Journal of Honai Math, 81), 43-56, April 2025
https:/doi.org/10.30862/jhm.v8i1.755

Learning proof of trigonometric identities with ChatGPT

Putri Fitri, Yusuf Hartono*, Meryansumayeka

Universitas Sriwijaya, Palembang, Indonesia

* Correspondence: yhartono@unsri.ac.id

Received: 13 November 2024 | Revised: 7 April 2025 | Accepted: 15 April 2025 | Published: 30 April 2025

Abstract

The integration of artificial intelligence (AI) into mathematics education has demonstrated potential in enhancing students' conceptual understanding and reasoning abilities, particularly in the context of mathematical proof. Despite these advancements, limited research has investigated the use of AI-based language models such as ChatGPT to support students in learning the logical structure of trigonometric identity proofs, which remain a challenging topic for many learners. To address this gap, this study introduces a novel learning trajectory assisted by ChatGPT aimed at improving students' abilities in constructing and understanding trigonometric proofs. Employing a validation study design, the research was conducted in three phases: experimental preparation, experimental design, and retrospective analysis. Data were collected through observations, interviews, document analysis, and written tests, and analyzed qualitatively. A total of 50 eleventh-grade students engaged in three learning activities: studying proofs with ChatGPT-generated explanations, interacting directly with ChatGPT to explore proof strategies, and independently solving proof tasks without AI assistance. The findings indicate that ChatGPT effectively supports students in comprehending the logical steps involved in proof construction, enhances their engagement with mathematical reasoning, and promotes deeper understanding of trigonometric identities. These results highlight the potential impact of conversational AI tools in fostering proof-based thinking and enriching mathematics instruction at the secondary education level.

Keywords: Chat-GPT, design research, identity trigonometry, learning design, proof

Introduction

The process of learning to prove trigonometric identities presents significant cognitive challenges for students, as it necessitates a comprehensive understanding of trigonometric principles coupled with proficient algebraic manipulation skills. The act of constructing,

simplifying, and verifying trigonometric identities requires systematic reasoning and logical consistency, making it one of the more demanding components within the secondary mathematics curriculum. Previous research and teacher interviews have highlighted that student in grade X generally lack prior exposure to trigonometric concepts at the elementary and junior high school levels, thereby underscoring the importance of establishing a solid foundational understanding before engaging with advanced topics such as trigonometric proofs (Astuti et al., 2024).

At the senior secondary level, trigonometry instruction often includes engagement with mathematical proofs, which are central to fostering both cognitive development and affective appreciation of mathematical reasoning (Aminudin et al., 2019; Lee, 2016). Proofs are not only instrumental for deepening conceptual understanding but also serve as essential tools in the development, structuring, and transmission of mathematical knowledge (Mahfudy, 2017). However, the inherently abstract and deductive nature of proof construction renders it a persistent source of difficulty for many students (Muzangwa & Ogbonnaya, 2024). Difficulties in proving trigonometric identities are frequently attributed to students' insufficient grasp of basic trigonometric relationships, improper application of identities and formulas, and weak algebraic reasoning. Empirical findings by Rohimah and Prabawanto (2020) reveal that students struggle with applying standard trigonometric formulas, interpreting interrelationships among trigonometric ratios, and executing algebraic transformations, such as factoring trigonometric quadratic expressions. Similarly, Arhin and Hokor (2021) found that students often make systematic errors during the transformation, processing, and encoding stages of solving trigonometric problems, reflecting deficits in both conceptual and procedural knowledge. The difficulty of engaging in proof-based reasoning, particularly in trigonometry, is further compounded by students' limited capacity for deductive reasoning, which is a critical skill in mathematical proof construction (Miyazaki et al., 2017; Pedemonte & Balacheff, 2016).

Proving trigonometric identities remains a persistent challenge for many students, primarily due to the complexity of the topic and the cognitive demands it places on learners. This area of mathematics is often perceived as particularly difficult because it necessitates a strong conceptual foundation in trigonometry and the ability to integrate algebraic reasoning with trigonometric principles (Mustamir, 2019; Rohimah & Prabawanto, 2019; Siyepu, 2015). Students frequently encounter difficulties in bridging their understanding of algebra with trigonometric identities, indicating a disconnection between procedural knowledge and conceptual insight (Rohimah & Prabawanto, 2019). Compounding this issue is the insufficient emphasis placed on proof-oriented tasks in secondary mathematics curricula, which may be attributed to the limited inclusion of such items in standardized assessments (Fatmahayati, 2019). Finally, Noto et al. (2019) affirm that although proof constitutes a vital aspect of mathematical competence, students rarely develop this skill adequately within current educational practices.

Furthermore, students tend to rely heavily on formulaic approaches, such as using function tables, without fully understanding or applying the underlying properties of

trigonometric functions. For instance, many students struggle to compute values such as *sin* 210° and lack the ability to recognize or relate different trigonometric relationships (Maknun et al., 2019). These tendencies suggest that students often engage with mathematics at a surface level focused on result-oriented tasks, whereas proof requires higher-order thinking, including analytical reasoning, synthesis of concepts, and critical evaluation of the validity of mathematical arguments.

