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Preservice Teachers Studying Video Narratives of Student Argumentation

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Abstract

National Standards in the teaching of mathematics call for teachers to pay attention to the nature of student problem solving and argumentation in learning mathematics. To attend to student argumentation, resources are needed in which student argumentation can be observed. This paper reports the result of a preservice-teacher intervention study that was designed to investigate preservice teachers' recognition of children's arguments about fraction comparisons. Teachers in the study described student argumentation in a video narrative before and after instruction, which included video narratives designed to highlight argumentation. Pre- and post-assessment of teachers' identification of student argumentation used a mixed-methods analysis to investigate growth in noticing more of children's argumentation across a video narrative that was created with 15 video clips that displayed student argumentation in the classroom. Results showed that growth occurred in cycles that described shorter argumentation stories within the longer narrative. On average, over 85% of the teachers consistently exhibited growth at the end of a cycle of 3 to 6 video clips that provided a complete story of several children's argumentation, in contrast to growth on specific clips that showed a particular child's argument. Results suggest the enhanced value of using video episodes that reveal complete stories of student argumentation in a classroom for teacher intervention.

Keywords: preservice teachers, children's argumentation, video narratives, reasoning, fraction comparisons

Introduction

Current standards in the teaching of mathematics (NCTM, 2000) and standardized tests emphasize student understanding of basic math concepts and applications. This goal creates challenges for teachers especially if they were taught math through memorization of rules and procedures. The standards call for student communication employing justifying ideas, creating conjectures, and exploring mathematical tasks together and advise that instructional programs cultivate a classroom environment in which students are encouraged to evaluate arguments, ask questions to clarify arguments, offer counterarguments, and develop new arguments that can be supported. The Standards requiring students to understand math concepts also place new demands on teachers to monitor and assess student knowledge by paying attention to arguments that support solutions as well as to collective class argumentation (Bieda & Lepak, 2014; Conner, et al., 2014a; Krummheuer, 1995; 2007; Whitenack & Yackel, 2002).

The purpose of this study was the determine how preservice teachers' noticing of student argumentation was affected by studying video narratives of students engaged in argumentation. The intervention described in this study was conducted with preservice teachers enrolled in a required semester-long undergraduate Math Methods course in an urban university in the United States. During the intervention, preservice teachers studied classroom video narratives of children involved in collaborative problem solving created to illustrate argumentation about fraction comparisons. How the studying of the video narratives affected teachers' noticing of argumentation was investigated through the collection of pre-and post-teacher assessments capturing the extent to which teachers were able to identify the claims and counterclaims made by the children, note the evidence for these claims, and recognize the form of the children's proof-like reasoning (Maher & Martino, 1996).

Theoretical Perspective

Argumentation and Reasoning

There is a strong correlation between reasoning and argumentation. Some researchers have noted that reasoning is a part of argumentation (Krummheuer, 1995; Sriraman & Umland, 2020), while others have described argumentation as "a special kind of reasoning" (Pedemonte, 2007; Wagner et al., 2014; Yankelewitz, 2009). Moreover, Whitenack and Yackel (2002) state both that "reasoning involves making mathematical arguments, in particular, explaining one's ideas to clarify those ideas for others" and that "explaining and justifying [that is, making arguments] are important aspects of reasoning". Whether the relationship between argumentation and reasoning is described as argumentation as a certain kind of reasoning, or reasoning as part of argumentation, researchers agree that they occur together.

This study utilizes the perspective that argumentation is the engagement of others as they provide evidence for the validity of their solutions to a problem and reasoning is the form of that justification. By student argumentation, we mean the engagement of students with each other to justify the solution to a problem. Maher and Martino (1996) and Tall et al (2012) show that the reasoning of even young children can be proof-like, taking many forms including counterexamples, cases, or induction. Tall and colleagues noted that students' reasoning develops from their own "embodied mathematical world" with the reasoning that is particular to the student's way of thinking, to an "all-inclusive one," in which reasoning incorporates more global conventions which leads toward the construction of a valid argument. In this view, arguments may or may not be valid, but argumentation is productive when it is the process in which a community works toward creating a valid argument.

Extant research supports the view that argumentation is essential to math learning and that the role of the teacher in promoting student argumentation is crucial (Ball et al., 2002; Maher, 2010; Maher et al., 2010; Yackel, 2002; Weber et al., 2008) with Krummheuer (1995) categorizing the role of a teacher as "imperative" and Whitenack and Yackel (2002) asserting that its importance "cannot be overstated." Often through guided questioning, teachers can promote argumentation by inviting children to explain, justify, and defend their ideas, and invite others to evaluate those ideas to foster the construction of productive argumentation. Teachers can encourage children to expand their arguments by asking them to offer additional evidence or to make explicit elements of the argument clear (Conner et al., 2014b; Maher & Martino, 1996; Zack, 1997). Bieda (2010) reminds us that when teachers are minimally involved in classroom discourse, very little argumentation is produced.

Argumentation in everyday life may suggest meanings that do not match a definition of argumentation used in mathematics. Hence, teachers need to be knowledgeable about what is meant by productive argumentation in classrooms so that they can promote this discourse and make sense of the justifications offered by children (Star & Strickland, 2008; Wagner et al, 2014; Whitenack & Yackel, 2002).

Using Video in Teacher Education and Teacher Noticing

Video is a valuable tool for capturing student learning and has the potential for teacher instruction. Towers (2007) and others (Brunvand & Fishman, 2006; Hmelo-Silver et al., 2013; Jacobs et al., 2010; Martinez et al., 2015; Palius & Maher, 2013; Van Es & Sherin, 2008) found that video can help preservice teachers to refocus their attention from teaching and what the teacher is doing to what students are learning. By studying video events, teachers have time to reflect, review episodes, and deepen their observations of student behavior (Sherin & Van Es, 2005; Van Es & Sherin, 2002; 2008). Researchers have reported that through studying videos of student learning, teachers can attend to the variety of forms of reasoning used by students engaged in collaborative problem solving (Maher, 2011; Maher et al., 2014; Palius & Maher, 2011).

Proficiency in adjusting instruction in real time to engage students in supporting their ideas and solutions requires that teachers can notice and correctly interpret events as they unfold, making the ability to notice a key feature in developing teaching expertise. Van Es & Sherin (2008) report difficulty for both practicing and preservice teachers in noticing important aspects of instruction in real time and that observing videos of classroom interactions has the advantage of supporting the recognition of details and critical events, in contrast to general activities, defined by Jacobs and colleagues (2010) as "professional noticing" or the intentional noticing particular to a profession. When studying video data, teachers can develop new ways of noticing and interpreting events of classroom episodes. Star and Strickland (2007) concur reporting that noticing skills can be improved through the purposeful viewing of videos.

Toulmin's Model to Analyze Argumentation

Argumentation consists of a series of statements, actions, tools, notation, drawings, models, images, or numerical data that take on different roles in the convincing process (Krummheuer, 1995; Yackel, 2002) and Toulmin's model (2003) is a useful tool to illustrate what the function, or role, each element has within the course of an argument. Krummheuer (2007) and others (for example, see Pedemonte, 2007; Van Ness, 2017; Van Ness & Maher, 2019) have adapted Toulimin's model to support the analysis of the argumentation stating that it "offers structure to the sometimes messy and often ill-defined construct of reasoning in school mathematics that is included in students' mathematical argumentation" (Conner et al, 2014a).

The basic elements of the adapted model include claims, data, warrants, and backing. Although Toulmin includes the qualifier and rebuttal in his original diagram, researchers that have adapted the Toulmin model for K-12 argumentation primarily focus on the elements of claim, data, warrant, and backing. A claim is a conclusion believed to be true and data are statements, facts, or information meant to be evidence in support of a claim. A warrant makes the inference between the claim and data explicit and may be presented to explain how the data supports the claim and to make the argument more convincing.

Backing, or general theories or generally accepted beliefs that explain why the warrant should be accepted as having authority, also might be included in the argument (Krummheuer, 1995). During an argument, an arguer might correct, refine, modify, replace, refute, or retract statements in the argument. The order in which elements of an argument are contributed varies. Also, statements within an argument are considered situationally since what might be considered as a claim, data, warrant, or backing can depend on the audience and the context in which these are offered. The distinction among the elements, then, may be determined by analysis of the entire interaction (Yackel, 2002). Thus, analyzing argumentation is a fluid and situationally dependent process (Krummheuer, 2007).

