Journal of Honai Math, 8(3), 517-530, December 2025
https:/doi.org/10.30862/jhm.v8i3.1073

Discovery Learning Worksheets for Diagnosing Errors in Function Concepts: A Qualitative Analysis in Early Calculus

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Received: 2025 | Revised: 21 November 2025 | Accepted: 30 December 2025

Abstract

This study investigates the difficulties experienced by first-year calculus students when interpreting function graphs and linking them to real-world contexts. Previous observations indicated that around 71.4% of students had difficulty accurately reading and representing functions graphically. To address this, the study used Discovery Learning-based worksheets for diagnosis and instruction to identify, categorise and reduce errors in students' understanding of functions and their graphical representations. A qualitative descriptive research design involving 21 first-semester students from the Mathematics Education Programme at Universitas Ahmad Dahlan was applied. Data were collected from the students' responses to the worksheets and analysed through iterative processes of data reduction, display and drawing conclusions. Student errors were classified into three categories: conceptual, procedural and technical. The findings revealed that conceptual errors were the most prevalent, particularly when distinguishing functions from general relations, interpreting discrete and continuous domains, and understanding the meaning and application of functions in real-life situations. The Discovery Learning worksheets encouraged students to actively construct concepts through exploration, reflection, and guided problem solving, while simultaneously providing insight into their reasoning processes and persistent misconceptions. This study concludes that Discovery Learning worksheets are effective in diagnosing errors and reinforcing concepts in calculus learning. These findings offer mathematics educators a practical pedagogical framework for identifying learning obstacles and strengthening students' foundational understanding of functions and graphical representations in introductory calculus courses.

Keywords: calculus education; discovery learning; error analysis; function concept; student worksheet; qualitative descriptive research.

Introduction

Functions constitute the foundational framework of calculus, providing the primary means of modelling relationships between variables and forming the foundation of limits, derivatives, and integrals (Zhou et al., 2024). To master functions, students must fluently navigate multiple representational forms symbolic, graphical, tabular and verbal and recognise invariant relationships across them (Martins et al., 2023). Nevertheless, learners at all levels frequently struggle to coordinate these representations, often failing to bridge the gap between formal mathematical definitions and contextual or visual interpretations (Ainsworth et al., 2006; Trujillo et al., 2023). These challenges often arise from fragmented conceptions of variables and covariational reasoning, hindering deeper comprehension of functional relationships (Moss et al., 2019).

These conceptual difficulties often continue into higher education, where students often reduce functions to mere algebraic formulas instead of understanding them as structured mappings between domains and codomains (Bardini et al., 2014; Gunawan et al., 2021). Such misconceptions result in consistent errors when identifying domains, interpreting graphical behaviours and relating independent to dependent variables (Milla et al., 2018; Uscanga et al., 2024; Soesanto & Dirgantoro, 2021). In university calculus contexts, these errors tend to fall into three interrelated categories: conceptual errors (e.g., conflating relations with functions or misunderstanding the one-to-one mapping property), procedural errors (e.g. misapplying solution sequences or omitting critical steps) and technical errors (e.g. computational inaccuracies or symbolic misnotation) (Ayuningsih et al., 2020; Hasibuan et al., 2022; Reskina et al., 2023). Systematic error analysis provides insight into students' underlying reasoning processes and cognitive gaps, enabling targeted instructional interventions (Xu, 2023).

In order to address these deeply rooted misconceptions, instructional approaches that prioritise active conceptual construction over passive reception are essential. Discovery Learning (DL) has been shown to be effective in reducing conceptual errors by structuring learning through guided exploration, identifying problems, processing data, verifying information and generalising (Delfita et al., 2017; Shanmugavelu et al., 2020). When implemented through purposefully designed student worksheets, DL provides learners with a structured pathway to reconstruct functional concepts while generating tangible evidence of their thought processes. These worksheets serve as both instructional media and diagnostic instruments, enabling educators to identify error patterns in real time, monitor cognitive progression and provide targeted feedback (Angreani et al., 2020; Collins et al., 2021).