In the context of the Fourth Industrial Revolution (Industry 4.0), rapid developments in computing and internet technologies have significantly transformed how information is accessed and processed, offering new opportunities for enhancing mathematical learning (Bonfield et al., 2020). Among these innovations, artificial intelligence (AI) has emerged as a prominent tool in education. AI refers to computational systems designed to perform tasks traditionally requiring human intelligence (Chen et al., 2020). One such advancement is ChatGPT (Generative Pre-trained Transformer), developed by OpenAI, which enables dynamic, real-time dialogue between users and AI, resembling instructional interactions in educational settings. ChatGPT holds considerable potential to enhance student learning by providing immediate feedback and facilitating access to complex content. Research by Mustafa (2023) indicates that 57.4% of students report increased engagement when learning is supported by ChatGPT, suggesting its capacity to foster active participation in the learning process. In particular, AI tools such as ChatGPT may offer effective scaffolding for students learning to prove trigonometric identities by providing guided explanations and step-by-step feedback (Wardat et al., 2023). Nevertheless, empirical studies specifically investigating the role and effectiveness of ChatGPT in supporting the learning of trigonometric proofs remain scarce. Thus, there is a pressing need to design and examine instructional strategies that integrate ChatGPT into the teaching and learning of trigonometric identity proofs, addressing this gap in current mathematics education research.

Methods

The present study employed a design research methodology, specifically a validation study type, to develop, implement, and evaluate an innovative pedagogical intervention in the domain of mathematics education. The intervention focused on enhancing students' understanding and proficiency in proving trigonometric identities through the integration of ChatGPT technology as a learning support tool. The research participants consisted of 50 eleventh-grade students (aged 17–18 years) from State Senior High School 1 South Indralaya, located in the Ogan Ilir district.

Data collection methods comprised classroom observations, semi-structured interviews, document analysis, and mathematical proficiency tests. The collected data were analyzed using a qualitative analytical approach, allowing for a comprehensive examination of the learning processes and students' conceptual development. The research was conducted over two iterative cycles, each encompassing three phases aligned with the validation study framework: preparation phase, experimental implementation, and retrospective analysis.

In the first cycle, the initial instructional design was constructed based on an extensive review of relevant literature, analysis of students' learning needs, and foundational principles of effective mathematics instruction. The instructional design included structured learning activities centered on techniques for proving trigonometric identities and incorporated ChatGPT as an assistive digital tool. An initial assessment was administered to evaluate students' baseline knowledge of trigonometric identities and their familiarity with using ChatGPT.

Subsequently, a Hypothetical Learning Trajectory (HLT) was formulated to guide the design of instructional activities. This HLT integrated specific learning objectives, anticipated students' reasoning processes, and strategies for addressing potential misconceptions, all contextualized within the topic of trigonometric identities. The research instruments—including the HLT, observation protocols, and assessment tasks—were subjected to expert validation and revised accordingly based on feedback. The outcomes and reflective analysis from the first cycle informed the refinement and development of the instructional components for the second cycle.

At this stage, the instructional designs and implementation plans were refined to focus on evaluating students' engagement in structured mathematical activities and their capacity for conjectural reasoning. The primary objective of this research was to formulate an effective instructional design that supports the development of students' competencies in proving trigonometric identities. A secondary objective was to enhance students' conceptual understanding through the integration of ChatGPT as an instructional aid. The experimental phase involved the validation of the Hypothetical Learning Trajectory (HLT) designed in the initial cycle. The first cycle served as a pilot study, while the second cycle involved a full classroom implementation of the revised learning design.

The second cycle aimed to investigate the extent to which students could meaningfully engage in the learning activities originally developed in the first cycle. In this phase, the revised HLT was implemented with the same cohort of 50 students from SMA Negeri 1 South Indralaya. Data collection methods included classroom observations, semi-structured interviews, document analysis, formative and summative assessments, and Focus Group Discussions (FGDs). These data were subsequently analyzed during the retrospective analysis phase to evaluate the effectiveness of the learning intervention.

Prior to the implementation of instructional activities, collaborative discussions were held between researchers and participating teachers to align the lesson plans and objectives. Post-lesson reflections were conducted at the end of each session to evaluate the instructional process and gather insights for further refinement. Following the second implementation, an advanced assessment phase was conducted to determine the accuracy and efficacy of the revised instructional design.

The data collected during this phase were subjected to qualitative analysis to assess the effectiveness of the Hypothetical Learning Trajectory (HLT), with particular emphasis on its integration with ChatGPT as a hybrid instructional tool. The analysis focused on identifying students' mathematical achievements and challenges, their patterns of interaction with

ChatGPT, and the extent to which the instructional design met their learning needs. Findings from the second research cycle yielded empirical evidence supporting the potential of a ChatGPT-assisted learning environment to facilitate students' conceptual understanding and procedural fluency in proving trigonometric identities. Finally, the research phases undertaken in this study are illustrated in Figure 1.

