

Exploring pre-service mathematics teachers' thinking for solving linear programming word problems

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Abstract

Solving word problems in linear programming presents significant challenges, not only for secondary school students but also for pre-service mathematics teachers. This study aims to investigate the cognitive processes of pre-service mathematics teachers in solving word problems related to linear programming. To achieve this objective, a comprehensive review of mathematics textbooks designed for pre-service teachers and secondary school students, as well as the corresponding curriculum, was conducted to identify an appropriate learning sequence for this topic. Subsequently, key problems were selected to facilitate learning, and predictions regarding the cognitive processes involved in solving these problems were formulated based on Newman's error analysis framework. Following this preparatory phase, an individual written assessment was administered to 27 pre-service mathematics teachers to examine their problem-solving approaches in linear programming word problems. The findings of this study include the identification of essential word problems in linear programming and a comparative analysis between the predicted and actual problem-solving processes exhibited by the participants. In conclusion, this study highlights the potential of cognitive process predictions in anticipating learning difficulties and informing instructional strategies. These insights can be leveraged to provide targeted support for pre-service teachers facing challenges in problem-solving and to develop pedagogical interventions aimed at enhancing their problem-solving skills.

Keywords: Algebra Education, Linear Programming, Newman Error Analysis, Pre-service Mathematics Teachers, Word Problems

Introduction

Solving word problems in mathematics is widely recognized as a challenging task, not only for secondary school students but also for pre-service mathematics teachers (Jupri & Drijvers,

2016; Pongsakdi et al., 2020; Siniguan, 2017; Verschaffel et al., 2020). Among the various mathematical topics that involve word problems, linear programming is particularly rich in real-world applications and problem-solving opportunities (Molnár, 2016; Rocha & Babo, 2024). Although this topic is covered in a dedicated course for pre-service mathematics teachers, commonly referred to as Linear Programming, research on strategies, challenges, and cognitive processes involved in solving linear programming word problems remains relatively limited (TLS & Herman, 2020). Mastery of this topic, especially in the context of solving word problems, is essential for pre-service teachers as they prepare for their future roles as mathematics educators.

Existing studies on pre-service mathematics teachers' ability to solve algebraic word problems indicate the need for improvement in critical thinking, creativity, and problem-solving skills (Getenet, 2024; Jupri & Hidayat, 2022; Jupri et al., 2021). The primary difficulties encountered in solving word problems include comprehending the problem, translating it into a mathematical model, and subsequently solving the model (Barham, 2020; Pagiling et al., 2020). Prior research on understanding and analyzing the process of solving algebraic word problems has frequently employed Newman's error analysis (Kurniati & Sagita, 2021; Wijaya et al., 2014). Therefore, it is valuable to extend the application of Newman's error analysis to explore the cognitive processes of pre-service mathematics teachers when solving linear programming word problems. By doing so, this study aims to provide deeper insights into the thinking patterns of pre-service teachers and contribute to the broader understanding of problem-solving challenges in mathematics education.

The main question of this study concerns the cognitive processes of pre-service mathematics teachers when solving linear programming word problems. To investigate this, the present study explores their thinking processes through the lens of Newman's error analysis. This analytical framework categorizes errors in problem-solving into five distinct types: reading, comprehension, transformation, process skills, and encoding (Newman, 1983; Patac & Patac, 2015; Thomas & Mahmud, 2021).

The following is a detailed description of these five error categories as defined by Newman (1983), Patac and Patac (2015), and Thomas and Mahmud (2021). Reading errors occur when individuals fail to correctly recognize words or mathematical symbols within a problem statement. Comprehension errors refer to difficulties in understanding the meaning or intent of a problem. Transformation errors arise when a word problem is incorrectly translated into a mathematical model. Process skill errors involve mistakes in executing mathematical procedures or operations. Finally, encoding errors pertain to inaccuracies in representing the mathematical solution in a coherent and acceptable written format.

By applying this framework, the study seeks to gain deeper insights into the specific difficulties encountered by pre-service mathematics teachers in solving linear programming word problems. This understanding can contribute to the development of targeted instructional strategies to enhance their problem-solving competencies.

