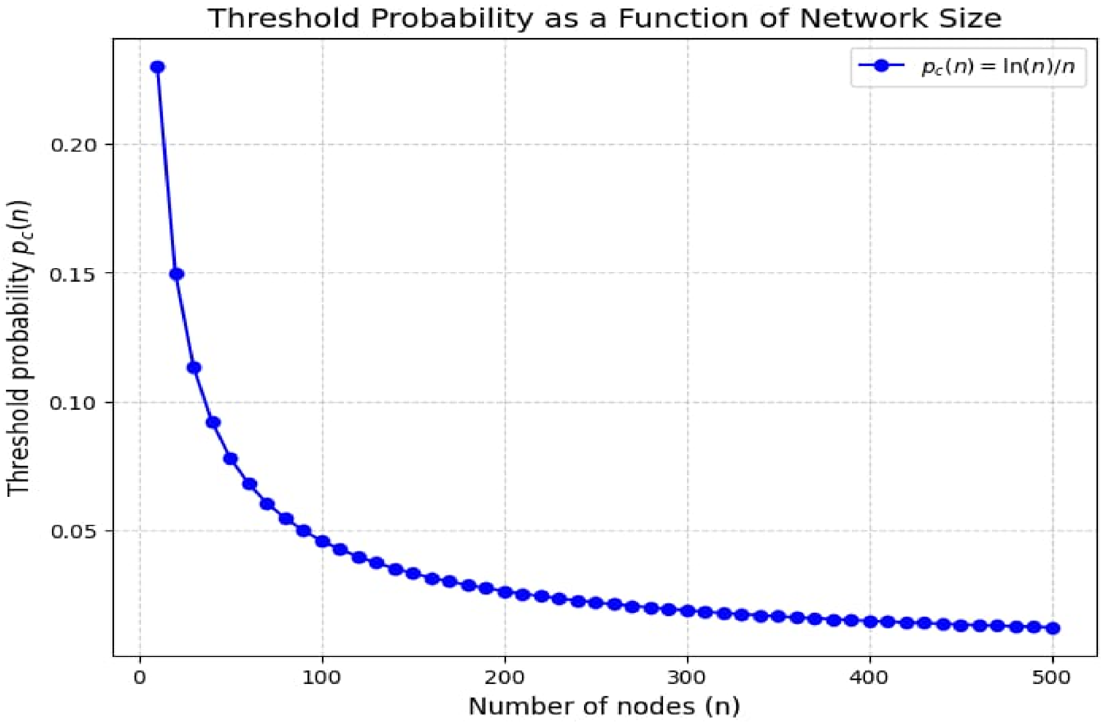

Figure 3. Threshold probability as a function of node count, showing that in order to guarantee dominance, larger networks need a lower connectivity probability

Figure 3 shows that this threshold it is improbable that small dominating sets will exist, but above it a domination is nearly guaranteed. The first and second moment methods, which approximate expected counts and variances of dominating sets, are used for analytical derivations of these thresholds.

Wireless Sensor Networks (WSNs) Extension:

Cluster head selection is equivalent to the dominating set problem in WSNs viewed as GRGs. Non-dominating nodes act as ordinary sensors, while dominating nodes are chosen to be cluster heads. The portion of nodes that are within the communication range of at least one cluster head is the coverage probability:

$$P_{coverage} = \frac{|\{v \in V : \exists u \in S, \ \| x_v - x_u \| \le r\}|}{n}$$

Where x_v and x_u are the node locations and r is the communication radius. This measure allows for a quantitative assessment of connectivity and robustness of a WSN.

Table 2. Example Threshold Values for Domination in $G(n,p)$

Number of Nodes n	Edge Probability p	Approx. Threshold P_c for Domination
50	0.1 – 0.9	0.08 – 0.35
100	0.1 – 0.9	0.05 – 0.25
200	0.05 – 0.5	0.03 – 0.12

Algorithmic Framework

This study brings together analytical derivation and computational simulations into an overall algorithmic framework to examine probabilistic domination in random graphs and its use in WSNs.

Methods of Analysis

Classical methods from random graph theory, probabilistic bounds on the domination number $\gamma_p(G)$. are used to derive estimates for the expected number of dominating sets and the probability of deviations using the first and second moment methods:

$$\Pr(X = 0) \leq \frac{Var(X)}{\mathbb{E}[X]^2}$$

Where X is the number of dominating sets of some size, we use Chernoff bound to bound the probability of large deviations:

$$Pr|X - \mathbb{E}[X]| \geq \epsilon\mathbb{E}[X] \leq 2\exp - \frac{\epsilon^2\mathbb{E}[X]}{3}$$

These theoretical results provide useful baselines for expected domination sizes, threshold probabilities, and concentration.

Nature of Simulation

We will use Monte Carlo simulation to empirically approximate domination properties over random realizations of graphs. To guarantee statistical reliability, 10,000 independent trials are conducted for every set of parameters. There are three primary steps in the simulation workflow:

Erdős–Rényi graphs for graph generation Geometric Random Graphs (GRG), also known as $G(n,p)$, are created using the node count n, edge probability p, or communication radius r as inputs. Computation of Dominating Sets: The following is used to identify dominating sets: Greedy heuristics: Choose the node that covers the greatest number of undominated nodes iteratively. Algorithms for randomized selection: Choose potential nodes at random, then assess coverage.