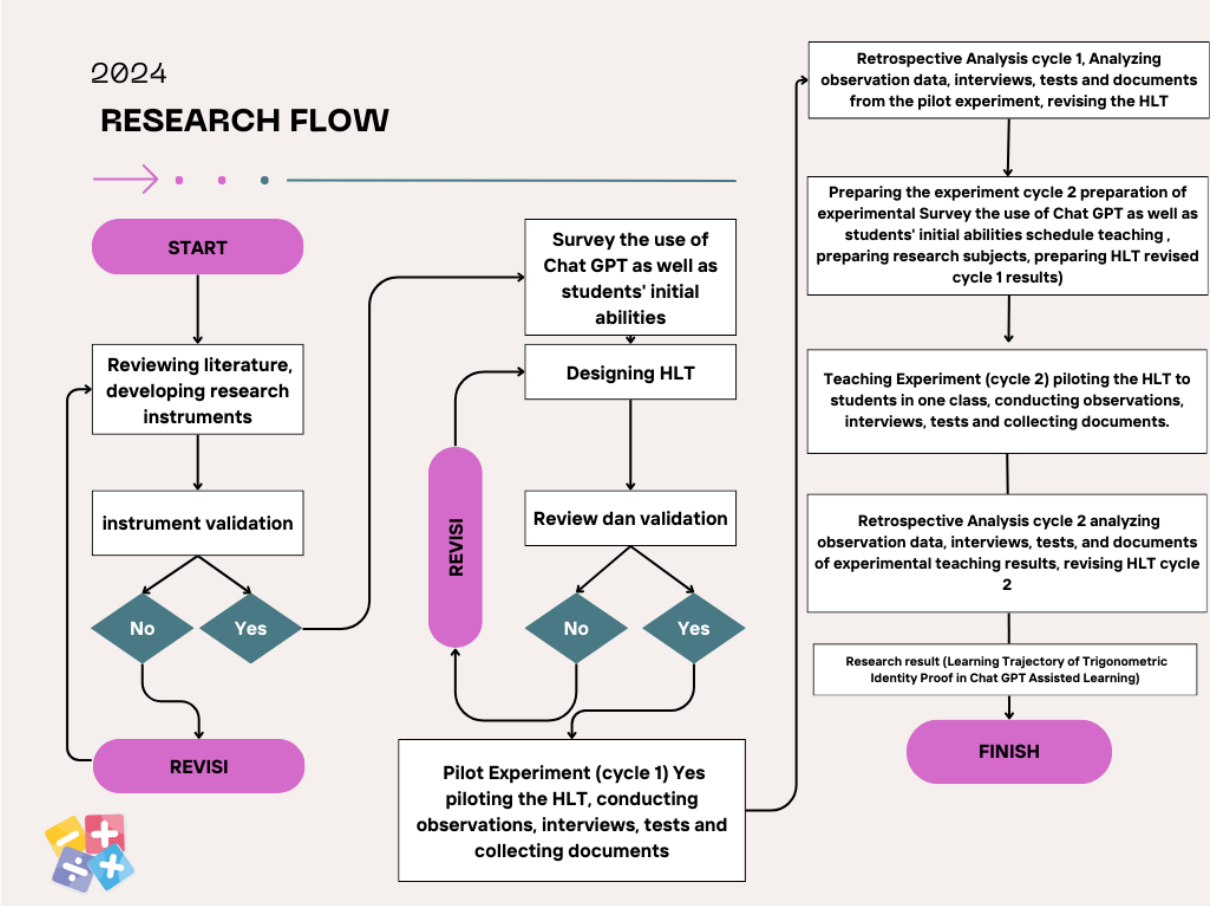

Figure 1. Research Flowchart

Results and Discussion

Prior to engaging in the activities, students underwent an initial assessment designed to gauge their understanding of trigonometric identity proofs. This assessment served as a baseline measure of their foundational knowledge. The subsequent learning process was organized into three distinct activities, each aimed at enhancing students' ability to prove trigonometric identities.

In Activity 1, students were provided with trigonometric identity problems, accompanied by solutions generated by ChatGPT. The objective was for students to review these solutions, analyze the reasoning behind each step, and articulate the logical process involved in the proof. Activity 2 involved a more interactive approach, where students worked through additional trigonometric problems with direct assistance from ChatGPT. During this activity, students received real-time feedback and corrections, which facilitated

active problem-solving and engagement with various methods of solution. In Activity 3, students were presented with a new identity proof problem and tasked with solving it independently, without any technological support.