Methods

A qualitative case study approach was employed in this research to examine the cognitive processes of pre-service mathematics teachers when solving word problems. The study was conducted in four sequential stages. First, a comprehensive review of mathematics textbooks and the corresponding curriculum for pre-service mathematics teachers was undertaken to gather information on the instructional sequence for linear programming. This stage facilitated an in-depth understanding of the scope and depth of the topic, ensuring its alignment with the preparation of pre-service teachers for teaching mathematics at the secondary school level. Additionally, secondary school mathematics textbooks were analyzed to ascertain the coverage of linear programming at this educational level. Given that the pre-service teachers will be instructing secondary school students, the present study was limited to exploring the aspects of linear programming relevant to this level.

Second, based on the findings from the initial stage, typical word problems related to linear programming were identified. Two representative tasks were selected to illustrate key problem-solving scenarios in linear programming, specifically those involving the determination of maximum and minimum values. Subsequently, predictions regarding the cognitive processes of pre-service mathematics teachers in solving these tasks were formulated through the lens of Newman's error analysis.

Third, an individual written assessment was administered to 27 pre-service mathematics teachers to evaluate their approaches to solving the selected linear programming word problems. The assessment lasted for approximately 30 minutes, and participants were prohibited from using electronic devices, such as smartphones or laptops, during the test. The participants, who were in their second year of a mathematics education program, were enrolled in a course on secondary school mathematics taught by the first author. Since they had previously completed a course on linear programming in their first year of study, they were deemed adequately prepared to participate in this research. Ethical considerations were addressed by obtaining formal approval from the mathematics education program and securing informed consent from all participants.

Finally, the written responses were analyzed using Newman's error analysis framework and compared with the pre-established predictions of pre-service teachers' cognitive processes. Newman's framework, which categorizes errors into five distinct types, as outlined in the preceding section, was utilized to gain insights into the strategies employed by pre-service teachers when solving word problems. The analysis was conducted by the first and second authors, with the findings subsequently reviewed and verified by the third author. The results of this analysis serve as a foundational basis for designing instructional sequences tailored to the teaching of linear programming for pre-service mathematics teachers.

Results and Discussion

This section examines the anticipated cognitive processes and actual solution strategies employed by pre-service mathematics teachers when solving linear programming word

problems. The subsequent subsections present a comparative analysis of the predicted and observed thinking processes of pre-service teachers in solving such problems, specifically in the context of determining maximum and minimum values. The analysis is conducted through the framework of Newman's error analysis.

Pre-service Mathematics Teachers' Thinking in Solving Maximum Value Problem

A representative problem selected for assessing pre-service mathematics teachers' ability to determine maximum values in linear programming word problems is presented in [Figure 1](#).

On land covering an area of 10,000 m², type A and type B houses will be built. For type A houses, 100 m² is needed. For type B houses, 75 m² is required. The maximum number of houses to be built will be 125 housing units. The profit from selling type A houses is IDR 60,000,000.00/unit and type B is IDR 40,000,000.00/unit. By providing sufficient description and explanation, determine the maximum profit that can be obtained from selling the house.

Figure 1. Task for determining the maximum value

It was anticipated that pre-service mathematics teachers would adopt a standard procedural approach in solving this linear programming problem. The expected solution process comprises the following steps: comprehending the problem statement, formulating a mathematical model, graphing the model in a Cartesian coordinate system, applying the corner point method to determine optimal values, and drawing appropriate conclusions. A visual representation of this anticipated solution process is provided in [Figure 2](#).

From the perspective of Newman's error analysis (Agustiani, [2021](#); Newman, [1983](#); Saleh et al., [2017](#); White, [2010](#)), a correct solution process would indicate that pre-service teachers successfully interpreted and understood the problem, accurately transformed it into a mathematical model, executed the solution process appropriately, and correctly encoded the final answer. However, potential errors were anticipated in the step involving the transformation of the word problem into a system of linear inequalities. Additionally, miscalculations during the computational process were expected. Prior research (e.g., Hickendorff, [2021](#); Jupri & Drijvers, [2016](#)) has highlighted that converting a word problem into an appropriate mathematical model is one of the most challenging aspects of solving such problems.