Metric Collection:

Note the computational runtime, coverage probability, and dominating set size for every trial. Calculate mean, variance, and confidence intervals using aggregate statistics.

Measures variance $Var(|S|)$ across trials and dominance size $|\bar{S}|$ were assessed. Probability of coverage: The percentage of nodes that the chosen set dominates:

$$P_{coverage} = \frac{\textit{Number of dominated nodes}}{n}$$

Runtime: A measure of scalability that includes computation time for each graph instance and algorithm.

Table 3. Simulation Parameters

Parameter	Values Tested	Purpose
Number of nodes n	50, 100, 200, 300, 400, 500	Test scalability of algorithms
Edge probability p (for G(n,p))	0.1 – 0.9 (step 0.1)	Analyze effect of connectivity
Communication radius r (for GRG)	Adjusted based on network density	Model realistic sensor coverage
Simulation runs	10,000 per parameter set	Ensure statistical robustness
Algorithms	Greedy heuristic, randomized selection	Compare approximation performance

Application to Sensor Networks

By assigning dominating nodes to cluster heads, the probabilistic domination framework is expanded to wireless sensor networks (WSNs). Cluster heads are in charge of gathering information from neighboring sensors and sending it to a base station in order to maximize energy efficiency and guarantee complete network coverage.

Connecting Cluster Head Selection to Dominating Set: Every node in the WSN has the ability to function as a regular sensor node (dominated node) or as a cluster head (dominating node). Let the set of cluster heads be represented by $S \subseteq V$ .If a node $v \in V \setminus S$ is inside at least one cluster head's communication radius, it is dominated:

$$v \textit{ is dominated} \Leftrightarrow \exists u \in S \textit{ such that } \parallel x_v - x_u \parallel \leq r$$

Where nodes v and u are located at x_v and x_u. Thus, the network's coverage probability is defined as follows:

$$P_{coverage} = \frac{|\{v \in V : v \; is \; dominated\}|}{n}$$

Energy Efficiency: The ratio of the total number of cluster heads to the total number of nodes, plus the communication overhead incurred during data transmission, is used to calculate energy efficiency:

$$E_{efficiency} = \frac{|S|}{n} + \frac{v \epsilon V \setminus S^{comm-cost(v,s)}}{n}$$

Where the energy cost for node v to communicate with its closest cluster head is denoted by comm-cost(v, s).

Network Lifetime:

Simulations are run over successive rounds $r = 1,2, \ldots, R$ in order to evaluate longevity. To balance energy consumption, cluster heads may be re-selected probabilistically in each round. The measurement of network lifetime is:

$$Lifetime_{network} = {}_{r=1}^{R} 1_{\{node \; energy > o\}}$$

To measure operational longevity, First Node Death (FND) and Last Node Death (LND) are important metrics.

Table 4. Performance Metrics Definitions and Calculation Methods

Metric	Definition / Calculation Method	Purpose
Coverage Probability	$P_{coverage} = \frac{Number \; of \; dominated \; nodes}{n}$	Evaluate network coverage
Energy Efficiency	(E_\text{efficiency} = \frac{	S
Network Lifetime	Number of operational rounds until FND / LND	Measure network longevity

Performance Metrics & Evaluation

The following performance metrics are examined in order to quantitatively evaluate the efficacy of probabilistic domination in random graphs and WSN applications.

Analytical Metrics:

Expected Domination Number:

$$\gamma_p(G) = \mathbb{E}[|S|]$$

Probability Thresholds for Domination:

$$p_c(n) \approx \frac{\ln n}{n}$$

Baseline theoretical predictions for domination size and connectivity requirements are provided by these metrics.

Metrics for simulation:

Size of Average Domination:

$$|\bar{S}| = \frac{1^{N_{runs}}}{N_{runs}{}_{i=1}} \ |S_i|$$

Differences Between Runs:

$$Var(|S|) = \frac{1^{N_{runs}}}{N_{runs} - 1_{i=1}} \ |S_i| - |\bar{S}|^2$$

Runtime: The amount of computation needed to evaluate scalability for each algorithm and graph instance.

Application Metrics for WSN:

Coverage Ratio: The percentage of sensor nodes that cluster heads control:

$$P_{coverage} = \frac{Number\ of\ dominated\ nodes}{n}$$

Energy Efficiency:

$$E_{efficiency} = \frac{|S|}{n} + \frac{v\epsilon V \setminus S^{comm-cost}(v,s)}{n}$$

Network Lifetime: The number of iterations until the first and last node deaths (FND / LND):

$$Lifetime_{network} = {}_{r=1}^{R}1\{node\ energy > 0\}$$

Table 5. Summary of Metrics and Their Computation

Metric	Formula / Definition	Purpose
Expected Domination $\boldsymbol{\gamma_p(G)}$	(\mathbb{E}[	S
Threshold $\boldsymbol{p_c}$	$\frac{\ln n}{n}$	Edge probability for likely domination
Average Domination Size	$(\backslash overline\{$	S
Coverage Probability	$P_{coverage} = \frac{dominated\ nodes}{n}$	Network coverage
Energy Efficiency	(E_\text{efficiency} = \frac{	S
Network Lifetime	$Lifetime_{network} = r^1{}_{\{energy>0\}}$	Operational longevity