A diagram adapted from Toulmin (1958, 2003) and similar to diagrams used by others (See Conner et al., 2014a; Krummheuer, 2007; Pedemonte, 2007; Wagner et al., 2014) to organize argumentation was used to guide the data analysis for this study. This diagram is shown in Figure 1.

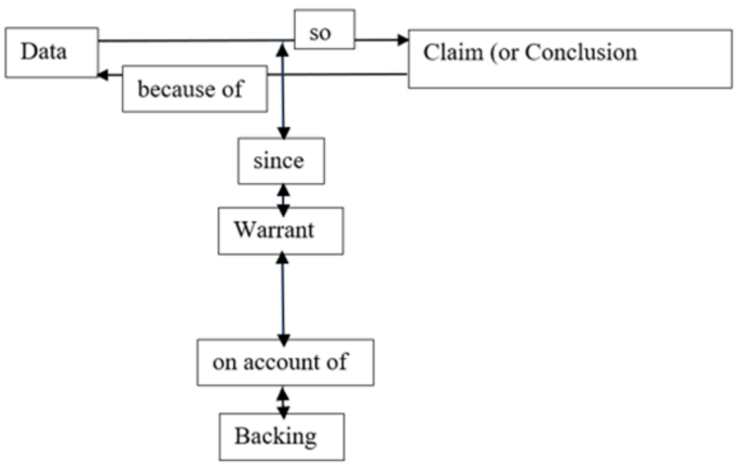


Figure 1. Diagram of elements and structure of argumentation adapted from Toulmin's model

The connecting rays and line segments in the diagram in Figure 1 represent the elements and connections in the argument that provide the structure. In this schematic representation of the argumentation structure, the arrow between the elements implies a specific flow of argumentation. For example, the flow of the argumentation might be, "claim because of data," where the claim is stated and then the data provided, or "data, so claim," where the claim is stated after the data is presented. Students' arguments can range from simple (claim, data) to complex, incorporating nested sub-arguments. Also, when data statements need to be further supported, they can function as both data in one argument and a claim in a sub-argument (Conner et al., 2014b; Krummheuer, 1995). For this study, data, warrant, and backing are referred to as the "evidence" for an argument.

Method

The study was designed to determine whether using video narratives, such as VMCAnalytics, could offer the support necessary for preservice teachers' recognizing of student argumentation. In particular, we investigated what argumentation teachers noticed when they described children's discourse when viewing a video narrative of children engaging in argumentation. Also, we explored to what extent teachers noticed more argumentation after studying other intervention video narratives that were created to highlight student argumentation. Finally, we explored whether there were differences in recognizing argumentation among the preservice teachers in noticing more argumentation. The research questions that guided the study included the following:

1. What do teachers notice about children's argumentation when provided with video narratives that show children engaged in argumentation?
2. Was there a change in teachers' noticing of the children's argumentation after studying video narratives created to highlight student argumentation, and if so, what was the nature of that change?
3. What was the nature of teacher differences, if any, in examining teacher growth across the series of 15 classroom videos, and how did those differences affect teachers' noticing of argumentation?

Setting and Population

The study took place during a semester-long secondary, graduate mathematics education course at a large northeastern university. The assignments for the study were given as part of the regular coursework. The participants in the study were preparing to be middle or high-school mathematics teachers. Eleven preservice teachers were enrolled in the course; nine participated in the study.

VMCAnalytics in the study

To present teachers with the opportunity to observe and study children engaged in argumentation, video narratives, or VMCAnalytics, were created. VMCAnalytics and the role they played in the study are described in the following sections.

The Video Mosaic Collaborative and the VMCAnalytic

Taking a lead role in utilizing next-generation technology, Rutgers University Libraries has developed a powerful community repository that "facilitates scholarly collaboration and communication" (Wilson & Jantz, 2011) by capturing the output of Rutgers University research, including digital video. Various artifacts from collections at the university are housed in a parent portal. Researchers from the Rutgers Digital Library of Sciences and Educational Psychology, in conjunction with mathematics education researchers who have studied the development of students' mathematical reasoning both longitudinally and cross-sectionally, have developed one of these collections, the Video Mosaic Collaborative repository (VMC; www.videomosaic.org), an open-source online resource that makes data from their studies publicly accessible. Videos from the VMC repository have been used successfully in courses, scholarly work, and research, and as resources for developing portfolio items for teacher education programs. The VMC video data are a subset of the 4,500 hours of classroom video captured over three decades from several research studies (Hmelo-silver et al., 2010; Maher, 2011). The videos have been used for the professional development of experienced teachers (Berenson, 2012) and with pre-and in-service teachers interested in studying student learning (Maher, Landis et al, 2010; Maher, Palius, et al, 2010).

Researchers have found that cyber-enabled video tools are effective in partitioning large amounts of video data into segments that are usable for teacher study and can support teachers' and researchers' observation of students learning and their reflection on that learning (Derry, 2007; Powell et al., 2003). A VMCAnalytic is a video narrative created via an award-winning RUanalytic video annotation tool (See Otto & Ralston, 2012) linked to the Rutgers Libraries repositories that allow users to link together and annotate chosen video clips of various lengths from selected segments of videos within the VMC video content. Each VMCAnalytic video narrative tells a unique story usually dealing with student classroom problem solving and is published on the VMC. Typical VMCAnaltyic video narratives consist of a collection of events that include short video clips partitioned into "critical events" (Maher & Martino, 1995; Powell et al, 2003) that each have a title and a description written to highlight specific details that can be observed in the video clip. The video narrative also has an overall title and general description. VMCAnalytics have been used by researchers to illustrate mathematical ideas and for educational purposes and to promote scholarly conversation, including being used in university courses in a variety of different ways (Agnew et al., 2010; Gomoll et al., 2015; Hmelo-Silver et al., 2013; Hmelo-Silver et al., 2014; Maher & Sigley, 2014; Sigley & Wilkinson, 2015).

Four VMCAnalytics were used as tools for the study

Informed by the literature supporting the effectiveness of video artifacts in teacher education and the value of using VMCAnalytics in teacher education research, four VMCAnalytic video narratives were created for this study to show children engaging in argumentation, as defined by Krummheuer (1995) and Toulmin (1958, 2003). All four video narratives were created to show video clip episodes of children making claims, providing data, using warrants, challenging and refuting claims, making counterclaims, and providing counterarguments from a single classroom session. Based on the findings of Hmelo-Silver et al (2014) that VMCAnalytics can be used as an effective tool for assessing understanding of learning, one of the video narratives created for the study was used as a tool to gather pre-assessment and post-assessment data on teachers' noticing of student argumentation.

Consistent with Brunvand and Fishman's (2006) findings that teacher noticing can be supported through the use of video, three other video narratives were created for the study and used as teaching tools. For the intervention, teachers were asked to study the video narratives, including watching the video clips and reading the titles and descriptions for each event of the children's argumentation.

The four video narratives that can be viewed at www.videomosaic.org, are described in the following two sections. Because teachers' descriptions in the assessment video narrative were the data analyzed for this study, more details are provided for it. Brief descriptions are provided for the other three video narratives. For a detailed analysis of these three video narratives, please see Van Ness (2017) as well as Fourth graders' argumentation about the density of fractions between 0 and 1 (http://dx.doi.org/doi:10.7282/T39K4CZC, Van Ness, 2015a), Eighth Grader Stephanie's Argumentation about Meaning for the Square of a Binomial using Algebraic Reasoning (http://dx.doi.org/doi:10.7282/T3FN180C, Van ness, 2015b), and Eighth Grader Stephanie's Argumentation about Meaning for the Square of a Binomial using Geometric Reasoning (http://dx.doi.org/doi:10.7282/T3QZ2CRF, Van ness, 2015c).