An analysis of the collected data revealed that 18 out of 27 pre-service mathematics teachers successfully solved the problem, while nine participants produced incorrect solutions. Generally, the solution strategies employed by those who answered correctly aligned with the predicted approach, as illustrated in [Figure 2](#). A typical correct solution process involved systematically identifying known and unknown quantities by defining variables x and y , constructing a mathematical model in the form of a system of linear inequalities, solving the system by graphing the constraints in a Cartesian coordinate plane, conducting a corner point evaluation, and drawing a conclusion. [Figure 3](#) presents an example of a correct solution, which closely mirrors the predicted solution process.

Let x be the number of type A houses; and y be the number of type B houses. Based on the information within the task, we will obtain the following mathematical model.

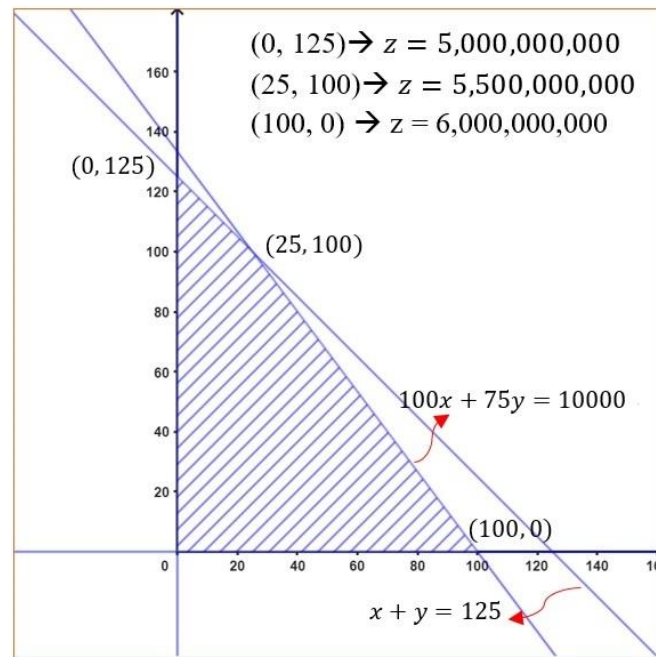
$$100x + 75y \leq 10000.$$

$$x + y \leq 125.$$

$$x \geq 0.$$

$$y \geq 0.$$

The objective function is $z = f(x, y) = 60000000x + 40000000y$.

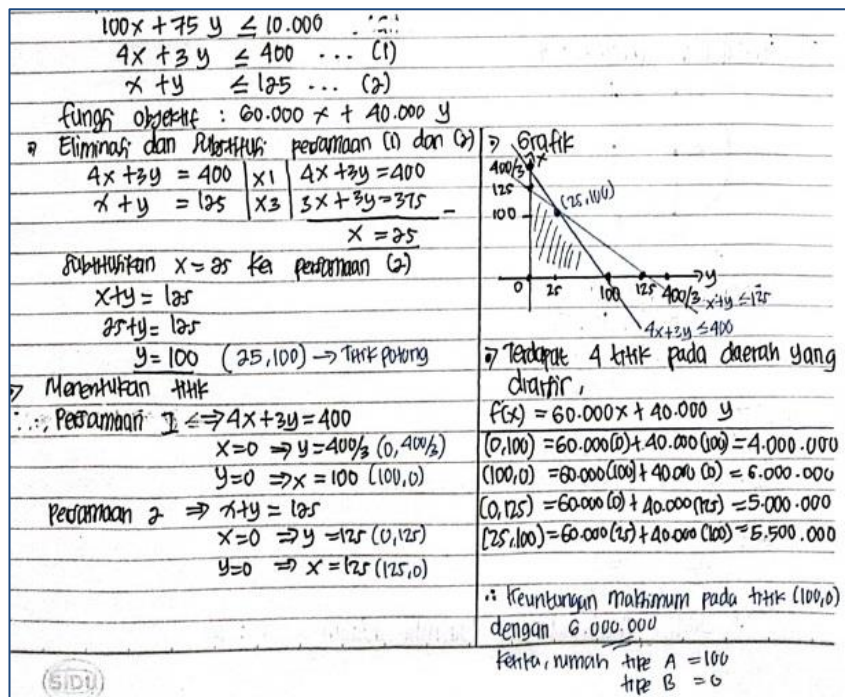


Based on the above solution process, it can be concluded that the maximum profit is IDR 6,000,000,000.00. This occurs when 100 and 0 type A and type B houses are built respectively.