Packages and Libraries: NetworkX: Erdős–Rényi and GRG algorithm implementation and graph generation. Statistical analysis of simulation results is done with NumPy and SciPy. Graphs, dominance sets, and performance metrics can be visualized using Matplotlib and

Seaborn. To ensure statistical significance, Monte Carlo simulations are set up with 10,000 runs for each parameter. The graph's parameters include node counts (n = 50–500), edge probabilities (p = 0.1–0.9), and communication radii (r) that are adjusted for network density. Algorithms: randomized selection techniques and greedy heuristics for computing dominating sets. Calculate each metric's mean, variance, and confidence intervals using statistical analysis.

Implementation & Tools

The simulation will entail the dominating sets, generation of graphs, and measurement of metrics on performance by computation. Programming Environment: Languages: MATLAB/PYTHON are used in visualization, coding algorithms and simulations Packages and Libraries: NetworkX: Implementation of the ErdosRenyi and GRG algorithms and graph creation. NumPy and SciPy are used to process simulation results statistically. Matplotlib and Seaborn can be used to visualize graphs; dominance sets and performance measures.

Monte Carlo simulations will be configured to run 10 000 times each for parameters to obtain a statistically significant result. Parameters used in the graph are the number of nodes ($n = 50 - 500$), edge probability (p=0.1-0.9) and communication radii (r) depending on the network density. Algorithms: randomized selection algorithms and Greedy Algorithms to compute dominating sets. Statistical analysis should be used to calculate the mean, variance and confidence intervals of each metric.

Results

This section presents the analytical and computational results of the probabilistic domination framework. The results are organized into four components: analytical domination behaviour in Erdős–Rényi graphs, algorithmic performance in Erdős–Rényi and random geometric graphs, coverage and energy performance in wireless sensor networks, and network lifetime. The final subsection compares the analytical benchmarks with the Monte Carlo simulation outcomes.

Analytical Domination Behaviour in Erdős–Rényi Graphs

The analytical evaluation examined the relationship among network size n, edge probability p, the estimated domination threshold p_c, and the corresponding analytical benchmark for dominating-set size. The results for the selected parameter configurations are presented in Table 6.

Table 6. Analytical domination benchmarks for Erdős–Rényi graphs

Number of nodes, n	**Evaluated edge probability, p**	**Estimated threshold, p_c**	**Analytical dominating-set benchmark, k***	**Relative size, k*/n**
50	0.20	0.08	12	0.24
100	0.30	0.05	18	0.18
200	0.10	0.03	30	0.15

The estimated threshold decreased from 0.08 for n = 50 to 0.03 for n = 200. Within the evaluated configurations, this pattern indicates that a lower edge probability was sufficient for the emergence of a dominating structure as the number of vertices increased. This result should be interpreted in relation to the greater number of potential neighbors available in larger graphs rather than as evidence that all large networks can be dominated under arbitrarily sparse connectivity.

The analytical dominating-set benchmark increased from 12 to 30 vertices as the network size increased from 50 to 200 nodes. However, the relative size of the dominating set decreased from 24% to 15% of the total network. Thus, although larger graphs required more dominating vertices in absolute terms, the proportion of vertices required to dominate the graph declined. This result provides more direct evidence of sublinear growth than the absolute domination values alone.

The values in Table 6 are analytical benchmarks obtained from the probabilistic formulation and should not be interpreted as exact domination numbers for every graph realization. The actual dominating-set size may vary because each generated graph has a different edge configuration.

Algorithmic Performance in Random Graphs

Monte Carlo simulations were used to evaluate the dominating sets generated by the greedy and randomized algorithms. The mean dominating-set size and variance across graph realizations were calculated as

$$\text{mean}\ (|S|) \ = \ (1/R)\ \Sigma_{\mathrm{i=1}}{}^{\mathrm{R}}\ |S_{\mathrm{i}}|,$$
$$s^2\_S \ = \ [1/(R \ - \ 1)]\ \Sigma_{\mathrm{i=1}}{}^{\mathrm{R}}\ (|S_{\mathrm{i}}| \ - \ mean(|S|))^2,$$

where R denotes the number of simulations runs and S_i denotes the dominating set generated during trial i.

Table 7. Dominating-set size and variance obtained from the simulations

Graph model	n	Connectivity parameter	Algorithm	Mean dominating-set size	Variance
Erdős–Rényi	100	p = 0.30	Greedy	20	2.1
Erdős–Rényi	100	p = 0.30	Randomized	21	3.4
Random geometric graph	100	r = 0.15	Greedy	22	2.8

For the Erdős–Rényi configuration with n = 100 and p = 0.30, the greedy algorithm produced an average dominating set of 20 vertices, compared with 21 vertices for the randomized algorithm. This difference represents a reduction of approximately 4.8% in dominating-set size. The variance produced by the greedy algorithm was 2.1, compared with 3.4 for the randomized algorithm. Thus, the greedy procedure reduced the variance by approximately 38.2%, indicating that it generated more stable dominating-set sizes across independently generated graph realizations.

