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Students field independent-dependent solving surface area of square pyramid: Commognitive perspective

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Abstract

Understanding students' cognitive processes in solving mathematical problems is crucial for improving instructional strategies and learning outcomes. However, limited studies have examined students' commognitive aspects in the context of geometric problem-solving, particularly in relation to cognitive styles such as Field-Independent (FI) and Field-Dependent (FD) tendencies. This study addresses this gap by analyzing the four aspects of commognition word use, visual mediators, narratives, and routines demonstrated by students when solving story problems on the surface area of a square pyramid. The study also explores the patterns of thinking and solution strategies employed by FI and FD students in approaching these problems. Conducted in Class VIII A at SMP Negeri 1 Sigi, the study involved two male students, one representing each cognitive style, to highlight differences in problem-solving approaches while controlling for gender. Data collection involved the Group Embedded Figures Test (GEFT) to determine cognitive style, validated problem-solving task sheets, and semi-structured interviews conducted in parallel with the written tasks. Data were analyzed using data condensation, data display, and conclusion drawing techniques. The findings indicate that FI students approach problem-solving with greater detail, clarity, efficiency, and accuracy, explicitly demonstrating all four aspects of commognition. In contrast, FD students exhibit clarity, efficiency, and accuracy but lack detail and thoroughness in their written responses. Both cognitive styles demonstrate all four commognitive aspects, with notable differences in the narrative component FI students explicitly write formulas, whereas FD students understand the formulas but do not record them in writing. These findings provide valuable insights into how cognitive styles influence mathematical problem-solving and commognitive development, offering implications for differentiated instructional strategies in mathematics education.

Keywords: Cognitive Style, Commognitive Perspective, Field Dependent, Field Independent, Square Pyramid

Introduction

Mathematics is an integral part of daily human life, encompassing both simple activities and complex technological advancements (Wulandari et al., 2021). The effectiveness of mathematics education is not solely reflected in students' academic achievements but also in their ability to develop and apply mathematical knowledge in real-world contexts. One of the essential competencies that support this process is mathematical communication skills. These skills play a crucial role in the learning process, as mathematical communication enables students to articulate their mathematical ideas through language, notation, or symbols. This ability allows them to comprehend, interpret, describe relationships, and translate contextual problems into mathematical models, both orally and in written form (Lubis et al., 2023). Thus, mathematical communication is a fundamental aspect of mathematics education that significantly influences students' learning experiences and outcomes.

However, empirical findings indicate that students' mathematical communication skills remain relatively low. According to Nowicki and Duke, as cited in Fatkhiyyah et al. (2019), students often struggle to interpret mathematical symbols, which hinders their ability to express ideas and construct mathematical solutions effectively. This issue is further reflected in Indonesia's performance in international assessments. In the 2015 Programme for International Student Assessment (PISA), the average mathematics score among OECD countries was 490, whereas Indonesia's score was 386, indicating the need for substantial improvements in mathematical competency (Fatkhiyyah et al., 2019). Furthermore, in 2018, Indonesia's mathematics competency score declined further to 379 (Fitri et al., 2023), highlighting a persistent gap between Indonesia and more developed nations. Given these findings, fostering mathematical communication skills is essential for enhancing students' understanding of mathematical concepts. Mathematics should not be perceived merely as an abstract system of symbols but as a meaningful language that facilitates problem-solving in everyday life.

The cognitive process is an internal activity within the central nervous system that occurs during thinking. Cognition is defined as an individual's mental process, particularly in relation to the understanding that the mind possesses internal states such as beliefs, desires, and intentions that can be examined within the framework of information processing, especially when it pertains to knowledge acquisition, expertise development, or learning (Ekawati et al., 2019). In addressing the challenges associated with low mathematical communication skills, particularly in solving story problems, Sfard's theory of commognition is considered highly relevant. This theory facilitates an in-depth analysis of how individuals communicate mathematically by emphasizing the interaction between communication and cognition in mathematics learning. The relevance of this theory lies in the alignment between the objectives of mathematical communication and the principles of commognition, which highlight the interconnectedness of communication patterns and cognitive processing. Moreover, the way individuals communicate in mathematical contexts significantly influences their conceptual understanding. Consequently, limitations in mathematical communication become particularly evident in story problem-solving, as these tasks require students to thoroughly analyze and comprehend given information to determine appropriate solutions. This study adopts Sfard's commognitive framework, which integrates communication and cognition (Zayyadi et al., 2020; Lefrida et al., 2021). The commognitive framework comprises four key components: keywords, visual mediators, endorsed narratives, and routines. The keyword component plays a crucial role not only in describing observations but also in constructing meaning through structured language and logical reasoning. As a result, students can articulate their thoughts in an organized and coherent manner, which is essential for effective problem-solving in mathematics.

Mathematical story problems, in this context, refer to problems that closely mirror real-life situations. These problems serve as an introductory stage for applying mathematical concepts to practical contexts before students engage with more complex problem-solving tasks (Mbari et al., 2024). Story problems necessitate problem-solving through numeracy skills (Syahda et al., 2021). However, Syahda et al. (2021) found that many students frequently make errors in solving such problems. A critical aspect of successful problem-solving is students' ability to accurately extract relevant information from a problem statement, as this facilitates a structured solution process. Each student employs a unique approach when processing the information presented in a mathematical problem, which is referred to as their cognitive style.