Figure 2. Prediction of a correct solution to a maximum value task

In addition to computational errors, two distinct types of errors were identified in the written solutions of pre-service mathematics teachers. First, some participants formulated an incorrect mathematical model by constructing a system of equations instead of a system of inequalities with two variables. This indicates a fundamental misunderstanding of the key concepts and terminology required to translate the problem statement into appropriate mathematical constraints. Second, while some participants successfully formulated the correct system of inequalities, they failed to represent the system graphically in a Cartesian plane to determine the feasible solution region. Consequently, they did not perform the corner point analysis, which is essential for identifying the optimal solution, leading to incorrect conclusions.

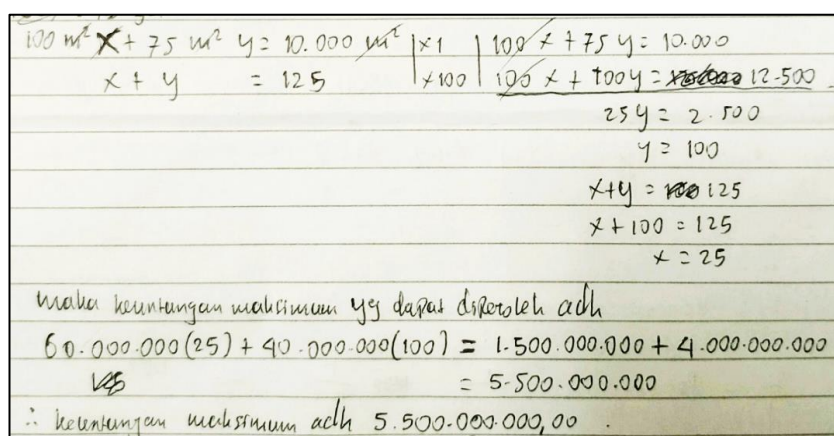
From the perspective of Newman's error analysis, the first type of error stems from difficulties in comprehending the problem statement and in transforming it into a proper mathematical model. Meanwhile, the second type of error arises from an incomplete execution of the solution process, as participants failed to follow the necessary procedural steps to reach a valid conclusion (Crouch & Haines, 2004; Jankvist & Niss, 2020; Newman, 1983).



Translation of the last part: The maximum profit, occurs at the point (100, 0), is 6,000,000 when the type A houses = 100, and type B houses = 0.

Figure 3. An example of correct solution for finding a maximum value

Figure 4 illustrates a representative example of the first type of error. As shown in the figure, the participant incorrectly formulates the mathematical model as a system of equations rather than a system of inequalities. By applying the elimination method, the participant obtains $x = 25$ and $y = 100$, which are then substituted into the objective function to yield a maximum profit of IDR 5,500,000,000.00. This result is evidently incorrect, as the erroneous formulation of the mathematical model leads to an inaccurate solution.



Translation:

Therefore, the maximum profit that is obtained:

$$\begin{aligned}
 60,000,000(25) + 40,000,000(100) &= 1,500,000,000 + 4,000,000,000 \\
 &= 5,500,000,000
 \end{aligned}$$

The maximum profit is 5,500,000,000.00

Figure 4. An example of incorrect solution for finding a maximum value

Pre-service Mathematics Teachers' Thinking in Solving Minimum Value Problem

A typical task involving linear programming word problems for determining minimum values is presented in Figure 5.

A farmer needs at least 16 units of substance A and 14 units of substance B to fertilize his vegetable garden. He obtained these two chemicals from liquid fertilizer and dry fertilizer. One flask of liquid fertilizer which costs IDR 40,000.00 contains 5 units of substance A and 3 units of substance B. Meanwhile, one bag of dry fertilizer which costs IDR 32,000.00 contains 3 units of substance A and 4 units of substance B. How many flasks of liquid fertilizer and how many bags of dry fertilizer should the farmer buy at the cheapest possible price?