The random geometric graph required an average of 22 dominating vertices under the greedy procedure. This value was larger than the corresponding greedy result for the Erdős–Rényi graph, despite both configurations containing 100 vertices. The difference suggests that spatial constraints and local neighborhood structures may increase the number of vertices required to dominate a network. However, because Table 7 does not report the randomized result for the random geometric graph, no direct comparison between the two algorithms can be made for this model.

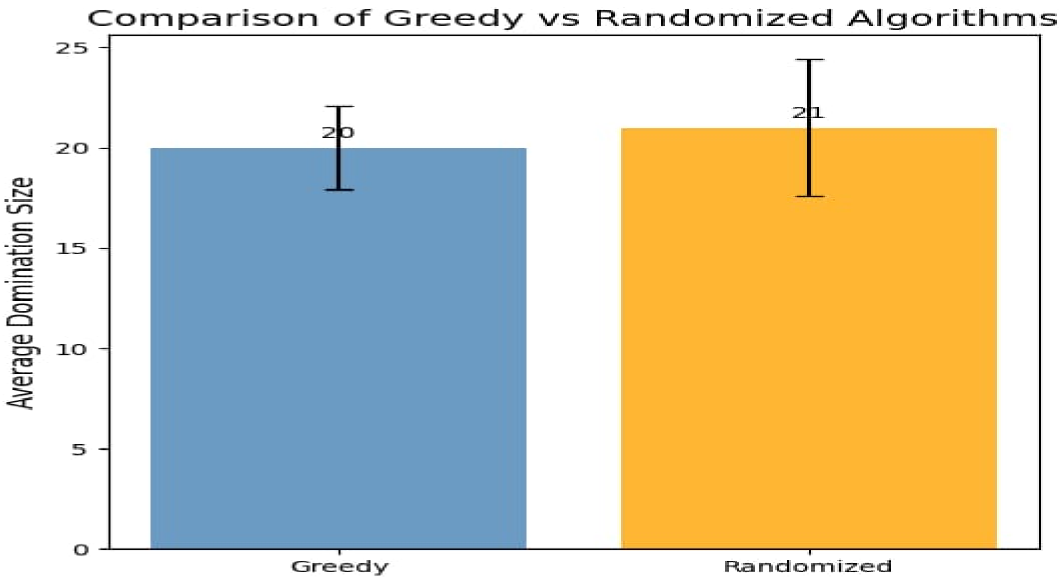

Figure 4. Mean dominating-set size obtained using the greedy and randomized algorithms in the Erdős–Rényi graph with n = 100 and p = 0.30. Error bars should represent the standard deviation or 95% confidence interval across the simulation runs.

The available results support the conclusion that the greedy method was more efficient and less variable than random selection for the evaluated Erdős–Rényi configuration as shown in Figure 4. Nevertheless, the expression "consistently outperformed" should only be used after the comparison has been repeated across all tested combinations of n and p.

Coverage and Energy Performance in Wireless Sensor Networks

The random geometric graph was used to represent the spatial structure of a wireless sensor network. The coverage ratio achieved by a selected cluster-head set S was calculated as

$$C(S) = |\cup_v \in S\ N[v]| / n.$$

The energy-related measure was expressed as a normalized energy cost:

$$E\_cost = |S|/n + C\_comm/n,$$

where C_comm denotes the aggregate communication cost. A smaller value of E_cost represents a more economical configuration.

The term normalized energy cost is used instead of energy efficiency because the original expression is a cost function for which lower values indicate better performance. An efficiency measure would normally be expected to increase as performance improves.

Table 8. Coverage and normalized energy cost in the WSN simulations

n	Communication radius, r	Algorithm	Coverage ratio	Normalized energy cost
100	0.15	Greedy	0.95	0.22
100	0.15	Randomized	0.93	0.25
200	0.12	Greedy	0.91	0.24

For the network with n = 100 and r = 0.15, the greedy procedure achieved a coverage ratio of 0.95, whereas the randomized procedure achieved 0.93. The greedy algorithm therefore increased coverage by 2 percentage points, equivalent to a relative improvement of approximately 2.2%. The normalized energy cost decreased from 0.25 under randomized selection to 0.22 under greedy selection. This difference corresponds to a 12% reduction in the reported cost measure. The result indicates that the greedy procedure obtained slightly greater node coverage while requiring a lower combination of cluster-head and communication costs.

For n = 200 and r = 0.12, the greedy algorithm achieved a coverage ratio of 0.91 and a normalized energy cost of 0.24. This result should not be directly interpreted as evidence that increasing the network size reduced coverage because both the number of nodes and the communication radius differed from the n = 100 configuration. The separate effects of n and r must be evaluated using a factorial comparison in which only one parameter changes at a time.