Despite the recognised potential of direct learning (DL) and worksheet-based instruction, few empirical studies in university-level calculus integrate these tools as structured frameworks for error analysis. Most existing research focuses on learning outcomes or skill enhancement, rather than systematically mapping how the instructional syntax of DL exposes and addresses specific conceptual, procedural and technical errors in understanding functions. This study aims to bridge this gap by employing a qualitative descriptive approach to analyse the types and sources of students’ errors relating to functions and function graphs when using a Discovery Learning-based worksheet. By aligning error categorisation with the sequential stages of DL, the study aims to provide actionable insights for refining calculus instruction and developing diagnostic learning materials that transform errors into opportunities for conceptual growth.

Methods

Research design

This study employed a qualitative descriptive design to systematically categorise students' errors in comprehending function concepts and their graphical representations. The design was structured around the six stages of the Discovery Learning (DL) model stimulation, problem statement, data collection, data processing, verification and generalisation to enable phase-by-phase mapping of conceptual, procedural and technical errors as they emerged during concept construction. This approach was chosen to provide an in-depth, context-rich analysis of students' cognitive processes, reasoning pathways and persistent misconceptions.

Participants and context

The research was conducted at the Ahmad Dahlan University (UAD) in Yogyakarta during the 2025/26 academic year. The participants were 21 first-semester students enrolled in the Mathematics Education Study Programme. The students were organised into five collaborative groups of four to five members each to facilitate peer interaction and group-based problem solving aligned with the DL instructional sequence. All participants were enrolled in the introductory calculus course, ensuring a shared academic baseline for the investigation.

Instruments and data collection

Data were collected through two complementary sources: (1) student-completed Discovery Learning (DL) worksheets and (2) structured classroom observations. The LKPD was explicitly designed to align with the six DL syntaxes and comprised contextualised mathematical tasks centred on functions and function graphs. Student responses provided direct documentary evidence of reasoning processes, error patterns and conceptual gaps. Concurrently, non-participant observations were conducted during group discussions to capture verbal reasoning, collaborative dynamics and real-time problem-solving strategies. These observational notes were recorded using a structured protocol aligned with the DL stages and the three target error categories. This enabled methodological triangulation between the students' written work and their interactive behaviours.

Data analysis procedure

The data analysis followed Miles, Huberman and Saldaña's (2018) interactive model comprising three iterative phases: data reduction, data display and drawing conclusions (see Figure 1). During the data reduction phase, the raw responses in the worksheets and the observational field notes were systematically coded and filtered to identify instances of conceptual, procedural and technical errors. Data irrelevant to the research focus was excluded, while instances of error were categorised and mapped to their corresponding DL syntax.

The data display phase involved organising the reduced data into analytical matrices and narrative summaries in order to visualise error distribution across instructional stages and identify recurring cognitive patterns. During the conclusion-drawing phase, the researcher interpreted error profiles in relation to students' mathematical reasoning, linking persistent misconceptions to specific instructional triggers within the DL sequence. The analysis proceeded iteratively until thematic saturation was achieved and comprehensive error typologies were established.

Figure 1. Data analysis stages (adapted from Miles et al., 2018).

Trustworthiness and ethical compliance

Methodological rigour was ensured through data triangulation, whereby worksheet responses were cross-verified with observational records. A mathematics education specialist conducted peer debriefing to validate error categorisation frameworks and mitigate researcher bias. All participants provided informed consent prior to data collection, and all materials were anonymised to protect student confidentiality. The study adhered to institutional ethical guidelines for educational research involving human participants.

Results and Discussion

Classroom implementation and collaborative dynamics

Learning activities were delivered using the six-step Discovery Learning (DL) model. Students were organised into five collaborative groups of four to five members. The structured worksheet guided learners through contextual exploration, peer negotiation and formal generalisation. As shown in Figure 2, students participated in active group discussions, whiteboard problem-solving and peer presentations, reflecting the DL model's focus on student autonomy and interactive knowledge construction. This collaborative environment naturally provided an opportunity to observe how students reasoned, where misconceptions emerged, and how peer scaffolding either reinforced or corrected procedural and conceptual errors.

