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Fast Forward Method on Single Spin with Rabi Frequency

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Abstract: *This research examines the application of the fast forward method to single spin with Rabi frequency. In this study, the electron spin dynamics are examined by accelerating adiabatic quantum dynamics. Through the concept of adiabatic will be obtained unchanged state of the system, at the beginning and end of the evolution of the system. This study aims to obtain an additional Hamiltonian on the concept of accelerated adiabatic quantum dynamics on single spin. The method used is the fast forward method developed by Masuda and Nakamura. The result of this research is to obtain the additional Hamiltonian "equation 54" and the driving magnetic field through the fast forward method "equation 56". The fast forward method is applied by first obtaining the eigenvalues of the Hamiltonian system. Furthermore, by reviewing the lowest energy state (ground state). It is concluded that this study obtained an additional Hamiltonian term with a driving magnetic field that ensures that a single spin can move from the initial state to the final state in a short time, while maintaining the characteristics of each energy level in the system.*

Keywords: *Fast forward, quantum dynamics, quantum spin, Rabi frequency, theoretical physics*

Metode *Fast Forward* pada Spin Tunggal dengan Frekuensi Rabi

Abstrak: Penelitian ini mengkaji penerapan metode *fast forward* pada spin tunggal dengan frekuensi Rabi. Pada penelitian ini ditinjau dinamika spin elektron dengan mempercepat dinamika kuantum adiabatik**.** Melalui konsep adiabatik akan didapatkan keadaan sistem yang tidak berubah, pada saat awal dan akhir evolusi sistem. Penelitian ini bertujuan untuk mendapatkan Hamiltonian tambahan pada konsep dinamika kuantum adiabatik yang dipercepat pada spin tunggal. Metode yang digunakan merupakan metode *Fast Forward* yang dikembangkan oleh Masuda dan Nakamura. Hasil penelitian ini yaitu mendapatkan Hamiltonian tambahan "persamaan 54" dan medan magnet penggerak melalui metode *fast forward* "persamaan 56". Metode *fast forward* diterapkan dengan terlebih dahulu mendapatkan nilai eigen dari sistem Hamiltonian. Selanjutnya, dengan meninjau keadaan energi terendah *(ground state)*. Disimpulkan bahwa penelitian ini diperoleh suku Hamiltonian tambahan dengan medan magnet penggerak yang memastikan bahwa spin tunggal dapat bergerak dari keadaan awal ke keadaan akhir dalam waktu yang singkat, dengan mempertahankan karakteristik tiap level energi pada sistem tersebut.

Kata kunci: Dinamika kuantum, *fast forward*, fisika teori, frekuensi Rabi, spin kuantum

INTRODUCTION

In nanotechnology to study very small object, the importance of a short time in producing a product for industrial purposes related to materials and devices at the atomic (spin). One way to shorten the product design time is to optimize and manipulate the product manufacturing time. Nanotechnology manipulation aims to adjust the potential that depends on the dynamics of the wave function. Therefore, the concept of accelerating a process to achieve equilibrium was found, and the adiabatic concept was proposed to produce a product quickly without changing the system's characteristics (Benggadinda & Setiawan, 2021). The concept of accelerating quantum dynamics by not changing the characteristics of each energy level of the systems is called adiabatic quantum dynamics.

The adiabatic process in quantum is often used to induce or prepare for the final state in a strong and controllable manner, the adiabatic process here is a slow change of the Hamiltonian state parameters. This adiabatic process does not change the eigenic state before and after the system takes place (Chen & Muga, 2010).

The adiabatic process can be carried out but with a long time. So it is still less efficient when used to make a product. To overcome this problem, a method is needed to accelerate adiabatic quantum dynamics. Some of the methods that are being develoved are the fast forward and shortcuts to adiabaticity (STA) methods (Jarzynski, 2013).

Shortcuts to adiabaticity (STA) is a technique designed to accelerate adiabatic processes in quantum systems. Physics research often requires the development of new methods for understanding natural phenomena. One interesting method is fast forward, which allows us to accelerate the evolution of physical systems by ignoring some steps of time (Guéry-Odelin et al., 2019).