Cognitive styles in learning contexts are generally categorized into two types: Field Dependent (FD) and Field Independent (FI). According to Yuliyani and Setyaningsih (2022), individuals with a field-dependent (FD) cognitive style tend to perceive information holistically, relying more on external references, and are generally more socially oriented. Conversely, individuals with a field-independent (FI) cognitive style tend to process information analytically, demonstrating the ability to structure and develop concepts independently. Those with an FD cognitive style are typically more global in their thinking, focusing on the overall environment and often being influenced by external factors. In contrast, FI individuals exhibit a higher degree of analytical thinking, perceive the environment as a set of distinct components, and are less affected by external influences. Given these distinct cognitive characteristics, an investigation into how students with different cognitive styles engage in mathematical communication when solving story problems is of significant interest. Therefore, this study aims to examine students' commognitive processes in relation to their cognitive styles to determine whether notable differences exist between field-dependent and field-independent learners in mathematical problem-solving.

Researchers conducted an interview with a mathematics teacher at SMP Negeri 1 Sigi to gain insights into students' difficulties in solving mathematical problems related to the surface area of three-dimensional shapes. The interview findings indicate that students frequently encounter challenges, particularly in understanding and applying mathematical symbols, interpreting notation, and comprehending the formulas required to solve such problems. Furthermore, students often struggle with accurately illustrating mathematical concepts, as evidenced by their difficulties in both representing problem scenarios and performing calculations related to the surface area of a square pyramid. Another notable challenge arises when students present their solutions, as their responses often exhibit confusion and errors in mathematical reasoning.

These findings suggest that students' mathematical communication and cognitive processing skills remain relatively low, as they have yet to develop the ability to effectively articulate and comprehend mathematical concepts, both verbally and in written form. This issue underscores the need for further investigation into students' learning processes concerning the surface area of pyramids. Given these observations, this study aims to analyze students' commognitive processes in solving story problems related to the surface area of a square pyramid, with a particular focus on cognitive style. Specifically, this research will examine how the four key components of commognition keywords, visual mediators, endorsed narratives, and routines manifest in students' solution strategies. Additionally, it will explore whether significant differences exist in the emergence of these aspects between students with field-dependent and field-independent cognitive styles.

Methods

This study employs an exploratory research method with a qualitative approach to investigate commognitive aspects, a subject that remains relatively underexplored. Exploratory research is characterized by diverse data collection methods such as interviews, observations, literature reviews, and group discussions along with a flexible research process that is not constrained by rigid hypotheses. Additionally, this approach provides a foundation for more in-depth and specific subsequent research.

The research subjects consisted of male students from class VIIIA at SMP Negeri 1 Sigi. Subject selection was conducted following the administration of the Group Embedded Figure Test (GEFT), a cognitive style test developed by Witkin in 1971, which has a reliability coefficient of 0.82 and has been widely validated in previous studies. Two subjects were selected: one with a field-dependent cognitive style and one with a field-independent cognitive style, both of whom were male. In addition to the GEFT results, subject selection was also based on recommendations from mathematics teachers, ensuring that the chosen students were capable of effectively articulating their thoughts during interviews.

The data collection process involved several techniques. First, the GEFT cognitive style test was administered. Second, students were given a written task containing a validated story problem on the surface area of a square pyramid. Third, semi-structured interviews were conducted during the problem-solving process, where researchers asked subjects to explain their reasoning after each step they completed.

Data analysis followed the iterative model proposed by Miles et al. (2014), which consists of three key stages: (1) data condensation, where data is synthesized and relevant information is identified through interviews and written task analysis; (2) data display, where the organized data is systematically presented to facilitate interpretation; and (3) conclusion drawing, where patterns, causal relationships, and overall insights are derived from the collected data.

To ensure the validity of the findings, this study employed two main validation techniques: data credibility and data dependability. Data credibility was established through four methods: (1) prolonged engagement with the research subjects to facilitate follow-up inquiries when additional information was needed; (2) persistent observation, which involved reviewing relevant references to support the study's findings; (3) peer debriefing, where discussions were conducted with other researchers and students on cognitive aspects to validate interpretations; and (4) member checking, wherein interview transcripts were shared with the subjects for review. If discrepancies were identified, clarifications and revisions were made accordingly to enhance the credibility of the research.

The dependability of the study was ensured through rigorous validation by the research supervisor, who reviewed all stages of the research process. Figure 1 presents the written test on the story problem concerning the surface area of a square pyramid, which was administered to the subjects.

> In the park there is a monument. The monument is a pyramid with a square base with a base length of 6 meters and a pyramid height of 4 meters. Then the monument wants to be repainted. If the cost of a painter to paint the monument is Rp 60,000 per m^2. How much does it cost to paint the monument without the base?

Figure 1. Written Test on the Surface Area of a Rectangular Limas Story Problem

Results and Discussion

Following the completion of the GEFT cognitive style test, researchers selected two subjects for further analysis. These subjects were then provided with a task sheet containing a story problem related to the surface area of a square pyramid. Concurrently, semi-structured interviews were conducted to explore the cognitive and communicative processes involved in their problem-solving strategies. The following section presents an analysis of the commognitive aspects that emerged during the problem-solving process, with a particular focus on the cognitive engagement of field-independent (FI) students in solving the story problem related to the surface area of a square pyramid.