3VMCAnalytic as a Pre- and Post-Assessment Tool

To assess teachers' noticing of argumentation in the children's discourse, Fraction Assessment Analytic, Fourth Grades' Argumentation about the comparison 1/2 and 1/3 1 was created (https://rucore.libraries.rutgers.edu/analytic/#open/type=analytic&id=730, 11 minutes 21 seconds) and used to collect pre-assessment and post-assessment data. This video narrative was created from video data collected in a year-long research study of fourth graders' exploration of fractions ideas (See Maher & Yankelewitz, 2017) and the video data used were taken from the 5th session of the first month of the study. Fifteen event clips taken from the 57-minute class session made up the VMCAnalytic video narrative and each event clip ranged from about 1 to 3 minutes in length. Event clips were chosen specifically because of the student argumentation that could be observed. As previously stated, most VMCAnalytic video narratives include an overall title and description and titles and descriptions for each event that are purposely crafted to focus the viewers' attention on the video activity. Because of the assessment nature of the Fraction Assessment Analytic, only an overall title and description were provided and were crafted to give general information about the video narratives, not specific information about argumentation. Additionally, no event descriptions were given and the event titles were purposefully vague, i.e., Event 1, Event 2, Event 3, and so on.

Van ness and Maher (2019) and others (Van ness & Alston, 2017; Yankelewitz, 2009; Maher & Yankelewitz, 2017) report that during the 57-minute class session reflected in the video narrative, the children were engaged in comparing the fractions 1/2 and 1/3, specifically challenged to determine which was larger and by how much. The children were encouraged to support and justify their solutions. They offered proof-like arguments, that is, arguments that follow a form of argument such as counterexample, cases, or induction for the specific case, using objects to formulate similarities and differences (Tall et al., 2012). For example, children claimed that 1/2 was greater than 1/3 by 1/6 because the rod that represented 1/2 was longer than the rod that represented 1/3 by a white rod, and, since 6 white rods were the same length as the rod train with the number name 1, the number name of the white rod was 1/6. These arguments evolved as they used models to validate and invalidate their arguments and the arguments of others (Van Ness & Maher, 2019).

Children's argumentation shown in the 15 events of the video narrative was focused around two Cuisenaire rod models that were offered in support of their solutions. In Model 1, a train (two or more contiguous rods) was formed using an orange and red rod and was given the number name 1, to represent the unit, with the dark green rod given the number name 1/2, the purple rod, 1/3, and the red rod, 1/6 (see Figure 2).

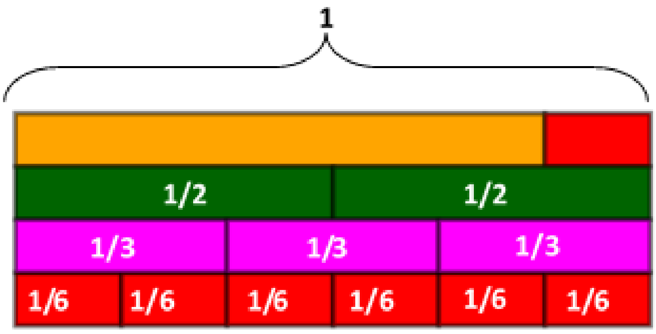


Figure 2. Model 1

In Model 2, unit, 1, was represented by the dark green rod, the light green rod was given the number name 1/2, the red rod was called 1/3, and the white rod was given the number name 1/6 (see Figure 3). A brief description of the Events 1 through Event 15 is provided below.

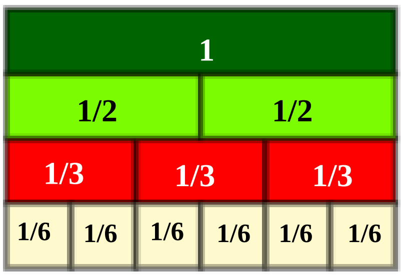


Figure 3. Model 1

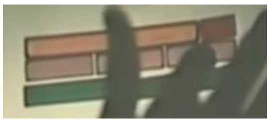
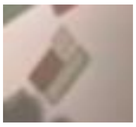
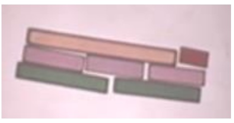
In Events 1–6 of the Fraction Assessment VMCAnalytic, children used Model 1 to develop their arguments. With this model, children stated that the dark green rod had the number name 1/2 and the purple rod had the number name 1/3. They placed the dark green 1/2 rod above the purple 1/3 rod and noted that the dark green 1/2 rod was longer than the purple 1/3 rod, concluding that 1/2 was greater than 1/3. They then quantified the difference between 1/2 and 1/3 using their models, showing that the dark green 1/2 rod was longer than the purple 1/3 rod by a red rod. Some children gave the red rod the number name 1/3, arguing that since three red rods are the same length as a dark green rod, each red rod has the number name 1/3 (this was an invalid argument based on confusion about the unit, and possibly combining Model 1 and Model 2). They then concluded that 1/2 was greater than 1/3 by 1/3. Other children challenged this result using Model 1 to show that the red rod is half the length of the purple 1/3 rod, and, since the purple rod has the number name 1/3 and half of 1/3 is 1/6, the number name of the red rod must be 1/6. They argued that 1/2 is greater than 1/3 by 1/6.

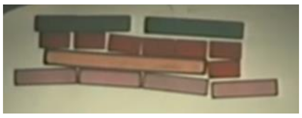
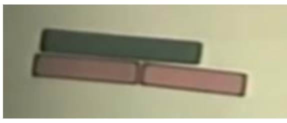
In Events 7–9, the argumentation transitioned to children using Model 2 to demonstrate their result, claiming that if the dark green rod were given the number name 1, the light green rod must have the number name 1/2, and the red rod would have the number name 1/3. They showed that the light green 1/2 rod was longer than the red 1/3 rod by a white rod. Some children referred to Model 2 and stated that the white rod had the number name 1, and so 1/2 was greater than 1/3 by 1. Other children disagreed, stating that 1/2 is greater than 1/3 by 1/6, supporting their counterclaim by showing that a train of six white rods is the same length as the dark green 1 rod; thus, each white rod has the number name 1/6.

In Events 10–15, the argumentation transitioned back to the arguments created using Model 1. Some children repeated their original argument that, using Model 1, the red rod has the number name 1/3 and so the model shows that 1/3 is greater than 1/3 by 1/3. Other children, referring to Model 1, repeated their argument that the red rod is half the length of the purple 1/3 rod, and so, since 1/6 is half of 1/3, 1/2 is greater than 1/3 by 1/6. Another child offered an argument that the red rod has the number name 1/6 by showing that six red rods are the same length as the train of orange and red rods with the number name 1. The children discussed the merits of these models and the various arguments, offering agreements and disagreements. Table 1 shows a summary and the models used in each event in the assessment analytics.

Table 1. Summary of assessment analytic events

Event	Model Used	Summary
Event 1		The facilitator presents the problem: Which is greater 1/2 or 1/3? Children claim that 1/2 is bigger than 1/3. No models are given as evidence.
Event 2		Children use rod Model 1 as evidence to support the claim that 1/2 is greater than 1/3.
Event 3		One child uses the model from event 2 as evidence to support that 1/2 is greater than 1/3.