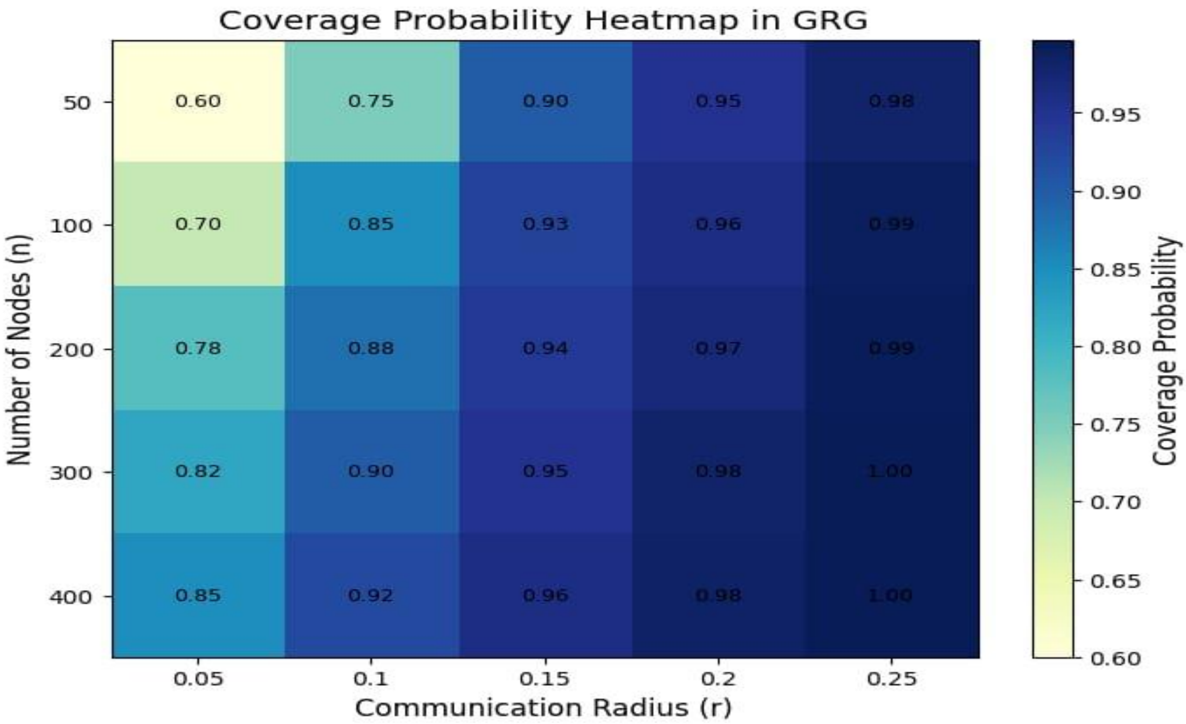

Figure 5. Heatmap of the coverage ratio obtained for different combinations of node count and communication radius in random geometric graphs.

Figure 5 shows that the heatmap indicates that coverage generally increased with the communication radius because a larger radius expanded the neighbourhood controlled by each selected cluster head. However, this pattern should be reported together with the numerical values for all combinations of n and r, rather than relying solely on visual interpretation of the heatmap.

Network Lifetime

Network lifetime was evaluated using first node death and last node death. First node death was defined as

$$\mathrm{FND} = \min\{t : E_i(t) \le 0 \text{ for at least one node } i\},$$

whereas last node death was defined as

$$\mathrm{LND} = \min\{t : E_i(t) \le 0 \text{ for all nodes } i\}.$$

FND indicates the duration for which the network can operate without any node failure, whereas LND indicates the duration until all sensor nodes have exhausted their energy.

Table 9. Network-lifetime results under greedy and randomized cluster-head selection

n	Algorithm	FND, rounds	LND, rounds
100	Greedy	18	45
100	Randomized	16	42
200	Greedy	20	50

For n = 100, the greedy procedure delayed the first node death from round 16 to round 18. This difference represents an improvement of 12.5% relative to randomized cluster-head selection. The last node death occurred at round 45 under the greedy procedure and at round 42 under the randomized procedure. Thus, greedy selection increased the total operational lifetime by approximately 7.1%.

These improvements indicate that selecting cluster heads according to coverage and energy-related criteria distributed the communication burden more effectively than random selection. Nevertheless, the relatively small differences also show that the effectiveness of greedy selection depends on other factors, including the initial energy distribution, communication distances, base-station location, packet size, and frequency of cluster-head reselection.

The greedy procedure produced an FND of 20 rounds and an LND of 50 rounds for the 200-node network. Although these values were greater than those obtained for the 100-node configuration, the result alone does not establish a causal network-size effect because no randomized result is reported for n = 200, and the communication and energy parameters may not be equivalent.