Table 1. HLT (Hypothetical Learning Trajectory)

Activity	Learning Objectives	Conjecture
Activity 1: Students learn questions and answers from ChatGPT.	To understand the steps involved in proving trigonometric identities with the assistance of technology.	Students will comprehend the proof steps and be able to explain the process clearly and accurately.
Activity 2: Students use ChatGPT as an assistant to answer questions.	To develop skills in breaking down problems using ChatGPT as a tool for guidance.	Students will complete the problem with ChatGPT's help, engage actively, and explore multiple solution approaches.
Activity 3: Students solve problems without ChatGPT's assistance.	To assess students' ability to apply proof strategies independently.	Students will solve the problems correctly, demonstrating a deep understanding without external aid.

Following the completion of these activities, the initial assessment will be re-administered to evaluate whether the learning design has effectively enhanced students' comprehension. The resulting data will be used to assess the effectiveness of the Hybrid Learning Tool (HLT) and the integration of ChatGPT in the learning process. The analysis of this post-intervention data will focus on identifying student achievements and challenges, exploring their interactions with ChatGPT, and evaluating the appropriateness of the learning design in addressing students' needs.

In Activity 1, students were presented with a trigonometric identity proof question, along with a solution generated by ChatGPT. They requested further clarification and asked for detailed explanations of the steps involved in the proof. This activity was designed to assist students, particularly those at the beginner level, in understanding the structured process of identity proof using technology. ChatGPT provided clear and detailed solutions, which allowed students to examine not only the steps of the proof but also the underlying reasoning behind each transformation or manipulation of the identity. Through this engagement, students were encouraged to develop critical thinking skills, especially in identifying key components of the proof. This methodical approach not only facilitated a deeper understanding of proof construction but also prepared students for more advanced problem-solving tasks. Moreover, this activity contributed to building a strong foundation in creating well-organized and logical proofs.

As illustrated in Figure 2, the majority of students demonstrated the ability to understand the trigonometric identity proofs presented. In the image, the second student is able to articulate additional steps in the proof process. However, difficulties arose when students were required to manipulate algebraic expressions or handle more complex

trigonometric identities. To further investigate these results, researchers conducted an interview following Activity 1.

Researcher : *"How did you find your experience with learning and answering questions using ChatGPT? Do you feel that your understanding improved after using it?"*

Student : *"Yes, it's very helpful! I can follow the steps in the proofs, especially for formulas like sine and cosine. However, when the problems become more complicated, I struggle to keep up."*

Researcher : *"What part do you find most challenging?"*

Student : *"The part where we need to combine multiple identities. I understand the steps initially, but when it comes to transforming the problem into a more complex identity, I get confused."*

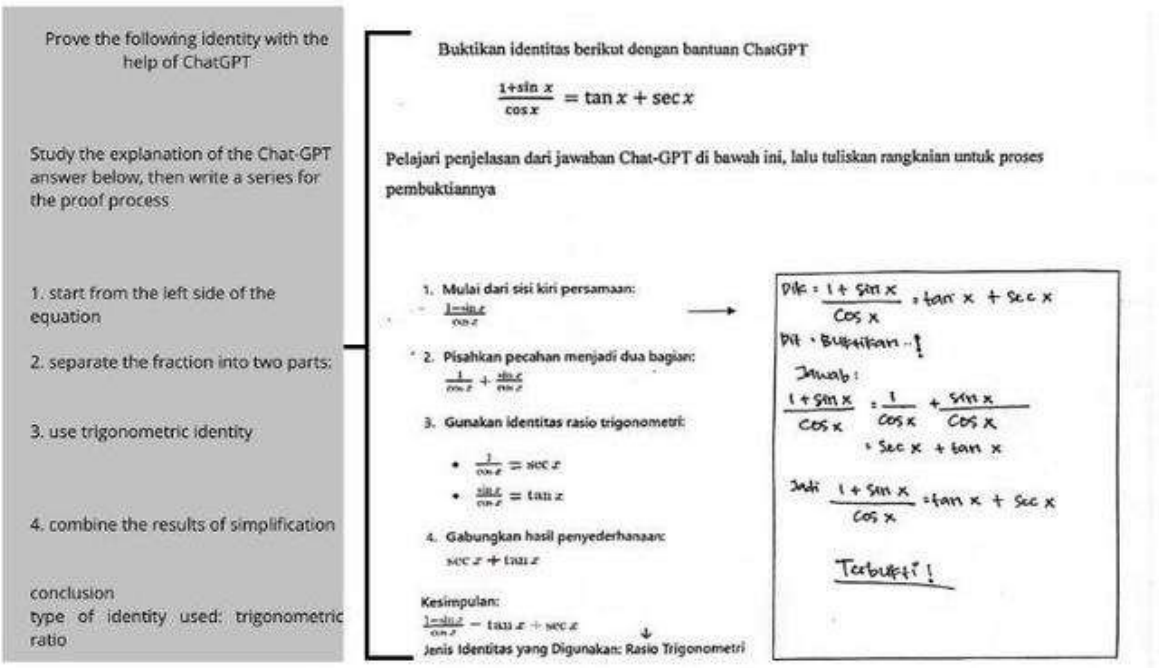

Figure 2. Student's Work in Activity 1

In Figure 3, students are seen explaining their work during Activity 1. This suggests that, while technology aids in the initial understanding of concepts, deeper comprehension requires additional instructional support.