Event 4		The facilitator asks children to consider how much bigger 1/2 is than 1/3. Children use the rods to claim that 1/2 is 1/3 bigger than 1/3.
Event 5	 	One child uses elements of rod Model 2 to support the idea that 1/3 is bigger than 1/2. Then children then use the same model from event 4 to show that since 3 red rods are the same length as a green rod, and since the 1/2 rod is 1 red rod longer than the 1/3 rod, 1/2 is greater than 1/3 by 1/3.
Event 6	 	A child offers a counterclaim that 1/2 is greater than 1/3 by 1/6. The child uses elements of Model 1 to create evidence that 1/2 is bigger than 1/3 by one half of 1/3, and so it is bigger by 1/6.
Event 7		A child claims that 1/2 is bigger than 1/3 by a white rod and uses elements of rod Model 2 to support the claim. Another child states that this is not a valid model to use because the size of the unit is different.
Event 8	 	Children refine their claim from event 7 to state that 1/2 is greater than 1/3 by 1 and use elements of rod Model 2 as evidence to support their claim. They give the white rod the number name 1.
Event 9		A child conjectures that in event 8, what is really meant is that 1/2 is greater than 1/3 by 1/6 and uses rod Model 2 as evidence, showing that the white rod has the number name 1/6. The children agree with the arguments presented.
Event 10		Children return to the earlier claim that 1/2 is greater than 1/3 by 1/3 and assert that the red rod has the number name 1/3 using rod Model 1.
Event 11		A child disagrees and presents a counteragument that supports the claim that the red rod has the number name 1/6 using the rods as evidence to show that a red rod is one half of the 1/3 rod and, since two 1/3 rods are longer than the 1/2 rod, 1/2 cannot be greater than 1/3 by 1/3. The other children modify their claim further to state that 1/2 is greater than 1/3 by both 1/3 and by 1/6.
Event 12		Another child disagrees with the claim that 1/2 is greater than 1/3 by 1/3 and presents a counterargument, stating that the 1/2 rod is not the same length as two 1/3 rods as would be the case if 1/2 were greater than 1/3 by 1/3. Elements of rod Model 1 are used to show that the length of the dark green 1/2 rod is longer than a purple 1/3 rod, but shorter than two purple 1/3 rods.
Event 13		Children continue their counterargument from event 12, stating that 1/2 could not be 1/3 greater than 1/3 because 1/3 longer is the length of two purple rods and that length is longer than the length of the dark green 1/2 rod, noting that there is a half of 1/3 left over and conjecturing that the amount left over is 1/6. A child then presents a second

		counterargument for the claim that 1/2 is greater than 1/3 by 1/3, arguing that 1/2 cannot be 1/3 greater than 1/3 because the dark green 1/2 rod is longer than one purple 1/3 rod (showing that 1/2 is greater than 1/3), but shorter than two of the purple 1/3 rods.
Event 14	 	Another child agrees with the counterclaim that the red rod has the number name 1/6 and uses the rods as evidence for the claim, showing that a train of 6 red rods is the same length as the train of rods with the number name 1. Another child agrees and shows that three sixths (representred by a train of 3 red rods) is the same length as 1/2 (represented by a dark green rod).
Event 15		Another child uses the rods as evidence to show that the difference between the dark green 1/2 rod and purple 1/3 rod cannot be 1/3 because the length of 2 thirds is longer than 1/2. The children continue the evidence for the argument using reasoning and stating that the dark green 1/2 rod is longer than the purple 1/3 rod and shorter than two purple 1/3 rods (two thirds), making the argument that 1/2 is less than 2/3 but more than 1/3.

Three Argumentation VMCAnalytics for the Instructional Intervention

Three other video narratives were created and used as teaching tools in the intervention. These video narratives were created with video clips chosen to highlight children's argumentation and were annotated to describe that argumentation in detail. A brief description of each as well as the link where each video narrative can be viewed is given below.

Intervention VMCAnalytic 1

Eighth Grader Stephanie's Argumentation about Meaning for the Square of a Binomial using Algebraic Reasoning (http://dx.doi.org/doi:10.7282/T3FN180C, 5 minutes 52 seconds). In this video narrative, 14 video-clip segments show eighth-grader Stephane developing arguments to support an algebraic representation of the binomial expansion. For more detailed description of this analytic, see Van Ness (2015a) and Van Ness (2017).

Intervention VMCAnalytic 2

Eighth Grader Stephanie's Argumentation about Meaning for the Square of a Binomial using Geometric Reasoning (http://dx.doi.org/doi:10.7282/T3QZ2CRF, 6 minutes). In this video narrative, eight video-clip events show eighth-grader Stephanie developing arguments to support a geometric representation of the binomial expansion. For more detailed description of this analysis, see Van Ness (2015b) and Van Ness (2017). Both Intervention VMCAnalytics 1 and 2 show how Stephanie explores representing (a+b)2 algebraically and by drawing a square with a side length of (a+b) units. In so doing, she makes and tests conjectures and claims, produces counterexamples, poses counterclaims, and refutes and modifies original conjectures, claims, and refines arguments.

Intervention VMCAnalytic 3

Fourth graders' argumentation about the density of fractions between 0 and 1 (http://dx.doi.org/doi:10.7282/T39K4CZC, 16 minutes, 50 seconds). In this video narrative, 18 events show a class of fourth-grade children engaged in argumentation about representing fractions on a line segment between 0 and 1. The events depicted in this VMCAnalytic were taken from a larger data set gathered as a result of a year-long research intervention involving fourth graders' exploration of fractions. The VMCAnalytic shows children's argumentation about whether infinitely many fractions can be placed between 0 and 1 on the number line. Claims, challenges to those claims, counterarguments, warrants, justification, and evidence are illustrated in the children's discourse. For a more detailed description of this analysis, see Van Ness (2015c) and Van Nees (2017).

Instructional Intervention

The instructional intervention included in this study lasted 8 weeks and consisted of three phases: pre-assessment data collection, teachers studying of argumentation VMCAnalytic video narratives, and post-assessment data collection. The three phases are described here and summarized in Table 2.

Table 2. Instructional intervention timeline

Phase	Week of Course	Teacher assignment
1	Week 1	Pre-assessment data collection with *Assessment VMCAnalytic*
2	Week 2 and 3	VMCAnalytic 1: *Eighth Grader Stephanie's Argumentation about Meaning for the Square of a Binomial using Algebraic Reasoning*
	Week 4 and 5	VMCAnalytic 2: *Eighth Grader Stephanie's Argumentation about Meaning for the Square of a Binomial using Geometric Reasoning,*
	Week 6 and 7	VMCAnalytic 3: *Fourth graders' argumentation about the density of fractions between 0 and 1*
3	Week 8	Post-assessment data collection with *Assessment VMCAnalytic*

Pre-Assessment Data Collection

In Week 1 of the intervention, preservice teachers were asked to view the Assessment VMCAnalytic. The video narrative included a general title and overall description, designed so that there were no event descriptions or descriptions of argumentation and only general event titles (e.g., Event 1, Event 2, and so on). Teachers were asked to write event descriptions and event titles specifically detailing the argumentation they observed in each event. Then they were instructed to write an overall description for all events that comprised 15 video clips of the video narrative. These titles and descriptions were collected as the study's pre-assessment data.

Studying VMCAnalytic Video Narratives

After the pre-assessment data were gathered, teachers spent the next 6 weeks studying the other three published video narratives which were specifically designed to highlight and describe children's argumentation with descriptions and titles crafted to describe the argumentation in students' discourse. "To study" was defined as watching the video events and reading the annotations that included the overall title and description, and the title and descriptions for each event.

During Weeks 2-3 teachers studied Intervention VMCAnalytic 1: Eighth Grader Stephanie's Argumentation about Meaning for the Square of a Binomial using Algebraic Reasoning. During Weeks 4-5 teachers studied Intervention VMCAnalytic 2: Eighth Grader Stephanie's Argumentation about Meaning for the Square of a Binomial using Geometric Reasoning, and during Weeks 5-7, teachers studied Intervention VMCAnalytic 3: Fourth graders' argumentation about the density of fractions between 0 and 1. During this time, teachers were provided with guiding questions and had the opportunity to discuss each analytic and the questions in an online forum. The instructor's role was minimal, limited to asking clarifying questions and encouraging participation. Table 3 provides the questions that teachers were given to use as a guide as they studied the video narratives.

Table 3. Guiding questions for studying argumentation video narratives

No	Guiding Questions
1.	Identify elements of argumentation that can be identified in this analytic.
2.	What are the claims being made by the children in the arguments presented? Who is making what claim?
3.	Identify evidence that the children use to support their claims.
4.	For the claims presented, identify those that are: (a) challenged; (b) modified and (c) refuted
5.	Was the argument resolved? Explain.

Post-Assessment Data Collection

In Week 8, teachers were instructed to again watch the Assessment VMCAnalytic and review the titles and descriptions they wrote before the intervention. They had an opportunity to make revisions and were instructed, again, to describe, in detail, the argumentation they saw for each event. Teachers were encouraged to revisit the three argumentation VMCAnalytics they studied and responded to the guiding questions from the online discussions. These revised descriptions were collected as post-assessment data. A summary of the timeline of the instructional intervention is provided in Table 4.