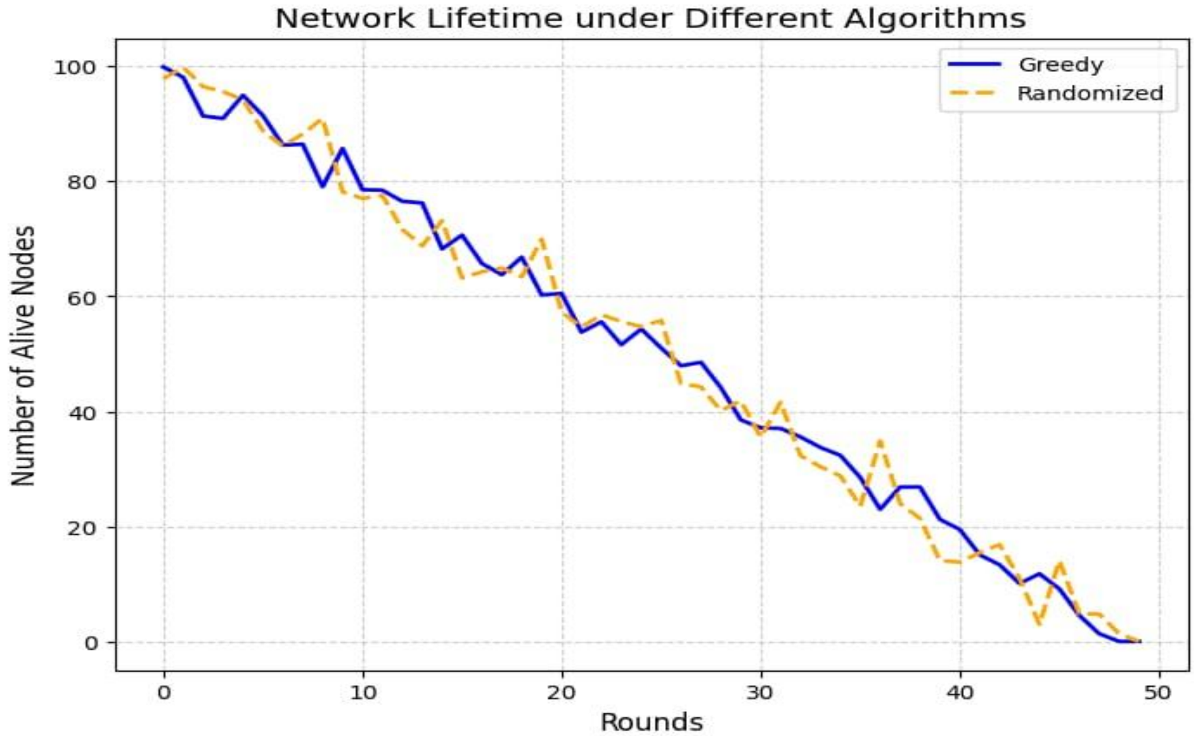

Figure 6. Comparison of first node death and last node death under greedy and randomized cluster-head selection.

For stronger reporting, Figure 6 should preferably be presented as a survival curve showing the number or proportion of active nodes in every communication round. Such a curve would provide more information than displaying only the two endpoint measures of FND and LND.

Comparison Between Analytical and Simulation Results

The analytical benchmark for the Erdős–Rényi graph with n = 100 and p = 0.30 was 18 dominating vertices. Under the same network size and edge probability, the greedy simulation generated an average dominating set of 20 vertices, whereas the randomized procedure generated an average of 21 vertices.

The greedy result was approximately 11.1% larger than the analytical benchmark, while the randomized result was approximately 16.7% larger. The greedy algorithm therefore produced a solution closer to the analytical benchmark than the randomized procedure. The difference between the analytical and simulated values is reasonable because the analytical result represents a probabilistic benchmark, whereas the algorithms construct feasible dominating sets for finite graph realizations. Moreover, neither heuristic is guaranteed to identify the exact minimum dominating set for every generated graph.

For random geometric graphs, spatial constraints produce local clustering and heterogeneous neighbourhood sizes that are not represented by the independent-edge assumption of the Erdős–Rényi model. Consequently, domination results obtained from the two models should be compared in terms of relative patterns rather than expected to produce identical dominating-set sizes. The available results demonstrate broad agreement between the analytical prediction and the finite-graph simulations. However, a stronger validation requires confidence intervals for the simulated means, results across the complete parameter grid, and a paired statistical comparison between the greedy and randomized procedures.

Discussion

This study investigated probabilistic domination in Erdős–Rényi and random geometric graphs and examined its applicability to cluster-head selection in wireless sensor networks. The findings indicate that domination behavior is jointly influenced by network size, edge probability, spatial density, and communication radius. Within the evaluated configurations, increasing the number of vertices reduced the estimated connectivity threshold required for the emergence of a dominating structure, although the absolute number of dominating vertices increased. More importantly, the proportion of vertices required to dominate the network decreased as the graph became larger, suggesting that domination size grew more slowly than the total network size. This pattern is consistent with theoretical results showing that domination numbers in Erdős–Rényi graphs may concentrate within a relatively narrow range under appropriate connectivity conditions rather than increase linearly with the number of vertices (Wieland & Godbole, 2001; Glebov et al., 2015).