The fast forward method is a method used to accelerate the time in making a product, including fast film projection on the screen (Aszhar et al., 2024; Nakamura et al., 2017). Nakamura and Masuda succeeded in developing a fast forward method on a relativistic system (Khujakulov & Nakamura, 2016), fast forward method in carnot machines (Masuda & Nakamura, 2022), and several other studies.

The adiabatic process in microscopic systems can be observed on electrons in the form of spin motion. However, this process often takes a very long time to ensure that the systems remains in an adiabatic state (Setiawan et al., 2023). The dynamics of electron motion in spin refers to how the spin properties of electrons affect the behavior and interactions of electrons in various systems (Manoukian, 2007). Electrons have spin $\left(\frac{1}{2}\right)$ which means it can be in two states, spin up $\left(+\frac{1}{2}\right)$ or spin down $\left(-\frac{1}{2}\right)$. The phenomenon of the Hamiltonian state on the adiabatic in single-spin dynamics refers to how the spin of electrons evolves in a system in which Hamiltonians, or total energy operators, change slowly over time (Petiziol et al., 2018).

Two-level system is a quantum system that has only two levels of energy or two quantum states (Ying et al., 2020). It gives two eigenvalues (energy) and two eigenvectors (Griffiths, 1961). The time-dependent Schrödinger equation for a two-state system can be expressed as:

$$H|\Psi\rangle = E|\Psi\rangle \tag{1}$$

With:

H : Hamiltonian of the system
Ψ : Wave function of the system
E : Eigenenergy

Attempts to move the spin of electrons are done by regulating the magnetic field. The magnetic field can cause the spin of the electron to move according to the direction of the magnetic field, namely by increasing the frequency of moving the magnetic field so that the energy also increases in controlling the spin movement (Berry, 2009). By regulating the magnetic field, we can control the energy difference between spin-up and spin-down.

In this study, the Fast Forward method is applied to a single spin, by first obtaining the eigenvalues of the Hamiltonian system. Furthermore, by reviewing the lowest energy state (ground state), additional Hamiltonian terms and a driving magnetic field were obtained that allowed a single spin to move from the initial state to the final state in a short time. Hamiltonian regularization terms and the driving magnetic field will maintain the system energy fixed in the ground state energy state (as the adiabatic state) during the accelerating process.

The original Hamiltonian is obtained from the Rabi frequency model. Rabi Frequency is the frequency at which the probability amplitude of two levels of atomic energy fluctuates in an oscillating electromagnetic field. Fast forward to Rabi Frequency on single spin is all about speeding up spin control and manipulation (Duan, 2022). Increasing the Rabi frequency can control the movement of the spin, which means adjusting the external field interacting with the spin (Layton et al., 2014).

In quantum systems to describe oscillations it is necessary Rabi frequency, for the discovery of magnetic resonance of the nucleus used in magnetic resonance imaging (Xie et al., 2017). Rabi frequency is necessary to support MRI (Magnetic resonance imaging) performance. In this case, fast forward can speed up the signal. Fast forward can speed up the sequence of scans on MRI by accelerating the movement of the magnetic field (Torres et al., 2022).

In previous studies, it was also found that the fast-forward method in the adiabatic system for spin allows Hamiltonian parameter changes to be carried out quickly while maintaining adiabatic properties. By reviewing the spin for example electrons with the fast-forward method will accelerate the resonance of the change of electron state, and the process of changing the direction of the electron with the magnetic field will be faster (Setiawan, 2019). This study is theoretical research that examines the literature that discusses the method of accelerating quantum dynamics adiabatic way (Panudju et al., 2024). This method of accelerating quantum dynamics is called the Fast-Forward method. First proposed by Masuda and Nakamura in 2010, this method modifies Hamiltonians by adding additional tribes to the early Hamiltonians referred to as regularized tribes. The goal is for the Schrodinger equation to remain time-dependent (Setiawan et al., 2023).

In this study, the author combines the fast forward method with Rabi frequency to accelerate the simulation of the physics by reviewing a single spin. This study focuses on the behavior of the quantum single spin system using the fast forward and Rabi frequency methods. The purpose of this study is to obtain a method in the form of additional Hamiltonian produced by the fast forward method on single spin quantum system with Rabi frequency.