Commognitive Aspects of Field-Independent (FI) Students in Solving the Surface Area of a Square Pyramid

When the FI subject completed the task involving a story problem on the surface area of a square pyramid, all four commognitive aspects were observed. These aspects are presented in Tables 1 to 4, which include transcripts of the subject's responses during interviews, along with the corresponding written solutions.

Keywords

To initiate the problem-solving process, the researcher provided the FI subject with a task and inquired about the key elements identified within the problem statement. The subject identified several keywords, including pyramid, square, right triangle, without base, square pyramid, hypotenuse, height of the upright side of the pyramid, area of the triangle, height, base, and Pythagoras. The following excerpt (Table 1) presents the interview dialogue between the researcher and the FI subject regarding the identification of keywords.

Table 1. FI Subject Conversation on Keywords

Code	Conversation
PDFIM5	After reading the problem, what do you get? Did you find any special information or words?
FIM5	What is known is that the pyramid is square, the length of the base is 6 meters and the height is 4 meters, the cost of painting is 60,000. What is asked is the cost of painting the monument without the base.
PDFIM6	Is there any more information you got?
FIM6	Nothing, sis.
PDFIM7	What is the first step you take after obtaining the information?
FIM7	I immediately described it, sis.
PDFIM8	What is the picture?
FIM8	Draw a right triangle, sis.
PDFIM9	What do you think the picture does in solving the problem?
FIM9	So that the shape is known, and the hypotenuse, sis.
PDFIM10	What is the use of the hypotenuse?
FIM10	Let me get the height of the upright side of the pyramid, because the hypotenuse is the same as the height of the upright side of the pyramid.
PDFIM11	What is the use of the vertical side of a pyramid?
FIM11	So that I can get the area of the triangle of the upright side of the pyramid, because I need the hypotenuse to get the area of the triangle.
PDFIM12	Okeee, now what do you think is the benefit of the picture you made?
FIM12	Let me get the height so I can find the triangle area.
PDFIM13	Did you use a formula in solving the problem?
FIM13	Yes, sis.
PDFIM14	What formula?
FIM14	The triangle area formula. Which is one-twelfth times the base times the height.
PDFIM15	Is there any other formula that you used?
FIM15	Ohh, I used the Pythagoras formula too, sis.
PDFIM16	Is there another formula?
FIM16	No, sis.

From the interview transcript in Table 1, it is evident that the FI subject was able to identify relevant keywords when solving the task. However, these keywords were not explicitly recorded on the subject's answer sheet. This finding aligns with research by Zayyadi et al. (2023), which observed that while students are generally proficient in identifying and utilizing keywords, they exhibit variation in how they document them. Some students rewrite known information, translate it into mathematical variables, or explicitly state the problem's key components, while others comprehend these aspects but do not record them in writing.

The FI subject demonstrated verbal mathematical communication skills, as evidenced by their ability to articulate problem-solving steps and reasoning during the interview. This verbalization supports cognitive processing by enabling the subject to analyze the problem and construct an effective solution strategy. The use of mathematical terminology also plays a crucial role in verbal communication, serving both as a medium for analogical reasoning and as an expression of conceptual understanding (Mujiasih et al., 2022).

Visual Mediator

Mathematical language can be conveyed through symbols and geometric figures, each serving a distinct function in mathematical communication. While symbols represent abstract mathematical operations, geometric figures provide a concrete representation of mathematical concepts. In written mathematical communication, these visual elements play a crucial role in facilitating understanding and problem-solving. The following section presents an analysis of the commognitive aspect of visual mediation, as observed in the FI subject's approach to solving a problem involving the surface area of a square pyramid. Table 2 provides the transcript of the researcher's conversation with the FI subject regarding the use of visual mediators.

Table 2. Conversation of FI Subject (Visual Mediator)

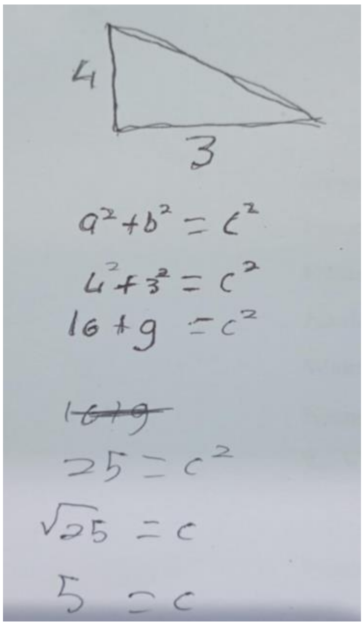
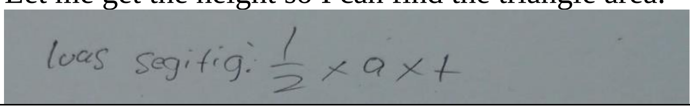
Code	Conversation
PDFIM7	What is the first step you take after obtaining the information?
FIM7	I immediately described it sis.
PDFIM8	What is the picture?
FIM8	Draw a right triangle, sis.
	