Figure 2. Students' activities during the learning process

Error Analysis Across Discovery Learning Syntaxes: Concert Ticket Context

The first worksheet task used a concert ticket pricing scenario to contextualise functions (Regular: Rp150,000; VIP: Rp250,000; VVIP: Rp400,000). Students were asked to determine whether this scenario represented a function (see Figure 3). Analysis of student responses across the DL syntaxes revealed distinct error typologies aligned with each cognitive stage.

Question 1:

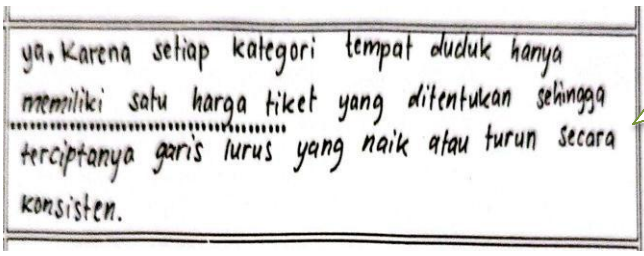
Perhatikan permasalahan berikut! Seorang penjual tiket konser menentukan harga tiket berdasarkan kategori tempat duduk. Harga tiket reguler Rp. 150.00,00., VIP Rp. 250.000,00., dan VVIP sebesar Rp. 400.000,00. Analisislah apakah permasalahan ini merupakan suatu fungsi?

Consider the following problem! A concert ticket seller determines ticket prices based on seating categories. The regular ticket costs Rp150,000, the VIP ticket costs Rp250,000, and the VVIP ticket costs Rp400,000. Analyze whether this problem represents a function.

Figure 3. The first function question on the student worksheet

Syntax 1: Stimulation

The guiding prompt "Can this situation be expressed as a function? Why?" activated prior knowledge. Four out of five groups correctly identified the scenario as a function, recognising that each seating category corresponds to a fixed price. However, Group 3 (Figure 4) made a conceptual error by stating that a function's graph must 'consistently rise or fall' in a straight line. This reflects a common misconception in representation: equating the formal definition of a function with linear graphical patterns. This finding is consistent with prior research on students' overgeneralisation of linear models in learning functions (Bardini et al., 2014; Trujillo et al., 2023).



Yes, because each seating category has only one fixed ticket price, resulting in a straight line that consistently rises or falls.

Figure 4. Answer of group 3 for question 1 (Simulation)

Syntax 2: Problem Statement

Learning activities were delivered using the six-step Discovery Learning (DL) model. Students were organised into five collaborative groups of four to five members. The structured worksheet guided learners through contextual exploration, peer negotiation and formal generalisation.

The students were tasked with formulating mathematical questions. Most groups successfully generated questions focused on identifying the domain, codomain and variable. However, Group 4 (Figure 5) shifted towards economic analysis (e.g., total revenue, price elasticity), indicating a procedural misalignment whereby applied reasoning preceded formal mathematical framing. This suggests that, without explicit guidance, learners may transition from mathematical abstraction to contextual interpretation before grasping foundational definitions.