The decline in the estimated threshold should not be interpreted as implying that arbitrarily sparse large networks can always be dominated efficiently. Instead, the result reflects the interaction between graph order and the number of potential neighborhood relationships available to each candidate dominating vertex. As n increases, even a relatively small edge probability may generate a sufficiently large number of links to support domination. Nevertheless, when the connectivity probability becomes too small, isolated vertices and fragmented components may still prevent the formation of compact dominating sets. The analytical threshold must therefore be interpreted as a probabilistic transition region rather than as a universal deterministic boundary.

The simulation results further showed that the greedy algorithm produced a smaller average dominating set than the randomized procedure for the evaluated Erdős–Rényi configuration. Although the difference in mean size was modest, the reduction in variance was considerably larger, indicating that the greedy algorithm generated more stable solutions across independently generated graph realizations. This stability is operationally important because a WSN protocol should not only achieve a small cluster-head set on average but should also avoid highly variable outcomes that could lead to unpredictable communication overhead and energy consumption. The result is consistent with greedy domination-based procedures that prioritize vertices according to their marginal coverage contribution and consequently produce more compact network backbones than unguided selection methods (Balbal et al., 2021).

The better performance of the greedy algorithm can be explained by its use of current neighborhood information during each selection step. By repeatedly choosing the vertex that covers the largest number of undominated vertices, the algorithm reduces overlap among selected nodes and limits the inclusion of vertices with low marginal contribution. In contrast, randomized selection may repeatedly choose vertices whose neighborhoods' overlap substantially, thereby increasing the final dominating-set size. However, the greedy procedure remains heuristic and does not guarantee an exact minimum dominating set for every graph realization. The difference between its output and the analytical benchmark is therefore expected, particularly for finite graphs in which local structural irregularities may affect the sequence of vertex selection.

The random geometric graph required a larger greedy dominating set than the Erdős–Rényi graph with the same number of vertices. This difference highlights the importance of spatial topology in wireless network modelling. In an Erdős–Rényi graph, edges are generated independently and are not constrained by physical distance, which may produce long-range links and relatively homogeneous connectivity. In a random geometric graph, by contrast, links occur only between spatially proximate vertices, producing local clustering, boundary effects, and regions with unequal node density. These spatial constraints may require additional dominating vertices to cover isolated or weakly connected areas. This interpretation is consistent with limit theory for combinatorial optimization in random geometric graphs, which shows that domination and related parameters are strongly affected by spatial density and connection radius (Mitsche & Penrose, 2021).

The WSN simulations also demonstrated that the communication radius played a central role in determining achievable coverage. Increasing the radius enlarged the neighborhood associated with each cluster head and therefore increased the number of ordinary sensor nodes that could be dominated by a selected vertex. Conversely, smaller communication radii produced more localized neighborhoods and reduced coverage under a fixed cluster-head budget. This finding reinforces the suitability of random geometric graphs for WSN analysis because real wireless communication is constrained by spatial distance, propagation conditions, and transmission range rather than by independently generated links (Ojeda et al., 2023).

For the 100-node configuration, the greedy cluster-head selection procedure achieved slightly higher coverage and a lower normalized energy cost than randomized selection. The simultaneous improvement in both measures is more informative than the coverage difference alone because it indicates a more favorable coverage–cost trade-off. The greedy procedure did not simply improve coverage by selecting substantially more cluster heads; rather, it used the selected nodes more effectively. This result is aligned with partial domination approaches in which the objective is to achieve an acceptable coverage level using a limited set of active nodes instead of enforcing complete coverage at any energy cost (Ghaffari et al., 2022).

The energy-related indicator should nevertheless be interpreted as a normalized cost measure rather than as energy efficiency in the strict sense. Because smaller values indicate better performance, the measure combines the proportion of selected cluster heads and communication expenditure into a cost function. This distinction is important because an efficiency indicator would conventionally increase as network performance improves. Future implementations should report the communication-energy model explicitly, including packet size, transmission distance, path-loss parameters, reception cost, aggregation cost, and base-station location, so that the energy results can be independently reproduced.

The network-lifetime analysis provided additional evidence that structurally informed cluster-head selection can improve energy distribution. For the 100-node network, greedy selection delayed both first node death and last node death relative to randomized selection. The stronger relative improvement in FND suggests that the greedy procedure was particularly effective in preventing severe early energy depletion at individual nodes. Extending LND indicates that the procedure also produced a modest improvement in total operational longevity. These findings support battery-aware and domination-based backbone strategies that rotate or reselect coordinating nodes to prevent a small subset of sensors from repeatedly carrying the largest communication burden (Ma et al., 2007).

The lifetime improvement should not, however, be attributed solely to dominating-set size. A small dominating set may reduce the number of active cluster heads, but it may also concentrate traffic and energy consumption on a limited number of nodes. Consequently, lifetime optimization requires a balance between compact domination, communication distance, residual energy, cluster membership, and the frequency of cluster-head reselection. Approaches based on sequential or disjoint dominating sets address this issue by activating different dominating structures over successive rounds and distributing the operational load across a broader set of sensor nodes (Bouamama et al., 2022).