Table 4. Instructional intervention timeline using the toulmin model as a tool for analyzing argumentation

Phase	Week of Course	Teacher assignment
1	Week 1	Pre-assessment data collection with *Assessment VMCAnalytic*
2	Week 2 and 3	VMCAnalytic 1: *Eighth Grader Stephanie's Argumentation about Meaning for the Square of a Binomial using Algebraic Reasoning*
	Week 4 and 5	VMCAnalytic 2: *Eighth Grader Stephanie's Argumentation about Meaning for the Square of a Binomial using Geometric Reasoning*
	Week 6 and 7	VMCAnalytic 3: *Fourth graders' argumentation about the density of fractions between 0 and 1*
3	Week 8	Post-assessment data collection with *Assessment VMCAnalytic*

Using Toulmin Model as a Tool for Analyzing Argumentation

As described previously, the Toulmin model (Toulmin, 1958; 2003) is a useful tool for diagram argumentation. Hence, once pre-assessment and post-assessment data in the form of teachers' descriptions of the video clips in the Fraction Assessment Analytic were collected, the Toulmin model was used to diagram the children's argumentation in the video events and then to create a diagram of each teacher's description of the argumentation in the video events, both for their pre- and post-assessments. It is important to point out that Toulmin's model was used as a guide by the researchers to describe the argumentation from the video events and was not included in the intervention with the teachers. Figures 4 and Figure 5 provide generic examples of how Toulmin's model was used to diagram argumentation in the study.

Figure 4. Data and claim with no connection

Figure 5. Data and claim with connections

The diagram in Figure 4 represents a situation in which a child stated Claim A and Data B, with no indication in the discourse that the data was supporting the claim. The diagram in Figure 5 represents a situation in which a child stated either "Claim A because of Data B" or "Data B so Claim A." In this case, the child used language that explicitly connected Claim A and Data B. The arrow represents the connection and suggests the structure of the argument, as well as the directionality of the statements. Notice that the data and claim in Figure 5 are connected with a solid arrow showing the structure of the argument. However, in Figure 4 there is no connection between the elements. The process of how diagramming was used in the study is described in the following two sections

Analysis of Children's Argumentation in the Fraction Assessment Analytic: The Standard Model

Researchers used the Toulmin model to diagram the structure and elements of the children's argumentation for each of the 15 video events in the Assessment VMCAnalytic. These diagrams were validated by at least two other experts in argumentation and proof to obtain reliability. The resulting diagrams encompassed all the argumentation demonstrated by children in the video clip segments and were used as a Standard Model for the argumentation that could be observed in the events of the video narrative. Figure 6 shows an example of the nature of the diagrams used to illustrate the argumentation in children's discourse. This diagram illustrates the argumentation evident in Event 6 of the Assessment VMCAnalytic.

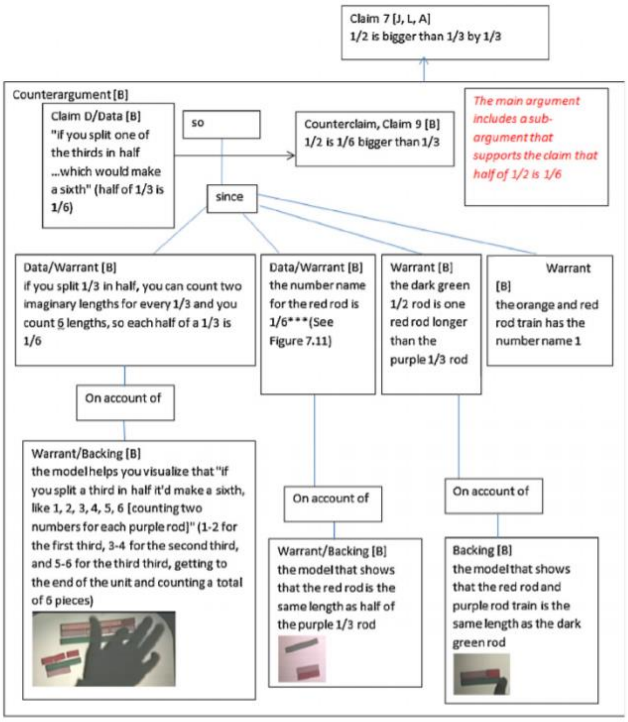


Figure 6. Diagram of the argumentation in event 6 of *fraction assessment analytic 1*

In the Event 6 video clip, the children present a counterargument to the claim that 1/2 is greater than 1/3 by 1/3. The counterargument includes the counterclaim that 1/2 is greater than 1/3 by 1/6. Evidence is presented in the form of rod models. Students reasoned that the rod that shows the difference between 1/2 and 1/3 is half as long as the rod that represents 1/6 and half of 1/3 is 1/6. It is interesting to note that, in the spirit of Krummheuer (1995) and Conner et al (2014a), this particular argumentation includes a nested sub-argument in which the data for the main argument becomes the claim, the warrant for the main becomes the data, and the backing for the main argument becomes the warrant. This is indicated in the diagram by the "Claim/Data," Data/Warrant," and so on. For a detailed discussion of the argumentation in the pre-assessment video narrative, including the diagrams (see Van Ness, 2017; Van Ness & Maher, 2019). The Standard Model that was developed to diagram the argumentation in the

Assessment VMCAnalytic was used as a key to track growth in teachers' descriptions, comparing their pre- and post-descriptions with the model for each of the 15 events.

Diagrams for Analysis of Teacher Growth

The teachers' pre-assessment and post-assessment descriptions and titles describing the children's argumentation became the "artifacts of practice" in the spirit of Van Es and Sherin (2008), and Toulmin's model was used to diagram these descriptions. To investigate the first research question and determine what teachers notice about children's argumentation when provided with narratives that show children engaged in argumentation, the teachers' descriptions of children's argumentation were analyzed. Attending very closely to the language teachers used, each of their descriptions was compared to the Standard Model for accuracy and then diagramed. Pre- and post-assessment descriptions were compared and changes relevant to the argumentation discussed were noted and interpreted. When elements of an argument were inferred rather than explicitly stated, the box with the implied element was created with dashed segments. If the structure of an argument was implied, the segments or arrows connecting elements were represented as dashed. When teachers mentioned elements of argumentation that might be true but were not specifically stated by the children in the event video clip, grey boxes with text were used. If a teacher's description included a reference to an implied element that was not included in the event, the element was represented as grey and dashed.

Figure 7 shows an example of one teacher's data set, including the teacher's pre-assessment description, post-assessment description, and the associated diagrams.

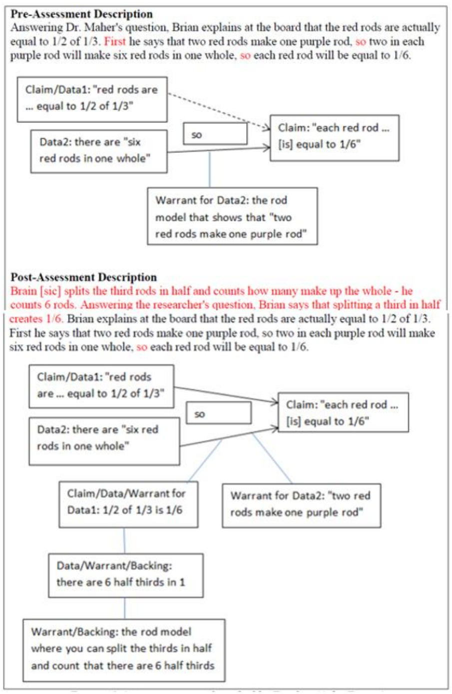


Figure 7. Example of one teacher's pre-assessment and post-assessment descriptions for event 6

To analyze the teachers' diagrams differences in each teacher's pre- and post-descriptions were described in detail in narrative form. For example, in Figure 7, in the pre-assessment, the teacher describes an argument with the claim: "Each red rod will be equal to 1/6" supported by data: "The red rods are actually equal to 1/2 of 1/3" and there are "six red rods in one whole" supported by warrant: the rod model that shows that two red rods make one purple rod. The teacher does not describe the support for the data (which is a claim in itself) that the red rods are equal to 1/2 of 1/3. Though uncertain, the use of "first" may indicate that the teacher intended the statement "the red rods are equal to 1/2 of 1/3," to support the claim that each red rod will be equal to 1/6, thus, the connection is implied, and a dashed arrow is used. Note that the "Data, so Claim" structure is indicated by the use of "so."