PDFIM9	What do you think the picture does in solving the problem?
FIM9	So that the shape is known and the hypotenuse, sis.
PDFIM10	What is the use of the hypotenuse?
FIM10	Let me get the height of the upright side of the pyramid, because the hypotenuse is the same as the height of the upright side of the pyramid.
PDFIM11	What is the use of the vertical side of a pyramid?
FIM11	So that I can get the area of the triangle of the upright side of the pyramid, because I need the hypotenuse to get the area of the triangle.
PDFIM12	Okeee, now what do you think is the benefit of the picture you made?
FIM12	Let me get the height so I can find the triangle area.
	

Table 2 illustrates that the FI subject was able to draw a right triangle as a written form of mathematical communication. However, the subject did not explicitly label the triangle. When asked about the benefits of the drawing, the subject indicated that it helped visualize the shape required for problem-solving. In solving the problem related to the surface area of a square pyramid, geometric representations are essential in translating the subject's conceptual understanding into a structured problem-solving approach. Research suggests that the use of visual representations in geometry significantly aids in developing solution strategies and identifying correct solutions (Mujiasih et al., 2022).

In this case, although the subject did not depict the entire square pyramid, the choice to represent the problem using a right triangle illustrates an effective use of visual mediators. Both right triangles and square pyramids serve as visual mediators, which function as cognitive tools in mathematical discourse (Mahlaba & Mudaly, 2022). The right triangle, specifically, facilitated the subject's calculation of the surface area of the square pyramid by providing a clear reference for determining necessary dimensions. According to Rahmatina and Pratiwi (2023), the use of various visual mediators is a critical factor in problem comprehension and plays a pivotal role in constructing logical reasoning and justification. Additionally, their study highlights that proficiency in utilizing visual mediators contributes to effective decision-making during problem-solving.

The subject demonstrated a clear understanding of the function of the right triangle within the problem-solving process. Notably, no calculation errors were made, and the subject successfully arrived at the correct answer. The ability to verbally explain the rationale for using the right triangle further supports the conclusion that the subject grasped the underlying mathematical concepts. Thus, it can be inferred that the visual mediator manifested through geometric representations was instrumental in solving the problem related to the surface area of a square pyramid. Moreover, the subject incorporated symbolic expressions such as the area formula for a triangle and the Pythagorean theorem, demonstrating a dual use of visual and symbolic mediation in mathematical reasoning. There are three primary classifications of visual mediators: iconic, concrete, and symbolic. Iconic mediators include graphs, diagrams, sketches, or drawings; concrete mediators involve tangible objects such as rulers or fingers for measurement and computation; and symbolic mediators encompass algebraic expressions and mathematical symbols (Lestari et al., 2021). The analysis of the FI subject's approach indicates the utilization of both iconic and symbolic visual mediators. The iconic visual mediator observed was the depiction of a right triangle, while the symbolic mediators included mathematical operators such as "+", "×", and "^{2}".

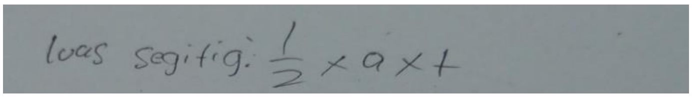
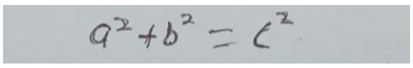
Previous studies suggest that learners continue to rely on visual mediators as communicative tools, with the choice of representation varying depending on the context of the mathematical content (Lestari et al., 2021). The findings of this study reinforce the notion that visual mediation serves as an effective cognitive strategy in mathematical problem-solving, particularly in geometry, were visualization aids in conceptual understanding and computational accuracy.

Narratives

In addressing the problem related to the surface area of a square-based pyramid, the FI subject utilized a narrative approach incorporating the Pythagorean theorem to determine the height of the upright side of the pyramid and the triangular area formula to calculate the area of the upright triangular face. However, the subject did not explicitly include the formula for the surface area of a square pyramid in the solution process. During the interview, the subject did not explicitly mention the application of the surface area formula for a square pyramid but demonstrated an implicit understanding by applying the relevant concepts in problem-solving.

According to Sfard (2020), narratives in mathematical discourse refer to mathematical facts that students employ in problem-solving, such as axioms, definitions, theorems, and formulas. Similarly, Mbhiza (2021) defines a narrative as a structured pattern of utterances related to mathematical objects and their interrelated processes, including mathematical definitions, theorems, and equations. Based on these theoretical perspectives, even if the subject did not explicitly write the formula, the ability to articulate the reasoning behind its application during the interview indicates the use of mathematical facts in the problem-solving process. Consequently, the FI subject demonstrated the narrative aspect of mathematical discourse in the employed solution strategy. The following is an excerpt of the FI subject's conversation with the researcher, presented in Table 3.

Table 3. FI Subject's Conversation (Narratives)

Code	Conversation
PDFIM13	Did you use a formula in solving the problem?
FIM13	Yes, sis.
PDFIM14	What formula?
FIM14	The triangle area formula. Which is one-twelfth times the base times the height. 
PDFIM15	Is there any other formula that you used?
FIM15	Ohh, I used the Pythagoras formula too, sis. 
PDFIM16	Is there another formula?
FIM16	No, sis.