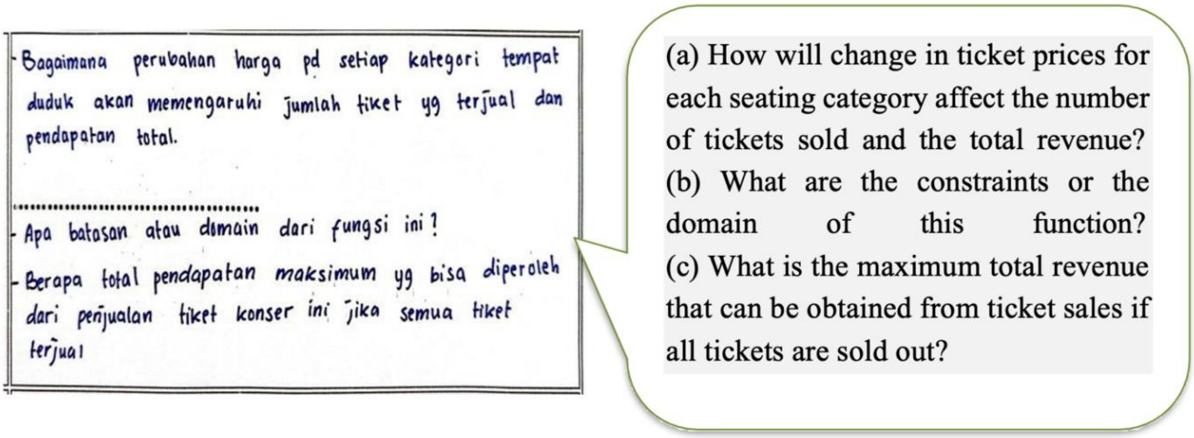


Figure 5. Answer of group 4 for question 1 (Problem Statement)

Syntax 3: Data Collection

All three groups correctly identified the one-to-one correspondence between ticket categories and prices. Group 2 made a conceptual error in justifying function status based on 'varying prices' rather than a unique mapping. Meanwhile, Group 3 (Figure 6) conflated 'varied' with 'distinct outputs', revealing a procedural error in symbolic notation. These findings are consistent with previous studies indicating that students frequently rely on superficial numerical differences rather than structural relational properties when defining functions (Uscanga et al., 2024; Sebsibe et al., 2019).

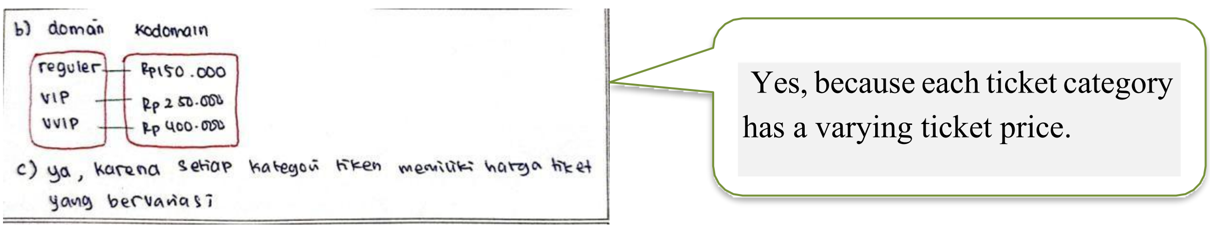


Figure 6. Answer of group 3 for question 1(b, c) (Data collection)

Syntax 4: Data processing

All groups correctly expressed the relationship algebraically and constructed function tables. However, when graphing, every group committed a conceptual error by connecting discrete points into a continuous straight line (Figure 7). This suggests an ongoing challenge in distinguishing between discrete and continuous domains, which is a well-documented issue in the early stages of learning about functions (Moss et al., 2019). The worksheet successfully identified this error by requiring multiple representational translations (context → table → equation → graph).

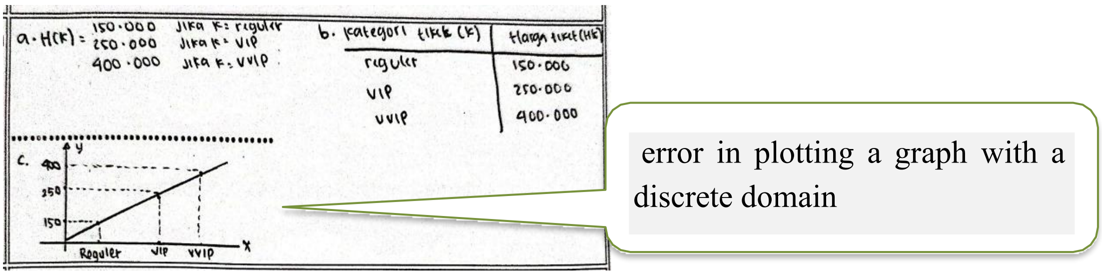


Figure 7. Answer of group 2 for question 1 (Data processing)

Syntax 5: Verification

The students confirmed that each input corresponded to exactly one output, thus satisfying the criterion of the vertical line test. However, when asked to interpret graphical patterns, most incorrectly labelled the discrete ticket-price graph as a 'linear relationship' (Figure 8). This conceptual-procedural hybrid error highlights the difference between recognising functional properties and understanding domain-specific graphical conventions.