In the post-assessment, the teacher adds detail that describes more elements and structure in the argumentation. The teacher adds: "Brain [sic] splits the third rods in half and counts how many make up the whole - he counts 6 rods. Answering the researcher's question, Brian says that splitting a third in half creates 1/6," which functions as a sub-argument for the claim that is being used as data, that the red rods are equal to 1/2 of 1/3. The elements of this claim serve multiple purposes. The statement that 1/2 of 1/3 is 1/6 is data in one argument and a warrant in the sub-argument. The statement that there are six half-thirds in one is a warrant in one argument and backing in the sub-argument. The description of the rod model where you can split the thirds in half and count that there are six half thirds is backing in the sub-argument. Within the sub-argument, however, there is another argument that one might call a "sub-sub-argument," with the claim that 1/2 of 1/3 is 1/6 as its claim. In this sub-sub argument, the statement that there are six half thirds in one also plays the role of data and the statement about the rod model is the warrant. The analysis, then, showed that the teacher described more of the children's argumentation in the post-assessment than in the pre-assessment, including making more explicit the connection between the data that the red rods are equal to 1/2 of 1/3 and the claim that the red rod has the number name 1/6.

Coding for Growth

Once differences were described in narrative form, each teacher's pre-assessment description was compared to the post-assessment description to answer the second research question and determine whether there was a change in teachers' noticing of children's argumentation after the intervention, and if so, what was the pattern of that change. For this study, if teachers noticed more of the argumentation in children's discourse, the change was coded as growth in the noticing of argumentation. If there was no change in the argumentation described by teachers, it was coded as no growth. For this study, growth in teachers' noticing of children's argumentation was defined as teachers describing more of children's argumentation, specifically, describing more elements of argumentation—claims, counterclaims, warrants, or backing—or more of the structure of an argument. The structure of an argument included how the elements of argumentation were connected in an event and how argumentation in one event was connected to argumentation in previous events, or both. Included in the structure of an argument were instances in which a claim or a counterclaim is connected either implicitly or explicitly to a previous claim or if an argument resulted in the modification or refutation of a claim.

Other factors that were considered in determining whether more argumentation was described by teachers from pre- to post-assessment included when the implicit was made explicit; when statements that were not spoken by the children were eliminated, when details clarified uncertain elements, when more of the children's actual words were used, or when additional information that was relevant to the argumentation in the event was described. To not overstate the growth in teachers' noticing of children's argumentation from pre-assessment to post-assessment, a conservative measure of growth was applied: any growth was coded as 1, whereas no growth was coded as 0. It is important to note that "growth" was distinguished from "change." If teachers changed their descriptions from pre-assessment to post-assessment in a way that did not indicate that they noticed more argumentation, a score of 0 was given. Figure 8 shows the pre-assessment description and post-assessment description from Figure 7 side by side.

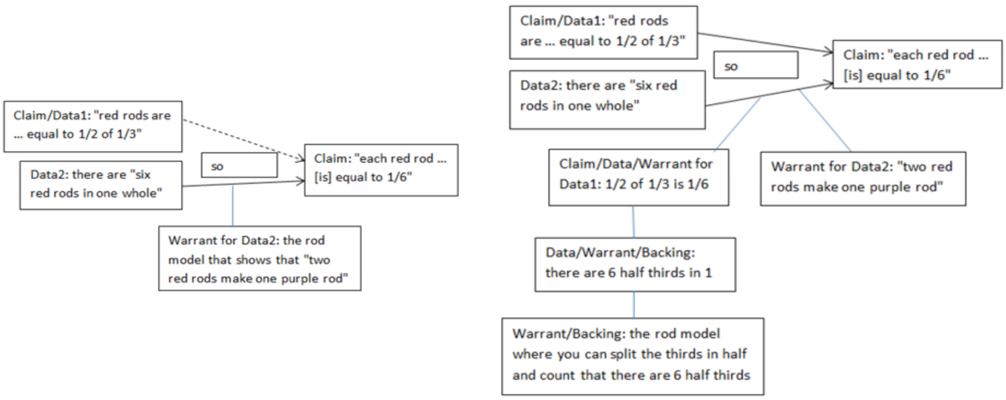


Figure 8. Side-by-side comparison of one teacher's pre- and post-descriptions

The post-assessment diagram to the right in Figure 8 shows that the teacher mentioned more elements of argumentation (as shown in boxes) and described more of how these elements were connected (as shown with connecting segments). Thus, a score of 1 is assigned to indicate growth.

Reliability for Teacher Description Diagrams

The diagrams created by the researchers for teachers' pre- and post-assessment descriptions were reviewed by trained graduate students and a mathematics researcher faculty for reliability. An Interrater reliability of 80% was achieved. Discrepancies, when they emerged were, for the most part, focused on what role children's statements played in the argumentation, rather than whether statements were elements of argumentation. As discussed by Krummheuer (1995, 2007) and others (Conner et al., 2014b; Yackel, 2002), diagramming children's argumentation can be challenging because arguments often do not include all of the elements of argumentation since some elements might be implied, rather than made explicit and the order in which elements are contributed to support the arguments might vary. Discrepancies that emerged were discussed and changes were made after negotiation.

Results

To further explore the second research question was there a change in the teachers' noticing of the children's argumentation, and if so, what was that change a variety of statistical analyses were applied to the growth data. These analyses are described in the following sections.

Teacher Growth in the Description of Argumentation

The preservice teachers' growth was averaged across 15 events. It was found that the growth ranged from 20% to 86.7% with an average growth of 52.6%. Additionally, growth for each event was averaged across the 9 teachers. The growth ranged from 22.2% in Event 1 to 88.8% in Events 6 and 15. These results are summarized in Table 5 and Table 6.

Table 5. Preservice teacher growth averaged across 15 video events

Teacher ID	1	2	3	4	5	6	7	8	9	Average Growth
% Of Events with Teacher Growth	80	53.3	53.3	20	46.7	33.3	40	86.7	60	**52.6**

Table 6. The mean growth rate for video events averaged across 9 preservice teachers

Video Events	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
% Teachers with Growth	22.2	44.4	44.4	44.4	33.3	88.9	44.4	44.4	77.8	22.2	66.7	66.7	55.6	44.4	88.9

Argumentation Cycles

To better understand the nature of the growth in teachers' noticing of argumentation, data from Table 5 the percent of teachers with growth in each video event was graphed. This graph is shown in Figure 9.

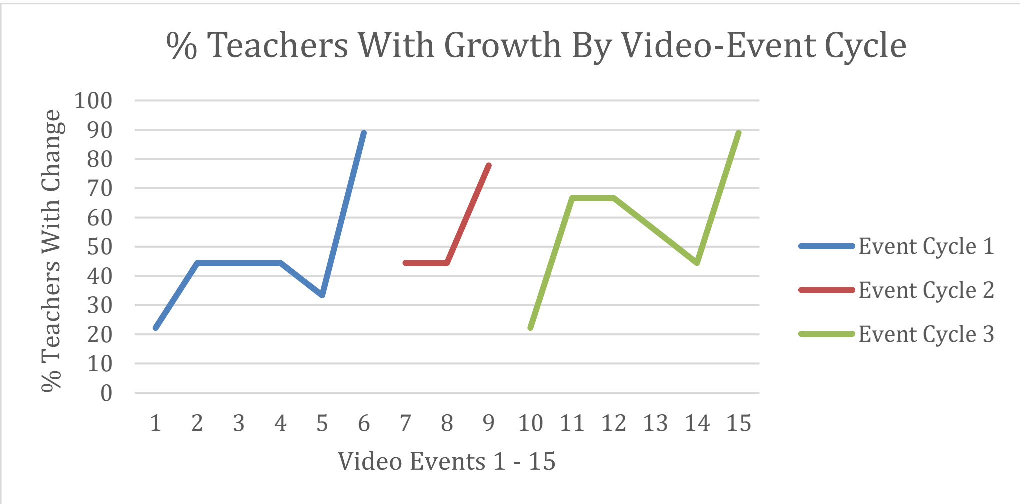


Figure 9. Percent of teachers exhibiting pre/post-study growth in the description of student argumentation for each of 15 video events

The graph suggested that peaks of growth in certain events appear to occur in three cycles: Cycle 1 – Events 1 through 6, Cycle 2 – Events 7 through 9, and Cycle 3 – Events 10 through 15. This observation prompted researchers to investigate the interrelationship of the events within each of the three cycles. This review is described below.