Figure 5. Task for determining a minimum value

As with the maximum value problem, it was anticipated that pre-service mathematics teachers would approach this task using standard procedural strategies for solving linear programming word problems. The expected solution process involves sequentially reading and comprehending the given information, formulating a mathematical model in the form of a system of inequalities with two variables, solving the model using the graphical method, performing corner point evaluations through calculations, and drawing a conclusion. A predicted correct solution is presented in Figure 6.

From the perspective of Newman error analysis, if pre-service teachers successfully follow this solution process, it indicates their ability to accurately read and comprehend the task, transform the problem into a valid system of inequalities, execute the solution process correctly, and properly encode the final solution (Halim & Rasidah, 2019; Newman, 1983; Noutsara et al., 2021).

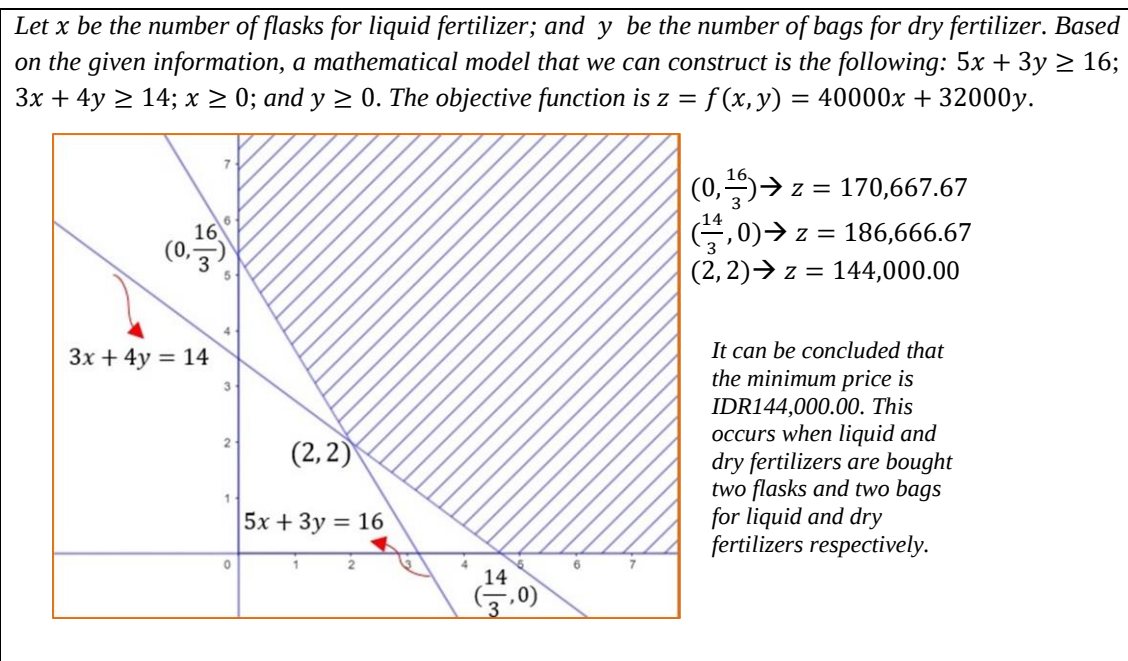


Figure 6. Prediction of a correct solution for a minimum value problem

An analysis of the collected data revealed that 20 out of 27 pre-service mathematics teachers correctly solved the task, while seven committed errors. Similar to the case of the maximum value problem, the solution strategies employed by those who arrived at correct answers aligned with the predicted solution shown in Figure 6. A representative correct solution involved identifying known and unknown variables, defining appropriate variables (x and y), constructing a mathematical model, solving the model graphically, performing corner point tests through calculations, and drawing a valid conclusion.