METHOD

This research is a basic research that examines the development of quantitative physical theories. This research is a literature study related to fast forward quantum theory for a single spin with Rabi Frequency analytically. Literature study is an activity to examine the theories underlying the research, both theories that are suitable for the field of science being researched and methodology (Panudju et al., 2024). This research was conducted from June to October 2024 at the Bengkulu University. This research consists of five stages, namely:

1. Preparation
   At this stage, the author prepares the research by collecting relevant and supporting literature such as books, journals, and other references related to quantum theory, Schrödinger equations, adiabatic quantum theorems, fast-forward methods, and Rabi Frequency.
2. Theoretical Studies
   Theoretical study of Schrödinger's equations using boundary conditions to find the wave function of Rabi frequency, then studied by the Rabi frequency model to obtain the frequency in moving the magnetic field which is used to determine the eigenvalue to solve the eigenvector solution. An eigenvector solution is used to describe the wave function in the system.

3. Results of the theoretical assessment
   The results of the theoretical study are in the form of wave functions that will be visualized in the form of graphs. The wave function will be used in finding the regularization term. The Fast forward method, which accelerates the dynamics of adiabatic quantum in a single spin by reviewing the lowest energy state (ground state), is obtained by Hamiltonian additional terms and a driving magnetic field that ensures that a single spin can move from the initial state to the final state in a short time. The results of the study of wave function theory on the fast forward equation with Rabi frequency.
4. Analysis and Discussion
   At this stage, the results obtained are in the form of a fast-forward method with Rabi frequency that produces regularization terms, driving magnetic fields, and an additional Hamiltonian by reviewing a single spin system to accelerate the spin motion dynamics that will be discussed systematically. Furthermore, the results of the analytical calculations will be compared with the Wolfram Mathematica 10.0 program.
5. Conclusion
   Each result and analysis of the discussion that was compiled was then concluded to answer all the problem formulations in this study.

RESULT AND DISCUSSION

In this study, a single spin system with Rabi frequency is considered. The original Hamiltonian equations of the Rabi frequency is:

$$H_0 = E_0 \cdot \sigma_0 + W_1 \cdot \sigma_X + W_2 \cdot \sigma_y + \Delta \cdot \sigma_z, \tag{2}$$

With:

σ_0 is the identity matrix, σ_x, σ_y, σ_z is Pauli's matrix, H_0 is an original Hamiltonian, E_0 is the energy constant, W is the angular frequency and Δ is the difference in energy.

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{3}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{4}$$

Substitution of the Identity matrix and the Pauli matrix on equations (2), to get an original Hamiltonian.

$$H_0 = \begin{pmatrix} E_0 + \Delta & W_1 - iW_2 \\ W_1 + iW_2 & E_0 - \Delta \end{pmatrix}. \tag{5}$$

The original Hamiltonian of the Rabi frequency model has been obtained in the equation (5). Original Hamiltonian would be used to find solutions from eigen value, with E defined as eigen value. To find the eigen value solution, the following formulation is used:

$$\text{determinant } (EI - H_0) = 0, \tag{6}$$

With:

I: Identity matrix.

$$\text{determinant} \left(E \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} E_0 + \Delta & W_1 - iW_2 \\ W_1 + iW_2 & E_0 - \Delta \end{pmatrix} \right) = 0, \tag{7}$$

$$\text{determinant} \begin{pmatrix} E - E_0 - \Delta & -W_1 + iW_2 \\ -W_1 - iW_2 & E - E_0 + \Delta \end{pmatrix} = 0, \tag{8}$$

$$(E - E_0 - \Delta)(E - E_0 + \Delta) - (-W_1 + iW_2)(-W_1 - iW_2) = 0, \tag{9}$$

Equation (9) can be simplified as follows:

$$E^2 - 2EE_0 + {E_0}^2 - \Delta^2 - {W_1}^2 - {W_2}^2 = 0, \tag{10}$$

$$(E - E_0)^2 = \Delta^2 + {W_1}^2 + {W_2}^2, \tag{11}$$

$$E = E_0 + \sqrt{\Delta^2 + {W_1}^2 + {W_2}^2}. \tag{12}$$

Equation (12) can be written as follows to generate eigen value as a solution E_+ (excited state) and E_- (ground state) which is referred to as energy,