In Activity 2, depicted in Figure 4, students were encouraged to interact directly with ChatGPT to address new trigonometric identity problems. The objective of this activity was to foster problem-solving skills and promote active engagement. Students were able to communicate with ChatGPT, posing questions and receiving immediate, real-time feedback. This direct interaction with the technology proved beneficial, as it allowed students to test their hypotheses and quickly identify any errors. Instant feedback facilitated the correction of conceptual misunderstandings and helped students develop a clearer understanding of the solutions. The iterative nature of this process not only reinforced their knowledge but also

expedited their mastery of trigonometric identities. The results from this activity indicate that students felt more confident in completing the tasks and showed significant improvement in their skills through the reflective process supported by ChatGPT.

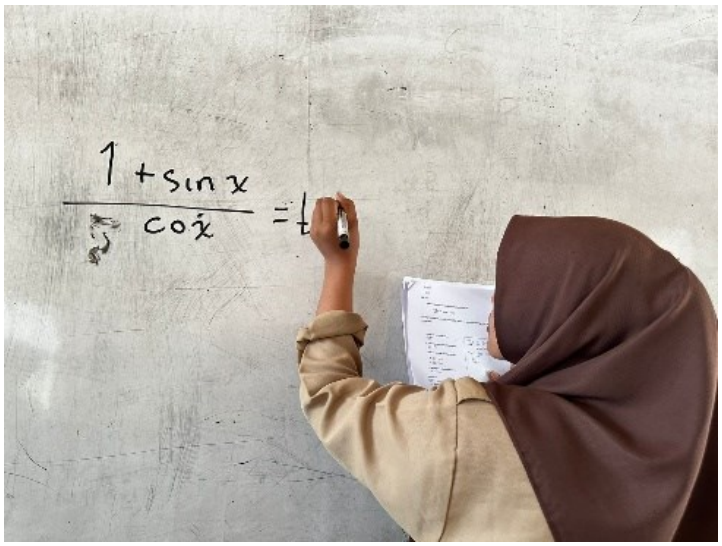

Figure 3. Student Presentations

Students utilize technology to explore various methods of proving trigonometric identities. They are encouraged to submit questions, repeat inquiries, or request further explanations for any parts of the process they find unclear. This approach mirrors Activity 1, where interviews with students were also conducted to gain deeper insights into their experiences.

Researcher : *"Now that you are using ChatGPT to answer questions directly, how do you perceive the impact of this tool on your learning?"*

Student : *"Yes, it's very helpful! I feel more confident when answering questions because ChatGPT guides me and alerts me if I make any mistakes. I also learn from each step it provides."*

Researcher : *"That's interesting. Has your approach to solving problems changed as a result of using ChatGPT? How often do you utilize it?"*

Student : *"Yes, I now understand the methods better. I can solve problems independently, though I still need assistance with some steps occasionally. If I don't fully understand something, I just ask ChatGPT again."*

Unlike the prior phases of learning that involved direct assistance from ChatGPT, the third phase of the study focuses on the challenges associated with applying knowledge independently. This phase is designed to assess how effectively students can demonstrate their understanding of trigonometric identity proofs without the aid of ChatGPT.

Building on the experience and insights gained through previous interactions and solutions provided by ChatGPT, students are expected to utilize more refined strategies and

exhibit greater confidence in solving problems autonomously. The objective of this phase is to evaluate the extent to which students have developed their skills in proving trigonometric identities independently.