However, errors were identified in the solutions provided by some pre-service teachers, in addition to basic calculation mistakes. The most common error was related to the formulation of the mathematical model. Instead of constructing the correct system of inequalities, some participants incorrectly formulated the constraints. Figure 7 presents an example of such an error, where the participant mistakenly wrote: $5x + 3y \leq 16$ and $3x + 4y \leq 14$ instead of the correct inequalities: $5x + 3y \geq 16$ and $3x + 4y \geq 14$.

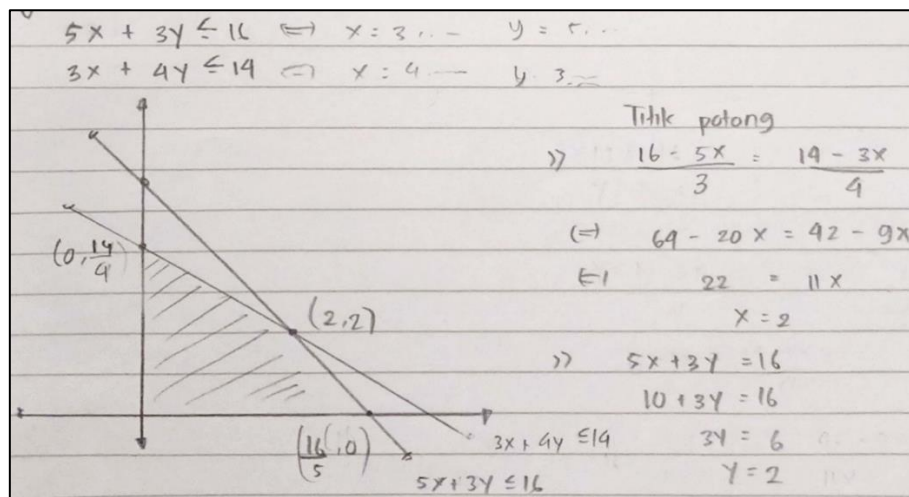


Figure 7. Example of an incorrect solution for the minimum value problem

From the perspective of Newman error analysis, this type of error suggests difficulties in reading and comprehending the problem statement, leading to an incorrect transformation of the given information into a mathematical model (Halim & Rasidah, 2019; Makgakga, 2023; Newman, 1983). The primary misconception observed was the misinterpretation of key terms, particularly in choosing between the "greater than or equal to" ($\geq$) and "less than or equal to" ($\leq$) symbols. As a consequence, the incorrect inequalities resulted in an erroneous feasible region and ultimately led to an incorrect solution.

A potential contributing factor to this error is the ambiguity of the phrase "...at the cheapest possible price." Several participants misinterpreted this phrase, associating it with the "less than or equal to" ($\leq$) condition rather than the correct "greater than or equal to" ($\geq$) condition. Addressing such linguistic ambiguities in teaching and learning practices could help mitigate these errors in the future.

Conclusion

This study provides two insights into the thinking processes of pre-service mathematics teachers when solving linear programming word problems. The findings indicate that their solution strategies predominantly align with standard procedural approaches, particularly the graphical method. The problem-solving sequence generally involves identifying known and unknown quantities, defining variables, formulating a system of inequalities, constructing a feasible region, applying the graphical method, performing corner point evaluations, and drawing conclusions. However, from a creativity perspective, the reliance on procedural strategies suggests a lack of innovative problem-solving approaches. This limitation may be attributed to the nature of the tasks, which provide minimal opportunities for diverse solution processes. Prior research (Basic et al., 2022; Jupri & Hidayat, 2022; Levav-Waynberg & Leikin, 2012) supports the notion that task structure influences problem-solving creativity. Given this, future research should further explore pre-service teachers' creative thinking in addressing linear programming problems, particularly in contexts that encourage alternative solution methods.

Additionally, the study highlights common errors in pre-service teachers' problem-solving processes, particularly in constructing mathematical models and executing solution steps. Besides routine calculation mistakes, two major errors were identified: incorrect formulation of mathematical models and improper application of solution procedures. From the perspective of Newman's error analysis (Newman, 1983), difficulties in comprehending tasks and transforming information contributed to errors in model construction, while challenges in executing algebraic procedures led to incorrect solutions (Thomas & Mahmud, 2021). These findings emphasize the need for further research into the specific difficulties pre-service mathematics teachers encounter in solving linear programming problems. Future studies should investigate instructional strategies that can enhance their conceptual understanding and problem-solving accuracy, particularly in overcoming model formulation and algebraic manipulation challenges.