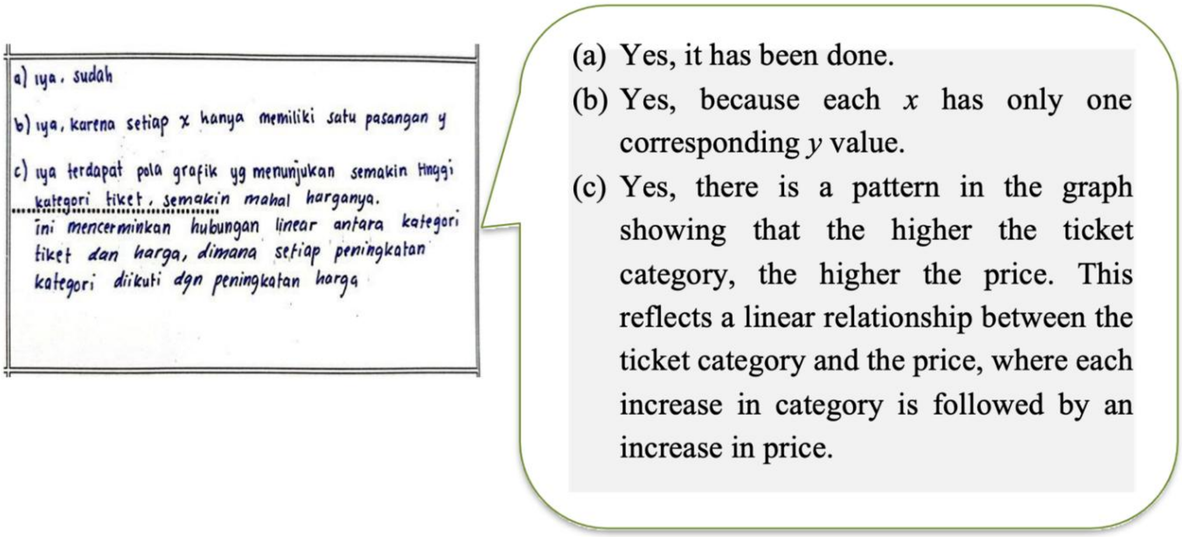


Figure 8. Answer of group 5 for question 1 (Verification)

Syntax 6: Generalisation

Most students provided accurate summaries of function definitions and graphical representations. However, Group 5 (Figure 9) defined a function as 'a relationship where each input is paired with an output', omitting the critical uniqueness constraint. This incomplete conceptualisation demonstrates how the final synthesis stage of DL reveals residual misconceptions that require targeted instructional reinforcement.

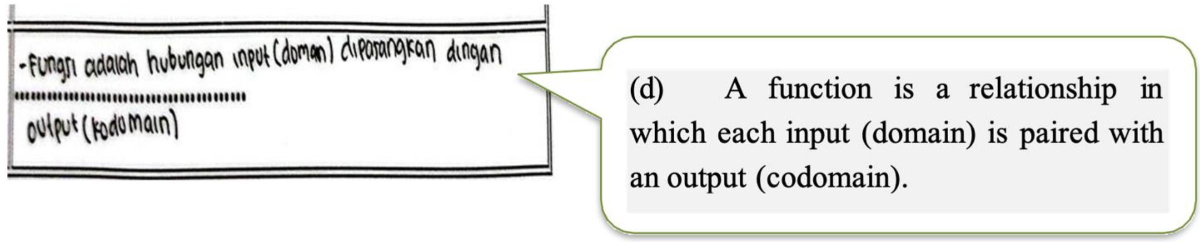


Figure 9. Answer of group 5 for question 1a (Generalization)