$$E_+ = E_0 + \sqrt{\Delta^2 + {W_1}^2 + {W_2}^2}, \tag{13}$$

$$E_- = E_0 - \sqrt{\Delta^2 + {W_1}^2 + {W_2}^2}. \tag{14}$$

Furthermore, from each of these eigenvalues, the lowest eigenvalue (ground state) will be selected, namely E_- to search for eigenvector. To get the eigenvector solution the following equation is used:

$$(EI - H_0)\ \Psi = 0, \tag{15}$$

With,

$$\begin{pmatrix} E - E_0 - \Delta & -W_1 + iW_2 \\ -W_1 - iW_2 & E - E_0 + \Delta \end{pmatrix} \begin{pmatrix} \Psi 1 \\ \Psi 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{16}$$

Substitution equation (14) in equation (16) to obtain the following equation:

$$\begin{pmatrix} -\Delta - \sqrt{\Delta^2 + {W_1}^2 + {W_2}^2} & -W_1 + iW_2 \\ -W_1 - iW_2 & \Delta - \sqrt{\Delta^2 + {W_1}^2 + {W_2}^2} \end{pmatrix} \begin{pmatrix} \Psi 1 \\ \Psi 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{17}$$

Then,

$$\left(\Delta - \sqrt{\Delta^2 + {W_1}^2 + {W_2}^2}\right)\Psi_2 = (W_1 + iW_2)\Psi_1, \tag{18}$$

and obtained,

$$\Psi_2 = \frac{(W_1 + iW_2)}{\left(\Delta - \sqrt{\Delta^2 + {W_1}^2 + {W_2}^2}\right)}\ \Psi_1. \tag{19}$$

With the principle of normalization, it can be written,

$$|\Psi_1|^2 + |\Psi_2|^2 = 1, \tag{20}$$

Substitution of equation (19) to equation (20), thus obtaining the following equation,

$$\Psi_1 = \frac{\Delta - \sqrt{\Delta^2 + {W_1}^2 + {W_2}^2}}{\sqrt{(w_1 + iw_2)^2 + \Delta^2 - 2\Delta\sqrt{\Delta^2 + w_1^2 + w_2^2} + \Delta^2 + w_1^2 + w_2^2}}, \tag{21}$$

Substitution of equation (21) to equation (19), thus obtaining the following equation,

$$\Psi_2 = \frac{(W_1 + iW_2)}{\sqrt{(w_1 + iw_2)^2 + \Delta^2 - 2\Delta\sqrt{\Delta^2 + w_1^2 + w_2^2} + \Delta^2 + w_1^2 + w_2^2}}, \tag{22}$$

With,

$$G \quad : \sqrt{(w_{1+i}w_2)^2 + \Delta^2 - 2\Delta\sqrt{\Delta^2 + w_1^2 + w_2^2} + \Delta^2 + w_1^2 + w_2^2} \tag{23}$$

Then,

$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} \begin{pmatrix} C_1(R) \\ C_2(R) \end{pmatrix} = \begin{pmatrix} \dfrac{\Delta - \sqrt{\Delta^2 + {W_1}^2 + {W_2}^2}}{G} \\ \dfrac{(W_1 + iW_2)}{G} \end{pmatrix}. \tag{24}$$

If we review the Hamiltonian on a spin system with the time parameter *R(t)* with a constant t-time. We can write,

$$H_0(R)\ \Psi(R) = E(R)\ \Psi(R), \tag{25}$$

With,

$$H_0(R) \begin{pmatrix} C_1\ (R) \\ C_2\ (R) \end{pmatrix} = E(R) \begin{pmatrix} C_1\ (R) \\ C_2\ (R) \end{pmatrix}, \tag{26}$$

The solution of Schrödinger's equation on the adiabatic state, with $R(t) = R_0 + \mathcal{E}t$ is an adiabatic parameter and $\mathcal{E} \ll 1$, We assume

$$\Psi(R(t)) = \begin{pmatrix} C_1(R) \\ C_2(R) \end{pmatrix} e^{-\frac{i}{\hbar}\int_0^t E(R(t))dt'} e^{i\mathfrak{z}(t)}, \tag{27}$$