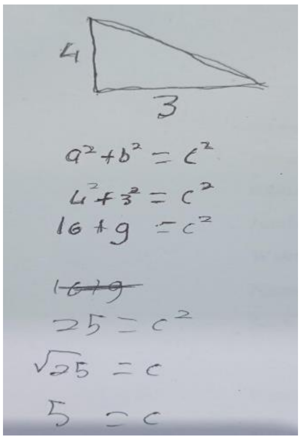
From Table 3, it is evident that the FI subject employed two formulas—the Pythagorean theorem and the formula for the area of a triangle—while solving the problem. Although the subject did not explicitly state the formula for the surface area of a square pyramid, the ability to apply these fundamental mathematical concepts suggests an understanding of the overall problem-solving process. In addition to problem-solving, narratives serve as an essential tool for assessing the depth of students' understanding. Lestari et al. (2021) emphasize that a supported narrative enhances students' comprehension, as the ability to articulate a concept appropriately indicates a deeper understanding of the given problem. Therefore, the FI subject's ability to explain the applied formulas and their relevance in solving the problem reinforces the presence of a mathematical narrative in the solution process.

Routines

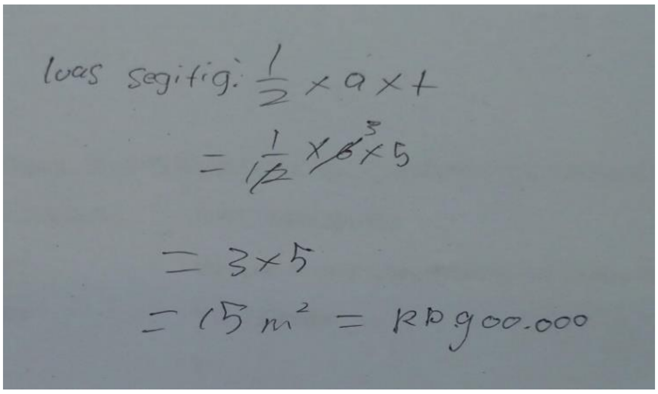
The routines demonstrated by the FI subject pertain to the structured steps taken to solve the given mathematical problem. The subject effectively articulated the solution process and documented it on the answer sheet. Additionally, another observed routine was the systematic execution of calculations. The following dialogue between the researcher and the subject, along with supporting pictorial evidence, is presented in Table 4.

Table 4. FI Subject Conversation (Routines)

<table>
<tr><th>Code</th><th>Conversation</th></tr>
<tr><td>PDFIM17</td><td>Ohh okay, now can you explain from the beginning of the completion process to the end?</td></tr>
<tr><td>FIM17</td><td>I can, first I draw what is known, it is known that the height of the pyramid is 4 meters, and the length is 6 meters. So, I divided the length in half, then I drew a right triangle like this picture. After that I looked for the c value from the formula, I got c is 5 meters.

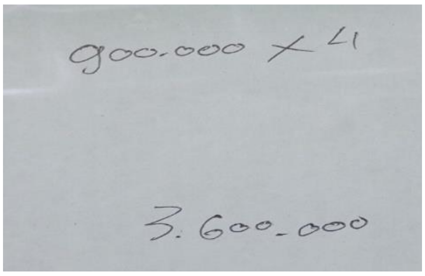


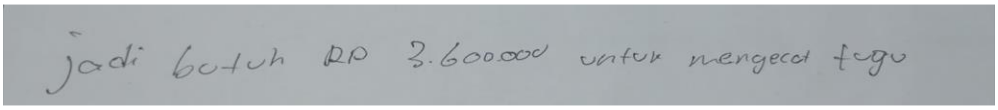
I entered the c value into the triangle area formula, I got an area of 15 m².
After that I multiplied it by 60,000, then I got the result of 900,000.



</td></tr>
</table>

After that I times 4. So, the answer is 3,600,000 sis.





PDFIM18	Why in times 4 sis?
FIM18	Because there are 4 upright sides of the pyramid.
PDFIM19	Okay, is there anything else you want to explain from your answer?
FIM19	No, sis.

From Table 4, it is evident that the FI subject successfully explained the problem-solving steps in a structured manner. The subject began by identifying key values and drawing a right triangle. However, the triangle was not labeled, and while the subject mentioned the base length as 6 meters, they wrote 3 meters (having halved the original length). Although the subject did not explicitly articulate the reasoning behind halving the length, the researcher inferred that this was a technique employed to facilitate problem-solving. Nevertheless, the subject exhibited some difficulty in explaining the rationale for this method.

Following the construction of the right triangle, the subject calculated the hypotenuse using the Pythagorean theorem, obtaining a value of 5 meters. The subject then applied the triangle area formula to determine an area of 15 m². Subsequently, the subject multiplied this value by the given cost per square meter (60,000), yielding a total of 900,000. The final step involved multiplying by four to account for the four upright triangular sides of the pyramid, leading to a total cost of Rp 3,600,000 for painting the monument.

The steps undertaken by the subject illustrate a clear procedural routine in problem-solving. Additionally, there were no computational errors present on the answer sheet. According to Mujiasih et al. (2022), routines in mathematical problem-solving pertain to systematic steps taken and are also reflected in the accuracy of calculations. It can be concluded that the FI subject successfully applied routines in solving the problem. Consequently, all four cognitive aspects keywords, visual mediators, narratives, and routines were evident in the subject's approach to determining the surface area of a square pyramid.