Error Analysis Across Discovery Learning Syntaxes: Rate-Based Contexts

The second worksheet presented two rate-based phenomena: an online ride service with an initial fee of Rp5,000 plus Rp2,000 per kilometre, and a fruit seller charging Rp10,000 per kilogramme. This emphasised linear functions, covariational reasoning and interpreting slope as a real-world rate. Student responses across the six Discovery Learning stages revealed a shift from initial conceptual recognition to procedural fluency; however, there were persistent difficulties in mathematical formalisation and contextual interpretation.

Question 2:

Perhatikan permasalahan berikut ! perhatikan fenomena berikut! 1) suatu perusahaan ojek Online mengenakan biaya awal Rp 5000, kemudian Rp 2000 untuk tiap kilometer berikutnya, 2) Seorang pedagang menjual buah dengan harga Rp 10000 per kilogram.

Observe the following problems! Consider the following phenomena:

1. An online transportation company charges an initial fee of Rp 5,000, then Rp 2,000 for each additional kilometer.
2. A fruit seller sells fruit at a price of Rp 10,000 per kilogram.

Syntaxes 1–3: Stimulation

At the stimulation stage, most students correctly identified both scenarios as functions, recognising the consistent input-output relationship between distance/weight and cost/price. However, Group 3 (Figure 10) made a conceptual error by describing the functional relationship rather than demonstrating it mathematically ("the base fare and additional charge per kilometre are clearly defined"). This response demonstrates an operational understanding of pricing structures, but does not articulate the formal criterion of unique mapping. This indicates a gap between everyday reasoning and mathematical formalisation.

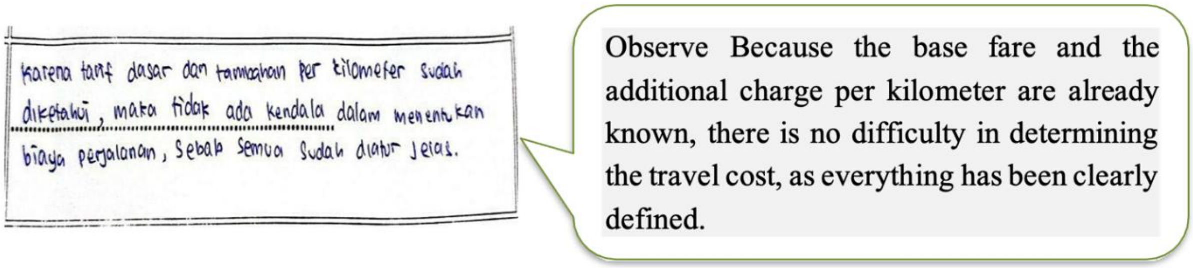


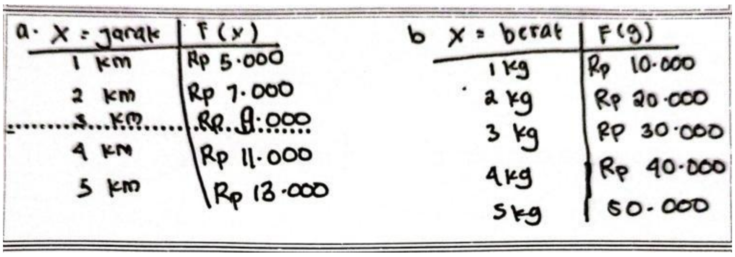
Figure 10. Answer of group 3 for question 1 (Stimulation)

Syntax 2: Problem statement

During the formulation of the problem statement, all groups successfully generated mathematically relevant questions, such as 'How can the price of fruit be expressed as a function of its weight?' and 'How can the cost of an online ride be expressed as a function of travel distance?'. This demonstrates an improved ability to translate contextual scenarios into formal mathematical questions, suggesting that the preceding stimulation stage effectively prepared students for structural thinking.