Despite these contributions, this study has several limitations. The research was conducted with a limited number of participants and tasks, restricting the generalizability of the findings. Furthermore, the absence of interviews prevented an in-depth exploration of pre-service teachers' thought processes, difficulties, and creative reasoning. A more comprehensive approach incorporating multiple data sources, such as think-aloud protocols and interviews, would provide deeper insights into their cognitive strategies. Future research should address these limitations by employing a larger sample size, incorporating diverse problem types, and utilizing qualitative methods to triangulate findings. These enhancements would contribute to a more robust understanding of pre-service mathematics teachers' thinking and problem-solving abilities in linear programming contexts.

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Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been covered completely by the authors.

References

- Agustiani, N. (2021). Analyzing students' errors in solving sequence and series application problems using Newman procedure. *International Journal on Emerging Mathematics Education*, 5(1), 23-32. <http://dx.doi.org/10.12928/ijeme.v5i1.17377>
- Barham, A. I. (2020). Investigating the development of pre-service teachers' problem-solving strategies via problem-solving mathematics classes. *European Journal of Educational Research*, 9(1), 129-141. <https://doi.org/10.12973/eu-jer.9.1.129>
- Basic, A., Arsic, B., Gajic, A., Parezanovic, R. Z., Macesic, D., & Petrovic, T. L. (2022). Creativity in teaching mathematics. *Human Research in Rehabilitation*, 12(2). <https://doi.org/10.21554/hrr.092203>
- Crouch, R., & Haines, C. (2004). Mathematical modelling: transitions between the real world and the mathematical model. *International Journal of Mathematical Education in Science and Technology*, 35(2), 197-206. <https://doi.org/10.1080/00207390310001638322>
- Getenet, S. (2024). Pre-Service teachers and ChatGPT in multistrategy problem-solving: Implications for mathematics teaching in primary schools. *International Electronic Journal of Mathematics Education*, 19(1), em0766. <https://doi.org/10.29333/iejme/14141>
- Halim, F. A., & Rasidah, N. I. (2019). Analisis kesalahan siswa dalam menyelesaikan soal cerita aritmatika sosial berdasarkan prosedur Newman [Analysis of student errors in resolving the problem of social arithmetic stories based on Newman procedure]. *Jurnal Pendidikan Matematika*, 2(1), 35-44. <http://dx.doi.org/10.30656/gauss.v2i1.1406>
- Hickendorff, M. (2021). The demands of simple and complex arithmetic word problems on language and cognitive resources. *Frontiers in Psychology*, 12, 727761. <https://doi.org/10.3389/fpsyg.2021.727761>
- Jankvist, U. T., & Niss, M. (2020). Upper secondary school students' difficulties with mathematical modelling. *International Journal of Mathematical Education in Science and Technology*, 51(4), 467-496. <https://doi.org/10.1080/0020739X.2019.1587530>