Commognitive Processes of Field-Dependent (FD) Students in Solving a Story Problem on the Surface Area of a Square Pyramid

When the FD subject engaged in solving a contextual problem involving the surface area of a square pyramid, various cognitive aspects were observed throughout the problem-solving process.

Keywords

To analyze the subject's understanding, the researcher assigned a task requiring the FD subject to identify key elements in the given problem. The subject was able to articulate several keywords, including "pyramid," "square," "right triangle," "without base," "square pyramid," "hypotenuse," "height of the upright side of the pyramid," "triangle area," "triangle," "cost," and "Pythagoras." Table 5 presents a transcript of the researcher's interaction with the subject during this phase of the study.

Table 5. FD Subject's Conversation (Keywords)

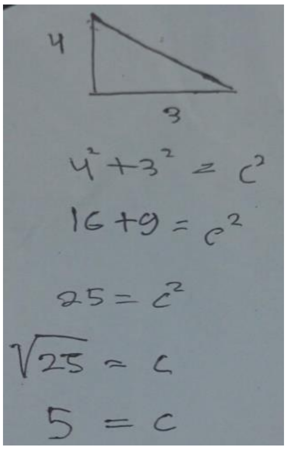
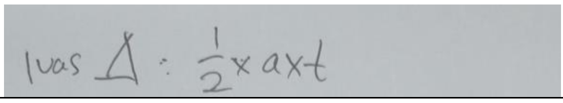
Code	Conversation
PDFDM5	After reading the question, what did you get? Did you find any special information or words?
FDM5	From the question, we were told to find the cost of painting a pyramid-shaped monument without a base. The same cost is 60,000 per meter, sis.
PDFDM6	After that, did you get any other information?
FDM6	It is known that the pyramid is square with a base length of 6 meters and a height of 4 meters.
PDFDM7	Do you have any more information?
FDM7	No more, sis.
PDFDM8	Now what is the first step you take to solve the problem?
FDM8	I took a picture, sis.
PDFDM9	What is the picture?
FDM9	I drew a triangle, sis
PDFDM10	What triangle is it?
FDM10	Right triangle, sis.
PDFDM11	What is the use of the right triangle?
FDM11	Makes it easier for me to find the hypotenuse, sis.
PDFDM12	What is the hypotenuse for?
FDM12	Find the area of the triangle
PDFDM13	What is the area of the triangle?
FDM13	The area of the upright side triangle of the pyramid.
PDFDM14	Now, what do you think is the benefit of drawing a right triangle?
FDM14	To make it easier for me to find the area of the upright side triangle, sis.
PDFDM15	Okay, did you use a formula in solving the problem?
FDM15	Yes, I used the Pythagoras formula and the triangle area.

Based on the conversation in Table 5, the FD subject successfully identified key terms related to the problem-solving task. However, certain critical information, such as explicitly stating "square pyramid" in the written response, was omitted. Additionally, while the subject verbally referenced the right triangle, its representation on the answer sheet was limited to a basic description. The height of the upright triangular face of the pyramid was denoted as t in the area formula, while the hypotenuse was symbolized as c in the Pythagorean theorem. Notably, the subject did not write down the Pythagorean theorem explicitly but demonstrated its application during the interview. This observation aligns with findings by Zayyadi et al. (2023), who reported that some students articulated essential keywords verbally but did not record them in their written solutions.

Visual Mediator

Mathematical language can be conveyed through symbols or geometric representations, both of which serve as fundamental forms of mathematical communication. While symbols encapsulate abstract mathematical operations, drawings provide a concrete depiction of mathematical concepts. Table 6 presents a segment of the researcher's dialogue with subject FD, along with supporting evidence regarding the role of visual mediators in problem-solving.

Table 6. Conversation of subject FD (Visual Mediator)

Code	Conversation
PDFDM8	Now what is the first step you take to solve the problem?
FDM8	I took a picture, sis.
PDFDM9	What picture?
FDM9	I drew a triangle, sis.
PDFDM10	What triangle is it?
FDM10	Right triangle, sis.
	
PDFDM11	What is the use of the right triangle?
FDM11	Makes it easier for me to find the hypotenuse, sis.
PDFDM12	What is the hypotenuse for?
FDM12	Find the area of the triangle.
PDFDM13	What is the area of the triangle?
FDM13	The area of the upright side triangle of the pyramid.
PDFDM14	Now, what do you think is the benefit of drawing a right triangle?
FDM14	To make it easier for me to find the area of the upright side triangle, sis.
	

As shown in Table 6, the FD subject employed a visual representation by drawing a triangle as a form of mathematical communication. However, the subject did not explicitly label the triangle. When asked about the purpose of the drawing, the subject explained that it facilitated the determination of the hypotenuse. In solving the problem of the surface area of a square pyramid, the use of geometric diagrams plays a crucial role in supporting students' conceptual understanding and problem-solving strategies. Previous studies have highlighted those visual representations significantly aid in mathematical reasoning and strategic problem-solving (Mujiasih et al., 2022).