$\mathfrak{z}$ is an adiabatic phase defined by:

$$\mathfrak{z}(t) = i\int_0^t dt' \left(C_1^* \frac{\partial C_1}{\partial t} + C_2^* \frac{\partial C_2}{\partial t} \right), \tag{28}$$

$$= i\varepsilon \int_0^t dt' \left(C_1^* \frac{\partial C_1}{\partial R} + C_2^* \frac{\partial C_2}{\partial R} \right), \tag{29}$$

So that equation (24) can be rewritten as:

$$\Psi(R(t)) = \begin{pmatrix} \dfrac{\Delta - \sqrt{\Delta^2 + W_1{}^2 (R(t)) + W_2{}^2}}{G} \\ \dfrac{(W_1 (R(t)) + iW_2)}{G} \end{pmatrix} e^{-\frac{i}{\hbar}\int_0^t E_0 - \sqrt{\Delta^2 + w_1^2(R(t)) + w_2^2}\, dt'}. \tag{30}$$

In order to maintain the adiabatic condition, the concept of regularization is used. The regularization term, $\widetilde{\mathcal{H}}$ obtained as

$$\widetilde{\mathcal{H}} \begin{pmatrix} C_1(R) \\ C_2(R) \end{pmatrix} = i\hbar \frac{\partial}{\partial R} \begin{pmatrix} C_1(R) \\ C_2(R) \end{pmatrix} - i\hbar \left(C_1^* \frac{\partial C_1}{\partial R} + C_2^* \frac{\partial C_2}{\partial R} \right) \begin{pmatrix} C_1(R) \\ C_2(R) \end{pmatrix}, \tag{31}$$

The second term on the right field in equation (31) as :

$$|C_1|^2 + |C_2|^2 = 1, \tag{32}$$

$$C_1 \cdot C_1^* + C_2 \cdot C_2^* = 1, \tag{33}$$

Then,

$$\frac{\partial}{\partial R} (C_1 \cdot C_1^* + C_2 \cdot C_2^*) = 0, \tag{34}$$

So that equation (31) can be rewritten like the following equation:

$$\widetilde{\mathcal{H}} \begin{pmatrix} C_1(R) \\ C_2(R) \end{pmatrix} = \begin{pmatrix} i\hbar \dfrac{\partial C_1(R)}{\partial R} \\ i\hbar \dfrac{\partial C_2(R)}{\partial R} \end{pmatrix}. \tag{35}$$

With,

$$a = \frac{\partial C_1}{\partial R}, \tag{36}$$

$$b = \frac{\partial C_2}{\partial R}, \tag{37}$$

In quantum mechanics, energy must have a real value because energy is measurable. In order for energy to have real value, the total energy operator in the sistem $\widetilde{\mathcal{H}}$ must be Hermitian. $\widetilde{\mathcal{H}}_{11} = -\widetilde{\mathcal{H}}_{22}$ and $\widetilde{\mathcal{H}}_{21}{}^* = \widetilde{\mathcal{H}}_{12}$ (Setiawan, 2019).

$$\begin{pmatrix} \widetilde{\mathcal{H}}_{11} & \widetilde{\mathcal{H}}_{12} \\ \widetilde{\mathcal{H}}_{21} & -\widetilde{\mathcal{H}}_{22} \end{pmatrix} \begin{pmatrix} C_1(R) \\ C_2(R) \end{pmatrix} = i\hbar \begin{pmatrix} a \\ b \end{pmatrix}, \tag{38}$$

$$\begin{pmatrix} \widetilde{\mathcal{H}}_{11} C_1(R) & \widetilde{\mathcal{H}}_{12} C_2(R) \\ \widetilde{\mathcal{H}}_{21} C_1(R) & -\widetilde{\mathcal{H}}_{22} C_2(R) \end{pmatrix} = \begin{pmatrix} i\hbar a \\ i\hbar b \end{pmatrix}, \tag{39}$$

$$\begin{pmatrix} C_1 & C_2 \\ -C_2 & C_1 \end{pmatrix} \begin{pmatrix} \widetilde{\mathcal{H}}_{11} \\ \widetilde{\mathcal{H}}_{12} \end{pmatrix} = \begin{pmatrix} i\hbar a \\ i\hbar b \end{pmatrix}, \tag{40}$$