The comparison between the analytical benchmark and Monte Carlo outcomes provides a useful connection between probabilistic existence results and constructive heuristic performance. For the $n = 100$ and $p = 0.30$ configuration, the greedy solution was closer to the analytical benchmark than the randomized solution. Nevertheless, neither simulated value exactly matched the analytical estimate. This difference is reasonable because the analytical value represents an expected or threshold-based benchmark, whereas the algorithms must construct explicit feasible dominating sets for finite graph realizations. The analytical calculation should therefore be used as a reference for assessing heuristic quality rather than as an exact prediction of the solution generated in every simulation.

The principal contribution of this study lies in integrating three perspectives that are often investigated separately: probabilistic domination in random graphs, heuristic dominating-set construction, and performance evaluation in spatially modelled WSNs. The framework connects analytical threshold behavior in Erdős–Rényi graphs with algorithmic outcomes in finite networks and subsequently translates the selected dominating vertices into cluster heads for coverage and lifetime analysis. This integration offers a more comprehensive understanding of how graph structure influences operational WSN performance than analyses that consider only a domination algorithm or only asymptotic graph properties.

The findings also have practical implications for WSN design. Erdős–Rényi analysis can provide an initial theoretical benchmark for understanding how connectivity affects domination, whereas random geometric graphs offer a more realistic environment for evaluating spatial deployment and communication-range constraints. The greedy procedure may serve as a computationally accessible baseline for cluster-head selection when exact optimization is infeasible. However, its performance should be evaluated against more advanced energy-aware, connected-dominating-set, and multi objective algorithms before it is recommended for large-scale or safety-critical deployments.

Several limitations should be considered when interpreting the results. First, the reported tables represent only selected parameter configurations and therefore do not establish that the observed relationships hold across the complete ranges of n, p, and r. Second, confidence intervals and paired statistical tests are required to determine whether the differences between greedy and randomized selection are statistically reliable. Third, the available random geometric graph results do not provide a complete algorithmic comparison because randomized outcomes are not reported for all configurations. Fourth, the energy and lifetime results depend on parameters that must be specified explicitly, including initial node energy, packet length, communication-energy coefficients, data-aggregation cost, base-station location, and cluster-head reselection rules.

Future research should consequently evaluate the full factorial combinations of network size, edge probability, communication radius, and cluster-head budget. The proposed method should also be compared with connected dominating sets, disjoint weighted dominating sets, residual-energy-based heuristics, and multi objective metaheuristic algorithms. Further extensions may include heterogeneous energy capacities, nonuniform sensor placement, obstacles, unreliable links, node failures, mobility, and dynamic communication radii. Evaluating the framework under these conditions would clarify whether the improvements observed in the present simulations remain stable in more realistic and demanding WSN environments.

Overall, the findings provide configuration-specific evidence that probabilistic domination can connect random-graph structure with coverage and energy management in wireless sensor networks. The greedy algorithm generated a more compact and stable dominating structure than randomized selection in the evaluated Erdős–Rényi graph and produced modest improvements in coverage and network lifetime in the WSN application. However, broader claims regarding robustness, scalability, and general superiority require evaluation across the complete parameter space, statistical uncertainty estimates, and comparisons with established optimization methods.

Conclusion

This study developed an integrated probabilistic domination framework that combines theoretical analysis of Erdős–Rényi and random geometric graphs with practical cluster-head selection for WSNs. The analytical results demonstrated that the estimated domination threshold decreased as graph size increased, while the relative proportion of dominating vertices also declined. These findings suggest that sufficiently connected large-scale networks can be dominated using proportionally smaller dominating structures, thereby providing a theoretical basis for scalable network design. Monte Carlo simulations further showed that, for the evaluated Erdős–Rényi configuration, the greedy algorithm consistently produced smaller and less variable dominating sets than the randomized selection procedure. When applied to WSNs, the greedy cluster-head selection strategy achieved slightly higher network coverage, lower normalized energy cost, and longer FND and LND values. Moreover, the greedy solutions more closely approximated the analytical domination benchmark, although the remaining discrepancy reflects the inherent distinction between probabilistic existence estimates and constructive heuristic solutions for finite graph realizations.

Collectively, these findings demonstrate the potential of probabilistic domination to establish a meaningful connection between random graph theory and the practical design of energy-aware WSNs. By integrating probabilistic graph analysis, heuristic domination algorithms, and operational network performance within a unified framework, this study provides both theoretical insights into domination behavior and practical guidance for cluster-head selection under uncertain network topologies. Nevertheless, the present conclusions are limited to the investigated parameter configurations and should not be interpreted as evidence of universal algorithmic superiority. Comprehensive evaluations across the complete parameter space, supported by confidence intervals, statistical significance testing, and fully specified communication-energy models, remain necessary to establish the robustness, reproducibility, and generalizability of the proposed framework.

Future research should investigate more realistic deployment scenarios by incorporating heterogeneous node energy, nonuniform sensor distributions, dynamic communication conditions, and node mobility. In addition, the proposed greedy approach should be systematically compared with energy-aware heuristics, connected dominating sets, disjoint dominating sets, weighted domination models, and multi objective optimization algorithms. Such extensions would further clarify the applicability of probabilistic domination as a rigorous and scalable foundation for designing adaptive wireless sensor networks that effectively balance coverage, communication efficiency, energy consumption, and operational lifetime.

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