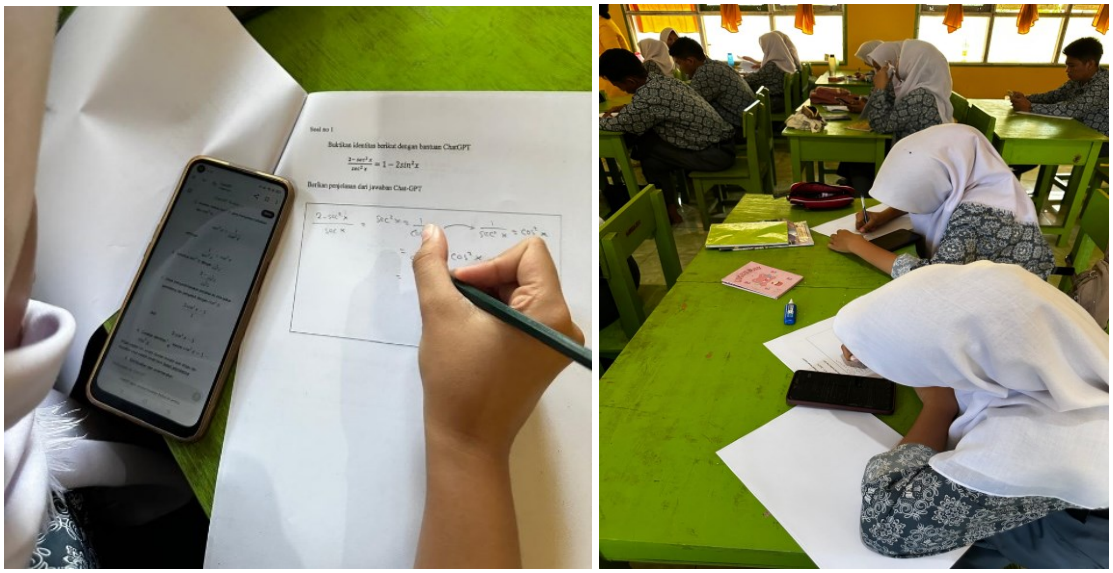

Figure 4. Activity 2

In the earlier stages of the study, students worked through questions with the assistance of ChatGPT (Activity 1) and received real-time support while solving problems using the tool (Activity 2). In contrast, during this phase, they are tasked with answering questions independently, without the aid of technology. Most students are able to solve the questions correctly on their own. Their prior interactions with ChatGPT have provided a solid foundation for understanding the proof process. However, some students still experience challenges, particularly in recalling the steps and managing complex proofs without direct prompts from a tool like ChatGPT.

Researcher : "How do you feel about completing the questions without the help of ChatGPT?"
Student : "At first, I felt nervous, but after practicing with ChatGPT, I gained more confidence. I learned to follow the steps that were taught, and I was curious to see if I could complete the questions without any issues."
Researcher : "I see. Do you believe that practicing with ChatGPT has helped you complete questions independently, without relying on technology?"
Student : "Definitely! I feel that I understand the concepts better now and can apply them on my own. I remember the steps that ChatGPT taught me, and that helps me complete the questions."

In Cycle 1, many students faced challenges in understanding and proving trigonometric identities. They frequently made errors in calculations and struggled with organizing the proof steps. Additionally, a significant number of students exhibited excessive dependence on ChatGPT, often copying answers without fully grasping the underlying concepts. As a result,

their active engagement in problem-solving remained limited, and the success rate in proving trigonometric identities was relatively low.

However, Cycle 2 saw notable improvements. Students demonstrated a better understanding of the concepts and were able to construct proofs more systematically and accurately. To reduce their reliance on ChatGPT, students were encouraged to attempt solving problems independently before seeking assistance from the AI. This approach led to increased student participation and confidence in problem-solving, as well as greater care in selecting the appropriate trigonometric identities for proofs.

The positive effects of this strategy were evident, with a higher number of students successfully proving trigonometric identities correctly and with fewer errors compared to Cycle 1. Learning became more effective as ChatGPT shifted from being a mere answer provider to a valuable tool for comprehension. Thus, the modified strategy in Cycle 2 not only improved students' understanding but also fostered greater independence, ultimately enhancing their performance in proving trigonometric identities.

In the third activity, while ChatGPT was not directly involved, its influence remained significant through the knowledge students had gained from previous interactions. The tool served as a support mechanism, offering quick explanations whenever students encountered obstacles. Its role in helping students develop their analytical skills was crucial in the proof process, demonstrating the utility of AI in enhancing both understanding and problem-solving efficiency.

Previous research by Lo (2023) indicated that ChatGPT can assist students in understanding complex material by engaging in human-like conversations. However, challenges may arise when the AI provides information lacking technical accuracy, particularly in mathematical contexts. Despite these limitations, ChatGPT has proven capable of delivering clear and structured solutions to trigonometric identity problems. By presenting logical, step-by-step explanations, it helps students grasp the foundational concepts behind proofs, reinforcing their conceptual understanding (Baidoo-Anu & Owusu Ansah, 2023). Students are able to identify key steps in the proof process, which they can then apply to other problems.

In the trigonometry learning process, ChatGPT's provision of immediate feedback when students make errors plays a critical role in deepening understanding and refining analytical skills. This instantaneous correction facilitates greater engagement with the material, enhancing student motivation to learn (Ifraheem et al., 2024). Furthermore, ChatGPT allows students to practice trigonometric problems independently, minimizing their dependence on teachers. The tool offers solutions that can be reviewed and understood multiple times, benefiting students who require additional time to master the material (Ipperciel, 2020). This self-paced learning model empowers students to manage their own learning trajectory based on individual needs. By providing initial guidance, ChatGPT encourages students to independently solve problems after practicing with provided examples.