We can solve equation (31) for $\widetilde{\mathcal{H}}$:

$$\begin{pmatrix} \widetilde{\mathcal{H}}_{11} \\ \widetilde{\mathcal{H}}_{12} \end{pmatrix} = \begin{pmatrix} C_1 & C_2 \\ -C_2 & C_1 \end{pmatrix}^{-1} \begin{pmatrix} i\hbar a \\ i\hbar b \end{pmatrix}, \tag{41}$$

$$\begin{pmatrix} \widetilde{\mathcal{H}}_{11} \\ \widetilde{\mathcal{H}}_{12} \end{pmatrix} = \begin{pmatrix} \dfrac{\Delta - \sqrt{\Delta^2 + W_1{}^2 + W_2{}^2}}{G} & \dfrac{(W_1 + iW_2)}{G} \\ \dfrac{-(w_{1+i} w_2)}{G} & \dfrac{\Delta - \sqrt{\Delta^2 + W_1{}^2 + W_2{}^2}}{G} \end{pmatrix}^{-1} \begin{pmatrix} i\hbar a \\ i\hbar b \end{pmatrix}, \tag{42}$$

Then we have,

$$\begin{pmatrix}\widetilde{\mathcal{H}}_{11}\\ \widetilde{\mathcal{H}}_{12}\end{pmatrix} = \begin{pmatrix} i\hbar a\frac{\left(\Delta-\sqrt{\Delta^2+W_1{}^2+W_2{}^2}\right)}{G} & - & i\hbar b\frac{(w_1+iw_2)}{G} \\ i\hbar a\frac{(w_1+iw_2)}{G} & + & i\hbar b\frac{\left(\Delta-\sqrt{\Delta^2+W_1{}^2+W_2{}^2}\right)}{G}\end{pmatrix}. \tag{43}$$

The equation of the matrix $\widetilde{\mathcal{H}}$ is:

$$\begin{pmatrix}\widetilde{\mathcal{H}}_{11} & \widetilde{\mathcal{H}}_{12}\\ \widetilde{\mathcal{H}}_{21}{}^{*} & -\widetilde{\mathcal{H}}_{22}\end{pmatrix}, \tag{44}$$

So that the matrix equation is obtained $\widetilde{\mathcal{H}}$, namely:

$$\begin{pmatrix} i\hbar a\frac{\left(\Delta-\sqrt{\Delta^2+W_1{}^2+W_2{}^2}\right)}{G}-i\hbar b\frac{(w_1+iw_2)}{G} & i\hbar a\frac{(w_1+iw_2)}{G}+i\hbar b\frac{\left(\Delta-\sqrt{\Delta^2+W_1{}^2+W_2{}^2}\right)}{G} \\ -i\hbar a\frac{(w_1-iw_2)}{G}-i\hbar b\frac{\left(\Delta-\sqrt{\Delta^2+W_1{}^2+W_2{}^2}\right)}{G} & -i\hbar a\frac{\left(\Delta-\sqrt{\Delta^2+W_1{}^2+W_2{}^2}\right)}{G}+i\hbar b\frac{(w_1+iw_2)}{G}\end{pmatrix}. \tag{45}$$

Furthermore, by using the fast forward equation to accelerate the system in an adiabatic state. The accelerated process is done using the time scaling factor α and adiabatic parameters ε (Setiawan et al., 2017).

$$\alpha(t)=\overline{\alpha}-(\alpha-1)\cos\left(\frac{2\pi}{T_{FF}}t\right) \tag{46}$$

Here $\nu(t)$ is a velocity function available from *α(t)* in the asymptotic limit,

$$\nu(t) = \lim_{\varepsilon\to 0,\tilde{\alpha}\to\infty} \varepsilon\alpha(t), \tag{47}$$

$$= \bar{v}\left(1-\cos\frac{2\pi}{T_{FF}}t\right) \tag{48}$$

The adiabatic quantum dynamics in equation (30) will be accelerated by multiplying the time parameter by assuming R(Λ(t)),

Where is an advanced time defined by:

$$\Lambda(t) = \int_0^t \alpha(t')\,dt', \tag{49}$$

Where $\bar{v}\ \lim_{\varepsilon\to 0,\tilde{\alpha}\to\infty}\overline{\alpha}\varepsilon$ (finite) is the mean of $\nu(t)$. Consequently, for $0 \le t \le T_{FF}$,

$$R(\Lambda(t)) = R_0 + \lim_{\varepsilon\to 0,\tilde{\alpha}\to\infty} \varepsilon(\Lambda(t), \tag{50}$$

$$= R_0 + \int_0^t \nu(t')\,dt', \tag{51}$$

$$= R_0 + \bar{v}\left[t-\frac{T_{FF}}{2\pi}\sin\left(\frac{2\pi}{T_{FF}}t\right)\right] \tag{52}$$