Cycle 1 (Events 1 through 6)

As described in previous sections, at the beginning of the session, children are asked to determine which is greater, 1/2 or 1/3, and by how much. In Cycle 1 (Events 1 through 6), children use Model 1, giving the train of orange and red rods the number name 1. From the first or Initial Event (Event 1) through the Intermediate Events (Events 2 through 5), children present arguments and agree that 1/2 is greater than 1/3 (Events 1 through 3) and then pose challenges and present arguments and counterarguments for the claim that 1/2 is 1/3 bigger than 1/3 (Events 4 through 5). The cycle ends in the final or Terminal Event (Event 6) with a counterclaim that 1/2 is bigger than 1/3 by 1/6, not 1/3, and a well-supported counterargument that focuses on the idea that the rod that represents how much bigger 1/2 is than 1/3 is half of the rod that has been named to represent 1/3 and one-half of 1/3 is 1/6, not 1/3. As teachers observed children's argumentation from the Initial Event to the Terminal Event, the growth in the descriptions from pre- to post-assessment increased.

Cycle 2 (Events 7 through 9)

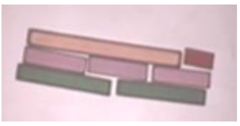
In Cycle 2 (Events 7 through 9), children begin a new cycle of argumentation in which they change what rod represents the unit. The new discussion centers around Model 2, in which the dark green rod represents unit, 1. In the Initial Event (Event 7) some children claim that 1/2 is greater than 1/3 by 1/6 and use Model 2 to support their claim; other children claim that 1/2 is greater than 1/3 by 1/3. By the end of the Terminal Event (Event 9), some children conjectured that 1/2 is greater than 1/3 by both 1/3 and 1/6. As with Cycle 1, children begin the argument by making a claim and as they present their argumentation throughout the cycle, the teachers notice more of children's argumentation from pre-assessment to post-assessment.

Cycle 3 (Events 10 through 15)

In Cycle 3, children return to using Model 1 (the model with a train of the orange and red rod as the unit) to support and refute the claims that have been made about how much bigger 1/2 is than 1/3. In the Initial Event (Event 10) of Cycle 3 children use the model to reiterate the invalid argument that, when the orange and red rod train has the number name 1, 1/2 is greater than 1/3 by 1/3. In the Intermediate Events (Events 11 through 14) claims and counterclaims are challenged with children arguing that 1/2 is not greater than 1/3 by 1/3, but rather, 1/2 is greater than 1/3 by 1/6 and supporting their counterclaims with arguments with the rod models. In the Terminal Event (Event 15) a child summarizes various arguments from the previous cycles and another child synthesizes the argumentation by concluding that, whether the orange and red rod train (Model 1) or the green rod, is given the number name 1 (Model 2), the difference between 1/2 and 1/3 is 1/6 and the argumentation is concluded. As with Cycles 1 and 2, in Cycle 3, the teachers' descriptions of the argumentation in the post-assessment compared to the pre-assessment captured more complexity of the children's argumentation in the Terminal Event than in the Initial and Intermediate Events.

Thus, the flow of the argumentation in the assessment analytic falls into three parts: (1) initial argumentation that when the orange and red rod train has the number name 1, 1/2 is greater than 1/3 by 1/3 (Cycle 1); (2) argumentation that when the dark green rod is given the number name 1, 1/2 is greater than 1/3 by 1/6 (Cycle 2); and (3) a return to the argumentation that when the orange and red rod train has the number name 1, 1/2 is greater than 1/3 by 1/3 and 1/2 is greater than 1/3 by 1/6 (Cycle 3). These critical events are summarized in Table 7.

Table 7. Summary of argumentation cycles

	Event	Model Used	Flow of Argumentation Cycles
Cycle 1	Event 1 through Event 6		Initial argumentation that when the orange and red rod train has the number name 1, 1/2 is greater than 1/3 by 1/3
Cycle 2	Event 7 through Event 9		Argumentation that when the dark green rod has the number name 1, 1/2 is greater than 1/3 by 1/6
Cycle 3	Event 10 through Event 15		Return to argumentation that when the orange and red rod train has the number name 1, 1/2 is greater than 1/3 by 1/3 and 1/2 is greater than 1/3 by 1/6

The preservice teachers showed the most growth in the noticing of argumentation in the final, culminating event in each of these cycles. Table 8 summarizes the cyclical nature of the teachers' growth.

Table 8. Teacher pre-to-post-assessment growth rate in describing student argumentation

	Argumentation Context	Event Number	Mean % Growth
Cycle 1	Children choose a train of orange and red rods as the unit	1-5	37.7
		6	**88.9**
Cycle 2	Children choose the green rod as the unit	7-8	44.4
		9	**77.8**
Cycle 3	Teacher making sense of the different children's argument solutions	10-14	51.1
		15	**88.9**

Additional Analyses

To address the third research question, whether there were teacher differences, if any, in examining teacher growth across the series of 15 classroom videos, and how those differences affect teachers' noticing of argumentation, two additional analysis platforms were used: The

Partition Platform and the Logistics Regression Platform of JMP Pro Statistical Discovery Software.

JMP Pro Partition Platform Analysis

The JMP Pro Partition Platform was employed to recursively partition the study data according to a relationship between predictors and the teachers' likelihood of growth in the description of the children's argumentation. This algorithm searches all possible splits of predictors creating a decision tree of partitions to best predict the percent of teachers with growth. In employing this platform, we included as a possible predictor the teacher growth pattern observed in Figure 9, namely, three hypothesized cycles of upward growth trends: the first upward trend occurring from video events 1 through 6; the second from video events 7 through 9; and the third from video events 10 through 15. The Y response input variable to the Partition Platform, 1 or 0, representing growth or no growth, was the input for each of the 15 video events for each of the 9 teachers in the study. For the independent X factor variable(s) associated with a Y value we input: (1) the identification of each teacher (Teacher Identification) labeled 1 through 9, (2) the order of the event as it appeared in the VMCAnalytic (i.e., 1 through 15); (3) The category of each video event as Initial/Intermediate or a Terminal Event within Video Cycles 1, 2, or 3 of Figure 9, and (4) The Video-Cycle Identification (Cycle Identification) of 1, 2, or 3 of the event as displayed in Figure 9.

The results of the Partition Platform analysis identified two factors associated with differing levels of teacher growth. The highest-level factor identified the dichotomy between Initial/Intermediate Events and Terminal Events observed in Figure 9. The second highest factor identified two different groups of teachers distinguished by different growth levels on the Initial/Intermediate Events – Teacher Group (TG) 1 with Teacher Identifications 1, 2, 3, 5, 8, and 9 having an average Initial/Intermediate event growth of 58.3% and Teacher Group 2 with Teacher Identifications 4, 6, and 7 having a lower average Initial/Intermediate event growth of 16.7%. In contrast, these two teacher groups averaged 83.3% and 88.9% growth, respectively, on the Terminal Events. The other predictor factors including Event Order Identification 1 – 15 and Event Cycle Identification 1 – 3 did not significantly improve the prediction model of teacher growth beyond the two significant predictor factors mentioned above.

Logistic Regression Analysis

Then, a logistic regression analysis was used to examine the children's argumentation using the growth predictors described above to estimate the parameters of a model for predicting each of 135 individual teacher probabilities of growth (9 teachers and 15 events per teacher). Based upon the reduced design factors derived from the Partition Analysis the logistics regression model made use of the following predictor factors: (1) Teacher Identifications nested within their respective teacher group identified by the Partition Platform analysis results as a test of the hypothesis of no difference in mean growth of teachers within the same teacher group, (2) Video Event Category of Initial/Intermediate Event or Terminal Event as a test of the hypothesis of no difference in the overall mean growth of teachers associated with the Initial/Intermediate Events compared to the Terminal Events, and (3) Teacher Groups Interaction with Video Event

Category of Initial/Intermediate or Terminal Event to test the null hypothesis that the growth differences of the two teacher groups are the same for both the Initial/Intermediate Events and Terminal Events (see Figure 10).