Syntax 3: Data Collection

During the data collection phase, students were tasked with creating function value tables for distances and weights ranging from 1 to 5 units. All groups completed this accurately (see Figure 11), which indicates a high level of procedural competence in generating discrete input-output pairs. The absence of errors at this stage suggests that tabular representation serves as an accessible bridge between contextual descriptions and algebraic structuring.



a. x = jarak	f(x)
1 km	Rp 5.000
2 km	Rp 7.000
3 km	Rp 9.000
4 km	Rp 11.000
5 km	Rp 13.000

b x = berat	f(g)
1 kg	Rp 10.000
2 kg	Rp 20.000
3 kg	Rp 30.000
4 kg	Rp 40.000
5 kg	50.000

Figure 11. Example of students' answers for Question 2 in the data collection stage

Syntax 4: Data Processing

In the data processing stage, students were required to translate tabular data into algebraic equations. As in the previous stage, all groups correctly formulated the linear functions (see Figure 12), demonstrating procedural fluency in symbolic representation. However, this uniform accuracy contrasts with earlier conceptual gaps, highlighting a common pattern in calculus instruction: students often become proficient in symbolic manipulation before fully understanding the underlying functional relationships.

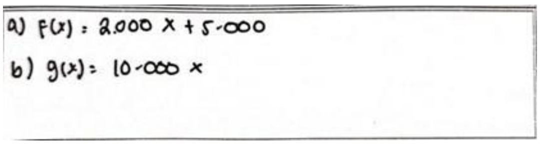
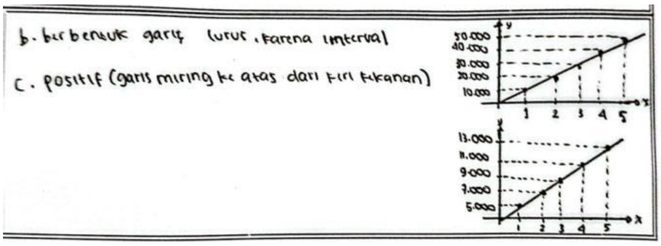


Figure 12. Example of students' answers for Question 2 in the data processing stage

Syntax 5: Verification

During the verification process, the students graphed both functions and interpreted their geometric properties. All groups accurately plotted the graphs and correctly identified them as straight lines. While most students correctly computed the gradient values (2,000 and 10,000), Group 2 (Figure 13) merely described the gradient as 'positive' (a line slanting upwards from left to right). This represents a procedural-geometric error: the students recognised the visual characteristic of slope, but failed to contextualise it as a rate of change. Not being able to articulate that the gradient signifies an increase in cost of Rp2,000 per kilometre or an increase in price of Rp10,000 per kilogram reveals a disconnect between algebraic computation and real-world covariational reasoning.

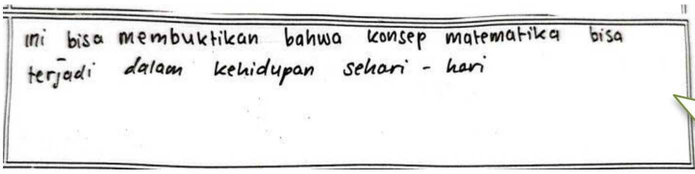


b). It is a straight line because of the interval.
c) Positive (a line slanting upward from left to right).

Figure 13. Answer of group 3 for question 1 (Verification)

Syntax 6: Generalisation

During the generalisation stage, students were prompted to synthesise their learning outcomes. While most groups produced mathematically appropriate conclusions, Group 5 (Figure 14) made a conceptual error by stating that 'mathematics occurs in everyday life'. While this conclusion was contextually true, it omitted core functional concepts such as domain, codomain, mapping properties and graphical representation. This overgeneralisation suggests that, without explicit instructional support, students may understand the contextual relevance of an activity, but miss the formal mathematical takeaways that are central to it.