The fast forward state is defined by:

$$\Psi_{FF}(t) = \begin{pmatrix}\frac{\Delta-\sqrt{\Delta^2+W_1{}^2 R(\Lambda(t))+W_2{}^2}}{G}\\ \frac{(W_1 R(\Lambda(t))+iW_2)}{G}\end{pmatrix} e^{-\frac{i}{\hbar}\int_0^t E_0-\sqrt{\Delta^2+w_1^2 R(\Lambda(t))+w_2^2}\,dt'}. \tag{53}$$

By using the new parameter R(Λ(t)), the additional Hamiltonian is obtained using the following equation:

$$\mathcal{H} = \nu(t)\,\widetilde{\mathcal{H}} =$$

$$\begin{pmatrix} \nu(t)\left(i\hbar a\frac{\Delta-\sqrt{\Delta^2+W_1{}^2R(\Lambda(t))+W_2{}^2}}{G}-i\hbar b\frac{(W_1R(\Lambda(t))+iW_2)}{G}\right) & \nu(t)\left(i\hbar a\frac{(W_1R(\Lambda(t))+iW_2)}{G}+i\hbar b\frac{\Delta-\sqrt{\Delta^2+W_1{}^2R(\Lambda(t))+W_2{}^2}}{G}\right)\\ \nu(t)\left(-i\hbar a\frac{(W_1R(\Lambda(t))-iW_2)}{G}-i\hbar b\frac{\Delta-\sqrt{\Delta^2+W_1{}^2R(\Lambda(t))+W_2{}^2}}{G}\right) & \nu(t)\left(-i\hbar a\frac{\Delta-\sqrt{\Delta^2+W_1{}^2R(\Lambda(t))+W_2{}^2}}{G}+i\hbar b\frac{(W_1R(\Lambda(t))+iW_2)}{G}\right)\end{pmatrix} \tag{54}$$

With:

m : $\Delta-\sqrt{\Delta^2+W_1{}^2 R(\Lambda(t))+W_2{}^2}$

d : $W_1 R(\Lambda(t)) + iW_2$

p : $W_1 R(\Lambda(t)) - iW_2$

Fast forward in relation to Rabi frequency on single spin is about accelerating spin control and manipulation through intelligent and optimized methods, thus enabling faster and more efficient operation in quantum systems (Layton et al., 2014). The Hamiltonian will be accelerated with the following fast forward equation:

$$H_{FF} = H_0\, R(\Lambda(t)) + \nu(t)\, \tilde{\mathcal{H}} \tag{55}$$

(Setiawan et al., 2017).

$$H_{FF} = \begin{pmatrix} (E_0 + \Delta) + \left(\nu(t)\left(i\hbar a\frac{m}{G} - i\hbar b\frac{d}{G}\right)\right) & (W_1(R(\Lambda(t)) - iW_2) + \left(\nu(t)\left(i\hbar a\frac{d}{G} + i\hbar b\frac{m}{G}\right)\right) \\ (W_1(R(\Lambda(t)) + iW_2) + \left(\nu(t)\left(-i\hbar a\frac{p}{G} - i\hbar b\frac{m}{G}\right)\right) & (E_0 - \Delta) + \left(\nu(t)\left(-i\hbar a\frac{m}{G} + i\hbar b\frac{d}{G}\right)\right) \end{pmatrix} \tag{56}$$

By selecting the parameters $\nu = 100$, the time for the final state (T). $T = 1$, $W_1 = R$, $W_2 = 1$, and $R = 2\nu\left(\frac{t}{2} - \frac{T\sin\left[\frac{2\pi t}{T}\right]}{4\pi}\right)$, $|C_1|^2$ and $|C_2|^2$ which evolved adiabatically can be described as:

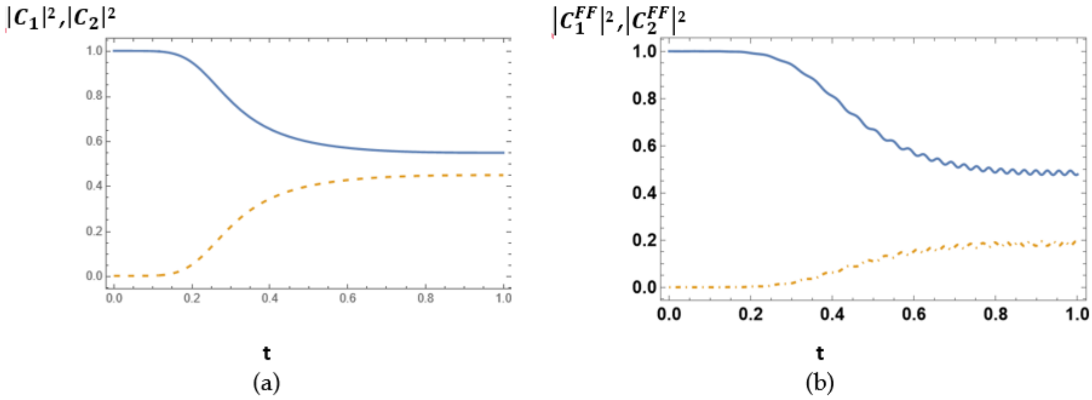

Figure 1. The figure of $[C_1]^2$ (solid line) and $[C_2]^2$ (dashed line) obtaining (a). from eigenvector (b). by solving TDSE

Figure 1 (a) shows a graph of the wave function in the initial state from eigenvector before the regularization term is added. Early state (solid line) $C_1\uparrow$ end $C_1\downarrow$ and at (dashed line) $C_2\downarrow$ end $C_2\uparrow$. Endpoints i.e. C_1 (0.550) and C_2 (0.450) Figure 1 (b) shows a graph of the wave function in the final state by solving TDSE after adding the regularization term or additional energy, by adding the parameterization $E_0 = 10$, $\Delta = 50$, shows that the spin moves with time scaling factor i.e. (T_{FF}) moving from a state of (solid line) $C_1^{FF}\uparrow$ end $C_1^{FF}\downarrow$ and (dashed line) $C_2^{FF}\downarrow$ end $C_2^{FF}\uparrow$. Endpoints i.e. C_1^{FF} (0.478), C_2^{FF}(0.192). In both figures show the starting and ending points are not much different, so adiabatic conditions can be maintained. This indicates that the spin is moving from the direction of up ↑ to down ↓ at the time of final position (Setiawan, 2019). In the fast-forward concept, the direction of the magnetic field changes to the opposite direction in time (T_{FF}) (Benggadinda & Setiawan, 2021). The adiabatic condition can be maintained in the figure because it shows the same beginning and end states with time parameterization.

The quantum adiabatic theorem is that if the system is initially in a certain eigenstate, it will remain in that eigenstate during the adiabatic process (Setiawan, 2019). The adiabatic process in both figures occurs when the external parameters of the Hamiltonian change slowly. Seen in figure 1 (a) the adiabatic process occurs and figure 1 (b) shows the adiabatic conditions that do not change and are maintained after adding an additional hamiltonian ($\mathcal{H}$) with the system accelerated through the fast forward Hamiltonian (H_{FF}) method. The Hamiltonian regularization term and the driving magnetic field will keep the system energy in the ground state energy state (known as the adiabatic state) during the accelerating process.

CONCLUSION

In this study we obtained the driving Hamiltonian by modifying the wave function as an adiabatic wave function then by solving the Schrödinger equation, we received the regularization term. By choosing the time scaling parameter we get the additional Hamiltonian ($\mathcal{H}$) "equation (54)" and the fast-forward Hamiltonian (H_{FF}) "equation (56)". It can be seen that the dynamics of the wave function obtained from the eigenvector and the wave function obtained by solving TDSE (with additional Hamiltonian $\widetilde{\mathcal{H}}$), still preserve the initial and final state.

The author recommends that future researchers can produce similar graphical images by trying more suitable parameterizations. The same figure will show the amount of energy that does not change when the process of adiabatic dynamics occurs. The addition of the number of spins is needed to stabilize this research.

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