- Jupri, A., & Drijvers, P. (2016). Student difficulties in mathematizing word problems in algebra. *Eurasia Journal of Mathematics, Science and Technology Education*, 12(9), 2481-2502. <https://doi.org/10.12973/eurasia.2016.1299a>
- Jupri, A., & Hidayat, A. S. (2022). Problem-solving approach and its impact on creative thinking ability of prospective mathematics teachers. *Jurnal Pendidikan Matematika*, 16(3), 257-268. <https://doi.org/10.22342/jpm.16.3.17820.257-268>
- Jupri, A., Usdiyana, D., & Sispiyati, R. (2021). Teaching and learning process for mathematization activities: The case of solving maximum and minimum problems. *Journal of Research and Advances in Mathematics Education*, 6(2), 100-110. <https://doi.org/10.23917/jramathedu.v6i2.13263>
- Kurniati, U., & Sagita, L. (2021). Error analysis using Newman procedures and the mathematical representation ability of pre service English teachers. *Anatolian Journal of Education*, 6(2), 135-156. <https://doi.org/10.29333/aje.2021.6211a>
- Levav-Waynberg, A., & Leikin, R. (2012). The role of multiple solution tasks in developing knowledge and creativity in geometry. *The Journal of Mathematical Behavior*, 31(1), 73-90. <https://doi.org/10.1016/j.jmathb.2011.11.001>
- Makgaka, T. P. (2023). Solving quadratic equations by completing the square: Applying Newman's error analysis model to analyse grade 11 errors. *Pythagoras*, 44(1), 1-11. <https://doi.org/10.4102/pythagoras.v44i1.742>
- Molnár, P. (2016). Solving a linear optimization word problems by using GeoGebra. *International Journal of Information and Communication Technologies in Education*, 5(2), 16-28. <https://doi.org/10.1515/ijicte-2016-0>
- Newman, M. A. (1983). *Strategies for diagnosis and remediation*. Sydney: Harcourt Brace Jovanovich.
- Noutsara, S., Neunjhem, T., & Chemrutsame, W. (2021). Mistakes in mathematics problems solving based on Newman's error analysis on set materials. *Journal La Edusci*, 2(1), 20-27. <https://doi.org/10.37899/journallaedusci.v2i1.367>
- Pagiling, S. L., Tembang, Y., Palobo, M., & Munfarikhatin, A. (2020). Analysis of pre-service primary teachers' difficulties in solving word problem. In *Proceedings of the 4th International Conference on Learning Innovation and Quality Education* (pp. 1-5). <https://doi.org/10.1145/3452144.345225>
- Patac, L. P., & Patac Jr, A. V. (2015). An application of student self-assessment and Newman error analysis in solving math problems. *Recoletos Multidisciplinary Research Journal*, 3(1). <https://doi.org/10.32871/rmrj1503.01.17>
- Pongsakdi, N., Kajamies, A., Veermans, K., Lertola, K., Vauras, M., & Lehtinen, E. (2020). What makes mathematical word problem solving challenging? Exploring the roles of word problem characteristics, text comprehension, and arithmetic skills. *ZDM*, 52, 33-44. <https://doi.org/10.1007/s11858-019-01118-9>
- Rocha, H., & Babo, A. (2024). Problem-solving and mathematical competence: A look to the relation during the study of linear programming. *Thinking Skills and Creativity*, 51, 101461. <https://doi.org/10.1016/j.tsc.2023.101461>

- Saleh, K., Yuwono, I., As'ari, A. R., & Sa'dijah, C. (2017). Errors analysis solving problems analogies by Newman procedure using analogical reasoning. *International Journal of Humanities and Social Sciences*, 9(1), 17-26.
- Siniguian, M. T. (2017). Students' difficulty in solving mathematical problems. *International Journal of Advanced Research in Engineering and Applied Sciences*, 6(2), 1-12.
- Thomas, D. S., & Mahmud, M. S. (2021). Analysis of students' error in solving quadratic equations using Newman's procedure. *International Journal of Academic Research in Business and Social Sciences*, 11(12), 222-237. <http://dx.doi.org/10.6007/IJARBSS/v11-i12/11760>
- TLS, D., & Herman, T. (2020). An analysis of pre-service mathematics teachers' Desmos activities for linear programming lesson. *International Journal of Pedagogical Development and Lifelong Learning*, 1(1), 1-10. <https://doi.org/10.30935/ijpdll/8312>
- Verschaffel, L., Schukajlow, S., Star, J., & Van Dooren, W. (2020). Word problems in mathematics education: A survey. *ZDM*, 52, 1-16. <https://doi.org/10.1007/s11858-020-01130-4>
- White, A. L. (2010). Numeracy, Literacy and Newman's Error Analysis. *Journal of Science and Mathematics Education in Southeast Asia*, 33(2), 129-148. <http://handle.uws.edu.au:8081/1959.7/557229>
- Wijaya, A., van den Heuvel-Panhuizen, M., Doorman, M., & Robitzsch, A. (2014). Difficulties in solving context-based PISA mathematics tasks: An analysis of students' errors. *The Mathematics Enthusiast*, 11(3), 555-584. <https://doi.org/10.54870/1551-3440.1317>