When learning activities are well-designed, students can eventually solve problems without assistance, indicating enhanced analytical skills and a deeper understanding of the

concepts (Alam, 2021). Supporting this, research by Taani and Alabidi (2024) found that teachers who integrated ChatGPT into their teaching reported positive outcomes, including improved teaching effectiveness, increased student engagement, and better comprehension of complex concepts. Similarly, Zawacki-Richter et al. (2019) highlighted the role of AI-based technology in fostering independent learning, allowing students to control the pace and direction of their studies based on personal needs. This aligns with the use of ChatGPT in educational settings, as the technology offers adaptable support tailored to students' varying skill levels.

Overall, this research contributes to the literature by illustrating how ChatGPT can be an effective tool for learning mathematics, particularly in mastering trigonometric identities. It provides new insights into the application of AI-based technologies in education and underscores their potential for broader implementation in diverse learning environments. The findings of this study indicate that ChatGPT supports students in understanding trigonometric identities more effectively. With AI's step-by-step guidance, students gain greater confidence and independence in solving problems. For educators, the study offers valuable perspectives on how AI can be utilized as a teaching tool, providing alternative explanations and supplementary exercises to enhance student learning.

Conclusion

ChatGPT plays a significant role in supporting the learning of trigonometric identity proofs through three distinct activities. In the first activity, ChatGPT acts as a valuable resource, assisting novice students by providing solutions and clarifying their understanding of the concepts. In the second activity, it functions as an interactive tutor, offering real-time feedback that helps students refine their comprehension and problem-solving skills. In the third activity, the experience gained from interacting with ChatGPT fosters student independence, enabling them to solve problems autonomously without relying on technological assistance.

Research has shown that ChatGPT can be an effective tool for enhancing students' understanding of trigonometric identity proofs, particularly in navigating the proof process and addressing complex multi-step problems. While its use significantly supports students in grasping the necessary concepts, further practice is required for students to achieve full mastery and independence. Despite these benefits, the study does not explore alternative instructional methods that may offer additional benefits, such as group discussions, direct teacher guidance, or the integration of interactive visual media. Furthermore, the study primarily measures student comprehension following a relatively brief intervention period.

For future research, it is recommended to investigate the combination of ChatGPT with group discussion-based learning strategies, teacher-led instruction, or the incorporation of visual media such as graphics and animations. These methods could potentially enhance students' understanding of trigonometric identity proofs. Additionally, the study highlights the challenges encountered during the transition from traditional educational methods to those

incorporating technological tools, underscoring the need for a balanced approach that integrates both traditional and technological learning strategies.

Acknowledgment

The author wishes to express gratitude to the Directorate of Research, Technology, and Community Service for their financial support of this research through the Master's Thesis Research Grant, under Contract Number: 090/E5/PG.02.00.PL/2024.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been covered completely by the authors.

References

Alam, A. (2021). Possibilities and apprehensions in the landscape of artificial intelligence in education. *2021 International Conference on Computational Intelligence and Computing Applications (ICCICA)*, 1–8. https://doi.org/10.1109/ICCICA52458.2021.9697272

Aminudin, M., Nusantara, T., Parta, I. N., Rahardjo, S., As'ari, A. R., & Subanji. (2019). Engaging problems on trigonometry: why were student hard to think critically? *Journal of Physics: Conference Series*, *1188*(1), 012038. https://doi.org/10.1088/1742-6596/1188/1/012038

Arhin, A., & Hokor, H. (2021). Analysis of high school students' errors in solving trigonometry problems. *International Journal of Mathematics Education, 19(2), 135-150*. https://www.mathsciteacher.com/download/analysis-of-high-school-students-errors-in-solving-trigonometry-problems-11076.pdf

Astuti, S. W., Wati, A. S., & Ayuwanti, I. (2024). Analisis kesalahan siswa dalam menyelesaikan soal cerita trigonometri berdasarkan Newman. *MATHEMA: jurnal pendidikan matematika*, *6*(1), 208-223. https://doi.org/10.33365/jm.v6i1.3417

Baidoo-Anu, D., & Owusu Ansah, L. (2023). Education in the era of generative artificial intelligence (ai): Understanding the potential benefits of ChatGPT in promoting teaching and learning. *Journal of AI*, *7*(1), 52–62. https://doi.org/10.61969/jai.1337500

Bonfield, C. A., Salter, M., Longmuir, A., Benson, M., & Adachi, C. (2020). Transformation or evolution? Education 4.0, teaching and learning in the digital age. *Higher Education Pedagogies*, *5*(1), 223–246. https://doi.org/10.1080/23752696.2020.1816847

Chen, L., Chen, P., & Lin, Z. (2020). Artificial Intelligence in education: A review. *IEEE Access*, *8*, 75264–75278. https://doi.org/10.1109/ACCESS.2020.2988510

Fatmahayati, W. (2019). Proses berpikir siswa SMA dalam menyusun bukti matematis. *GAUSS: Jurnal Pendidikan Matematika*, *2*(2), 63-73. https://doi.org/10.30656/gauss.v2i2.1790