Notably, the subject did not sketch a complete square pyramid but only a right triangle. Despite this omission, the drawing was instrumental in determining the surface area of the pyramid, demonstrating the subject's understanding of its function. Additionally, throughout the problem-solving process, the subject did not make calculation errors and successfully arrived at the correct answer. This indicates that the use of visual mediators specifically, drawings was integral to solving the problem involving the surface area of a square pyramid. Furthermore, the subject incorporated symbolic representations, including the formula for the area of a triangle and the Pythagorean theorem.

Visual mediators in mathematical discourse are categorized into three types: iconic, concrete, and symbolic visual mediators (Lestari et al., 2023). Iconic visual mediators include graphs, diagrams, sketches, and other representational images. Concrete visual mediators involve real or tangible objects, such as rulers, fingers, or other physical tools used to aid calculation. Symbolic visual mediators consist of mathematical symbols and algebraic expressions (Lestari et al., 2021). According to Mahlaba and Mudaly (2022), visual mediators function as cognitive tools that assist in mathematical reasoning. However, while visual representations enhance students' ability to conceptualize problems, supplementary symbolic mediators, such as algebraic operations, are also necessary for effective mathematical discourse (Rahmatina & Pratiwi, 2023).

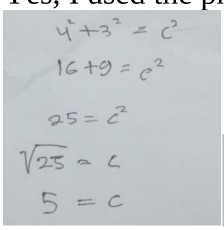
In the context of this study, the FD subject utilized both iconic and symbolic visual mediators. The iconic visual mediator was represented by the right triangle drawing, which supported the subject in understanding the problem structure and applying mathematical concepts. The symbolic visual mediators included mathematical operations such as addition (+), multiplication (×), and exponentiation (^{2}). The integration of these visual mediators contributed to the subject's ability to process and solve the given mathematical problem effectively.

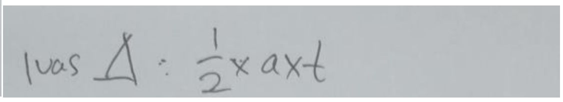
Narratives

In solving the surface area problem of a square-based pyramid, subject FD employed a narrative approach that involved the application of the Pythagorean theorem to determine the height of the triangular face and the area formula for calculating its surface area. Although the Pythagorean theorem was not explicitly written on the subject's answer sheet, the dialogue between the researcher and the subject confirmed its use. Table 7 presents an excerpt from this conversation, highlighting the subject's verbal explanation of the formulas utilized.

Table 7. FD Subject's Conversation (Narratives)

Code	Conversation
PDFDM15	Okay, did you use a formula in solving the problem?
FDM15	Yes, I used the phytagoras formula and the triangle area.





PDFDM16	That's it, no other formula?
FDM16	Yes, that's all I use

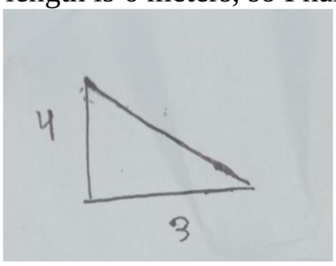
As demonstrated in Table 7, subject FD applied two mathematical formulas the Pythagorean theorem and the triangle area formula in solving the problem. Despite the absence of explicit written calculations involving the Pythagorean theorem on the answer sheet, the subject's verbal explanation indicated a conceptual understanding of its application. The ability to articulate the reasoning behind mathematical operations reflects the presence of a narrative in problem-solving.

Narratives in mathematical discourse play a crucial role in assessing students' comprehension levels. According to Lestari et al. (2021), a well-structured explanation demonstrates a student's understanding of mathematical concepts. By verbally describing the process and justifying the selection of specific formulas, students reinforce their cognitive engagement with the problem. In this context, subject FD's narrative response provides evidence of conceptual understanding, as he correctly identified and applied the appropriate mathematical formulas to derive the solution. This study underscores the importance of narrative explanations in mathematics education, particularly in evaluating students' reasoning processes. While written solutions provide direct evidence of computational accuracy, verbal narratives offer additional insights into students' conceptual grasp and problem-solving strategies.

Routines

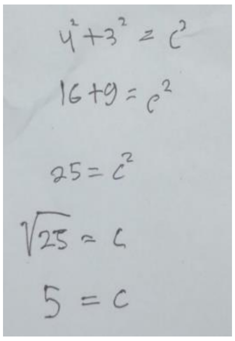
The routines demonstrated by subject FD are closely related to the structured steps undertaken in solving the given problem. The subject was able to systematically explain and document the solution process on the answer sheet. Additionally, another routine that emerged in subject FD's approach was the execution of the calculation process. Table 8 presents an excerpt from the conversation between the researcher and subject FD, providing evidence of the step-by-step problem-solving approach.

Table 8. FD Subject Conversation (Routines)

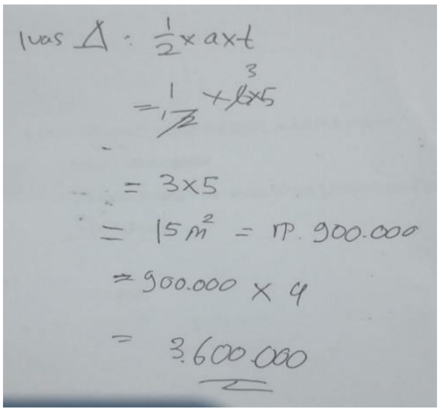
Code	Conversation
PDFDM17	Can you explain the completion process from start to finish?
FDM17	Yes, first I read the question, then I got a square pyramid, so I drew a triangle first, then the height is known from the question to be 4 meters, while the length is 6 meters, so I halved the length. 
PDFDM18	Why is it divided into two?
FDM18	Because of the triangle that I made, the length is half of 6 meters, so I divided it by two.