This shows that every mathematical concept can occur in everyday life

Figure 14. Answer of group 5 for question 1 (Generalization)

Synthesis: Pedagogical implications and theoretical alignment

Integrating Discovery Learning worksheets as diagnostic instruments revealed three critical insights. Firstly, conceptual errors predominated, particularly when distinguishing between discrete and continuous domains, formalising the uniqueness property of functions, and interpreting gradients as contextual rates rather than geometric slopes. Secondly, procedural errors primarily emerged during representational translation, where students correctly manipulated symbols or tables, but misaligned them with graphical or contextual meanings. Technical errors (computational or notational) were minimal, suggesting that foundational arithmetic skills were not the primary barrier.

These findings demonstrate that DL worksheets function as instructional media and structured error-mapping tools. Each stage of the process acted as a cognitive checkpoint: the initial stimulation revealed prior conceptions, the problem statement revealed framing tendencies, the data collection and testing process uncovered relational misunderstandings, the verification process highlighted representational gaps and the final stage of generalisation assessed readiness for formalisation. This phased approach is consistent with the theoretical premise of DL that guided discovery renders tacit reasoning visible, facilitating targeted remediation (Delfita et al., 2017; Firdaus & Wilujeng, 2018).

Unlike traditional error analysis, which often identifies mistakes after assessment, this study's approach, embedded within DL syntaxes, offers a novel pedagogical framework for real-time misconception detection. The results corroborate prior research emphasising the need for explicit instruction on domain types, covariational reasoning and mathematical communication in calculus foundations (Bardini et al., 2014; Uscanga et al., 2024). In practice, mathematics educators should integrate DL worksheets with targeted scaffolding at the verification and generalisation stages, explicitly contrasting discrete and continuous representations and contextualising them.

Conclusion

This study demonstrates that students' errors in understanding function concepts and graphical representations occur at all stages of the Discovery Learning (DL) model, with the greatest difficulties arising during the data processing and verification phases. A prevalent misconception was the tendency to depict discrete-domain functions as continuous straight lines, indicating a fundamental misunderstanding of domain type and its graphical implications. Students also struggled to interpret gradient as a contextual rate of change, coordinate independent and dependent variables, and apply precise mathematical notation. Overly general conclusions further indicated a gap between procedural completion and formal mathematical reasoning.

The systematic integration of DL syntaxes served as an effective diagnostic framework, rendering tacit reasoning processes visible and exposing persistent misconceptions. While structured exploration and peer collaboration facilitated initial concept discovery, the absence of explicit scaffolding during graph construction and symbolic generalisation meant that procedural habits overrode conceptual precision. The phased design of the worksheet successfully transformed errors into instructional opportunities, enabling targeted reflection and the iterative refinement of functional understanding.

These findings highlight the dual usefulness of Discovery Learning worksheets as pedagogical tools and diagnostic instruments in calculus instruction. In practice, mathematics educators should explicitly contrast discrete and continuous representations, contextualise gradient interpretation and reinforce formal function definitions during the verification and generalisation stages. Theoretically, the study confirms that error analysis embedded within structured enquiry models can bridge the gap between intuitive reasoning and formal mathematical generalisation. This offers a replicable approach to addressing foundational gaps in early calculus learning.

As a qualitative descriptive study, however, the findings are bound to the context of the specific cohort and instructional setting examined. Future research should expand the diagnostic DL worksheet framework to other calculus topics, such as limits and derivatives, and investigate its adaptability across diverse student populations. Integrating digital graphing tools or automated feedback mechanisms into the worksheet design could enhance real-time error detection and conceptual remediation further. Longitudinal studies are also recommended to assess whether structured error analysis within inquiry-based learning improves functional reasoning over time.

Acknowledgment

We would like to express our sincere gratitude to Dr. Puguh Wahyu Prasetyo, M.Si., Head of the Mathematics Education Study Program at Universitas Ahmad Dahlan, for his motivation and review of this article. We also extend our appreciation to the first-semester Mathematics Education students of classes A and B, academic year 2025/2026, who were directly involved in this research.Describe acknowledgement if any.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this manuscript. All ethical issues, including plagiarism, academic misconduct, data fabrication or falsification, and duplicate publication, have been fully addressed by the authors.

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