Ifraheem, S., Rasheed, M., & Siddiqui, A. (2024). Transforming education through artificial intelligence: Personalization, engagement and predictive analytics. *Journal of Asian Development Studies*, *13*(2), 250–266. https://doi.org/10.62345/jads.2024.13.2.22

Ipperciel, D. (2020). Student centeredness as innovation. *The International Review of Information Ethics*, *28*(6), 1-12. https://doi.org/10.29173/irie382

Lee, K. (2016). Students' proof schemes for mathematical proving and disproving of propositions. *The Journal of Mathematical Behavior*, *41*, 26–44. https://doi.org/10.1016/j.jmathb.2015.11.005

Lo, C. K. (2023). What is the impact of ChatGPT on education? a rapid review of the literature. *Education Sciences*, *13*(4), 410-426. https://doi.org/10.3390/educsci13040410

Mahfudy, S. (2017). Strategi pembuktian matematis mahasiswa pada soal geometri. *JTAM: Jurnal Teori Dan Aplikasi Matematika*, *1*(1), 31-40. https://doi.org/10.31764/jtam.v1i1.101

Maknun, C. L., Rosjanuardi, R., & Jupri, A. (2019). From ratios of right triangle to unit circle: an introduction to trigonometric functions. *Journal of Physics: Conference Series*, *1157*, 022124. https://doi.org/10.1088/1742-6596/1157/2/022124

Miyazaki, M., Fujita, T., & Jones, K. (2017). Students' understanding of the structure of deductive proof. *Educational Studies in Mathematics*, *94*(2), 223–239. https://doi.org/10.1007/s10649-016-9720-9

Mustamir, A. (2019). Application of demonstration methods to improve learning achievement in cultural arts subject and skills of filter art graphic materials in class IXE Students of SMP Negeri 3 Surabaya. *Indonesian Journal of Contemporary Education*, *1*(1), 15–17. https://doi.org/10.33122/ijoce.v1i1.4

Mustafa. (2023). Aktivitas siswa dalam memecahkan masalah matematika dengan berpikir komputasi berbantuan Chat-GPT. *Mathema: Jurnal Pendidikan Matematika, 5*(2), 283-298. https://doi.org/10.33365/jm.v5i2.3469

Muzangwa, J., & Ogbonnaya, U. (2024). Visualization techniques for proofs: Implications for enhancing conceptualization and understanding in mathematical analysis. *Journal of Honai Math*, *7*(2), 347–362. https://doi.org/https://doi.org/10.30862/jhm.v7i2.603

Noto, M. S., Priatna, N., & Dahlan, J. A. (2019). Mathematical proof: The learning obstacles of pre service mathematics teachers on transformation geometry. *Journal on Mathematics Education, 10*(1), 117–126. https://doi.org/10.22342/jme.10.1.5379.117-126

Pedemonte, B., & Balacheff, N. (2016). Establishing links between conceptions, argumentation and proof through the ck¢-enriched Toulmin model. *The Journal of Mathematical Behavior*, *41*, 104–122. https://doi.org/10.1016/j.jmathb.2015.10.008

Rohimah, S. M., & Prabawanto, S. (2019). Student's difficulty identification in completing the problem of equation and trigonometry identities. *International Journal of Trends in Mathematics Education Research*, *2*(1), 34–36. https://doi.org/10.33122/ijtmer.v2i1.50

Rohimah, A., & Prabawanto, S. (2020). Students' difficulties in solving trigonometric equations and identities. *Journal of Physics: Conference Series, 1467(1), 012002*. https://doi.org/10.1088/1742-6596/1521/3/032002

Siyepu, S. W. (2015). Analysis of errors in derivatives of trigonometric functions. *International Journal of STEM Education*, *2*(1), 16. https://doi.org/10.1186/s40594-015-0029-5

Taani, O., & Alabidi, S. (2024). ChatGPT in education: Benefits and challenges of ChatGPT for mathematics and science teaching practices. *International Journal of Mathematical Education in Science and Technology*, 1–30. https://doi.org/10.1080/0020739X.2024.2357341

Wardat, Y., Tashtoush, M. A., AlAli, R., & Jarrah, A. M. (2023). ChatGPT: A revolutionary tool for teaching and learning mathematics. *Eurasia Journal of Mathematics, Science and Technology Education, 19*(7), 1-18 . https://doi.org/10.29333/ejmste/13272

Zawacki-Richter, O., Marín, V. I., Bond, M., & Gouverneur, F. (2019). Systematic review of research on artificial intelligence applications in higher education – where are the educators? *International Journal of Educational Technology in Higher Education*, *16*(1), 1-27. https://doi.org/10.1186/s41239-019-0171-0

Zetriuslita, Z., Nofriyandi, N., & Istikomah, E. (2021). The increasing self-efficacy and self-regulated through GeoGebra based teaching reviewed from initial mathematical ability (IMA) Level. *International Journal of Instruction*, *14*(1), 587–598. https://doi.org/10.29333/iji.2021.14135a