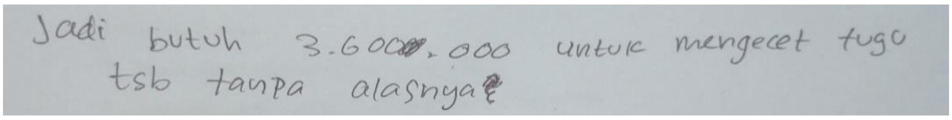
PDFDM19 Okee, please continue the explanation.

FDM19 Okee, please continue the explanation sister FDM19 Okay sis, after that after I drew it, I found the hypotenuse, I immediately entered what was known into the Pythagoras formula, then I got c which is 5 meters.



Then I entered the triangle area formula, and I got the area of 15 meters to the power of two. After that I multiplied it by the price from the question of 60,000. I got 900,000. After that I multiplied it by 4, I got 3,600,000.





PDFDM20 Why times by 4?

FDM20 Because there are 4 upright sides, the base shape is square.

PDFDM21 Okay, is there anything else you want to explain?

FDM21 No, sis.

As seen in Table 8, subject FD systematically followed a structured problem-solving routine. The subject began by reading the problem statement, identifying key elements, and explicitly mentioning critical keywords relevant to the task. The process continued with the construction of a diagram, though the subject did not label the drawing. The subject identified the base length as 6 meters but noted 3 meters, later clarifying that this resulted from halving the given length due to the properties of the right triangle within the square pyramid. Although the explanation lacked further detail on the reasoning behind this division, the conceptual understanding was evident. Following this, the subject calculated the hypotenuse by correctly applying the Pythagorean theorem, despite not explicitly writing the formula on the answer sheet. The computed hypotenuse of 5 meters was then used to determine the triangular face's area using the appropriate formula, yielding 15 m². Subsequently, the subject incorporated the given cost of Rp60,000 per square meter, arriving at a subtotal of Rp900,000. Finally, recognizing that the pyramid had four triangular faces, the subject multiplied the subtotal by 4, obtaining the final cost of Rp3,600,000.

The structured sequence of problem-solving steps, coupled with the absence of computational errors, highlights the presence of routines in mathematical reasoning. According to Mujiasih et al. (2022), routines in mathematical problem-solving encompass procedural steps and the accuracy of calculations. Moreover, as Rahmatina et al. (2022) emphasize, routines are integral to commognitive regulation, where inaccuracies in visual mediators can lead to erroneous routines.

A key distinction between the two groups was the FI subject's ability to provide detailed, clear, and efficient responses, while the FD subject required prompting questions to articulate their reasoning more comprehensively. This aligns with the findings of Yuliyani and Setyaningsih (2022), who reported that field-independent subjects tend to present precise, efficient, and detailed solutions independently, whereas field-dependent subjects exhibit a more social approach to problem-solving and require external cues to fully articulate their ideas. Similarly, the study by Amalia et al. (2020) highlighted that field-dependent subjects often experience difficulties in processing information, necessitating additional guidance to complete tasks effectively.

Finally, the findings suggest that subject FD effectively applied routines in problem-solving. Moreover, the analysis confirms the presence of all four commognitive aspects keywords, visual mediators, narratives, and routines in solving the surface area problem of a square pyramid. The study underscores the significance of structured reasoning processes in mathematics education and highlights the role of routines in facilitating conceptual clarity and problem-solving efficiency.

Conclusions

The findings of this study indicate that both FI and FD subjects demonstrated the presence of all four cognitive aspects keywords, visual mediators, narratives, and routines when solving the given task. The keywords identified in the responses of FI subjects included "pyramid," "square," "right triangle," "without base," "square pyramid," "hypotenuse," "height of the upright side of the pyramid," "area of the triangle" (referring to the triangular face of the pyramid), "height," "base," and "Pythagoras." Similarly, FD subjects utilized the keywords "pyramid," "square," "right triangle," "without base," "square pyramid," "hypotenuse," "height of the upright side of the pyramid," "triangular area" (referring to the triangular face of the pyramid), "triangle," "cost," and "Pythagoras."

Both groups employed visual mediators, specifically a diagram of a right triangle, to facilitate the calculation of the area of the pyramid's triangular face. Additionally, both FI and FD subjects incorporated narratives, primarily through the application of the Pythagorean theorem and the triangle area formula. However, while the FI subject explicitly documented both formulas on the answer sheet, the FD subject only demonstrated their application through verbal explanation in the interview. The routines aspect was also evident in both subjects' responses, as they were able to articulate the step-by-step problem-solving process during the interview.

Finally, future research is encouraged to examine the four cognitive aspects in greater detail across different mathematical topics. Additionally, researchers should consider designing questions that prompt students to provide more structured, explicit, and efficient responses. It is anticipated that the results of this study will serve as a valuable reference for educators and researchers seeking to enhance their understanding of cognitive processes in mathematical problem-solving.

Conflict of Interest

The authors declare no conflict of interest regarding the publication of this manuscript.

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