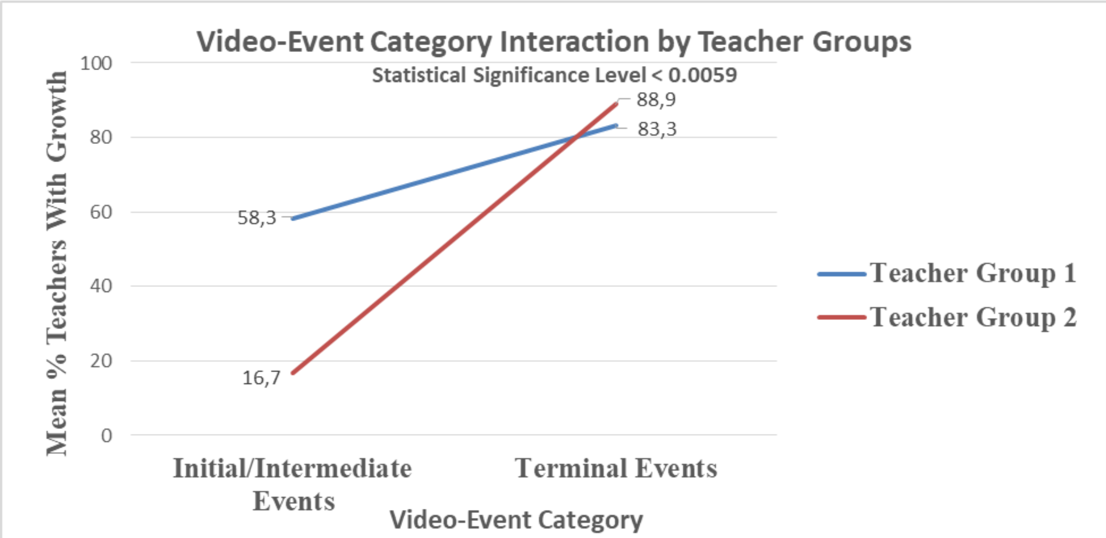


Figure 10. Interaction of teacher groups and video event categories

In addition to the factors identified in the Partition Analysis, the three-level Event Cycles factor was added to the Logistic Regression model as follows: (4) the interaction between Video Event Category and Video-Cycle Identification to test the null hypothesis that teacher growth differences of different levels of the Video Event Category do not significantly differ across Video Cycles 1, 2, and 3 (see Figure 11) and (5) the three-way interaction between Video Event Category, Video-Cycle Identification, and Teacher Groups (see Figure 12).

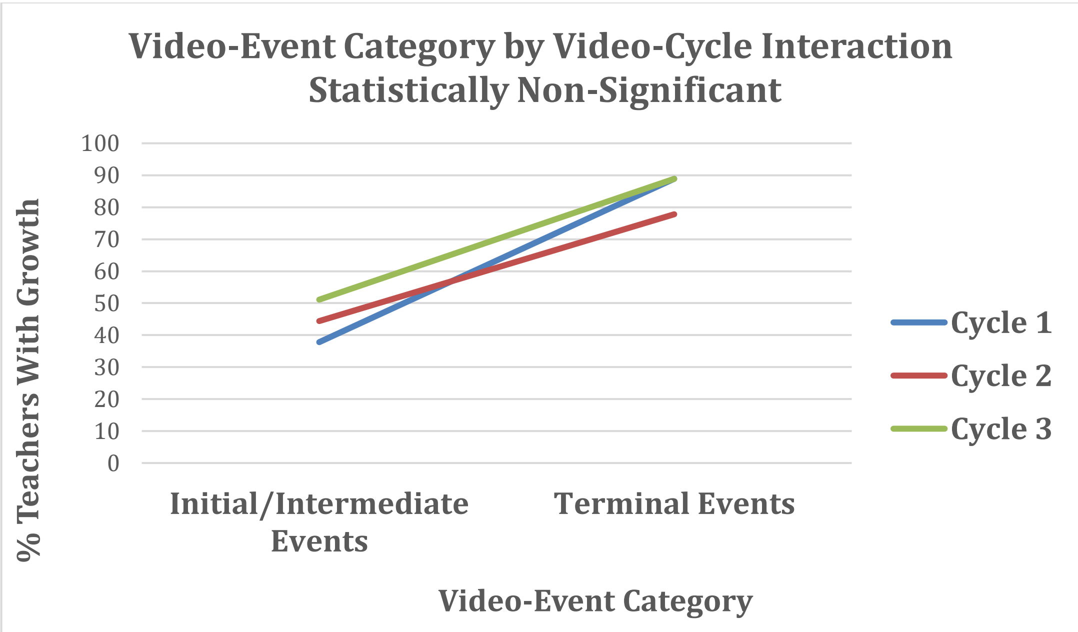


Figure 11. Relationship between video event category and event cycle ID

Figure 12 shows the video category by video cycle interaction which is consistent across Teacher Groups 1 and 2.

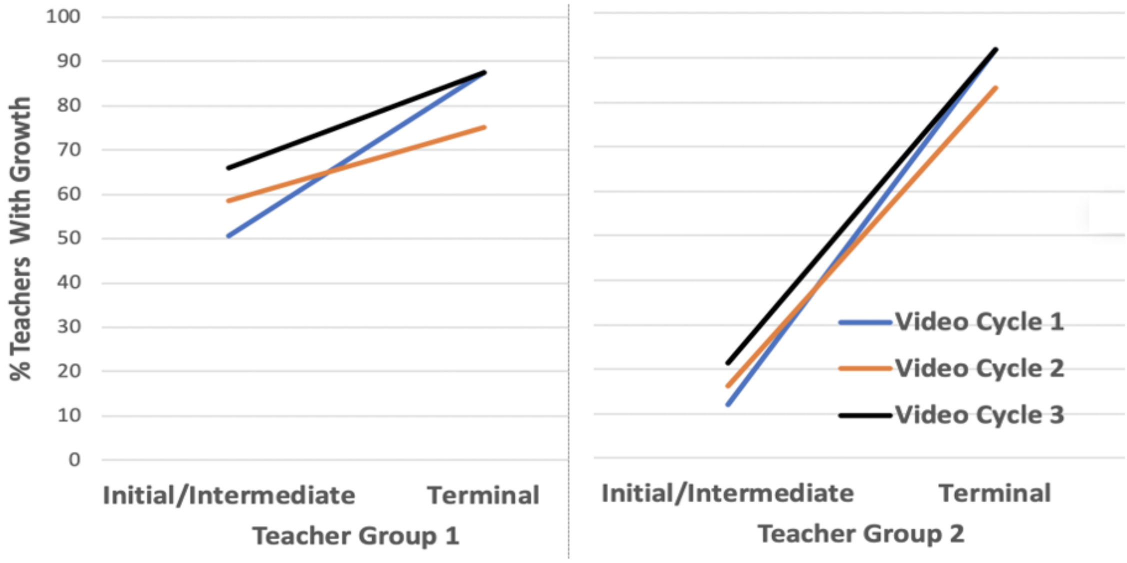


Figure 12. Three-way effect: video event category by video cycle ID by teacher groups

Table 9 provides the analysis results of the significance of the factors of the logistic regression model. Of the eight sources of the variation of teacher growth that comprise the Logistic Regression Model of Table 9, there are only two statistically significant sources of variation: Likelihood Ratio (LR) Test Effect (2) – Video Event Category of Initial/Intermediate versus Terminal Event Groups (EGs) (See Table 7) and LR Test Effect (3) - Interaction of Video Event Category and Teacher Groups (See Figure 10).

Table 9. Ordinal logistics fit: effect likelihood ratio testsTGs: teacher groups; EG: event group; A * B: the interaction effect between factors A and B

Likelihood Ratio (LR) Test Effects	N parm	DF	LR Chi-Square	Prob > Chi-Square
Teachers within TGs	7	7	12.76	0.0783
Video Event Category of Initial/Intermediate or Terminal Video EG	1	1	25.13	<0.0001
Video Event Category * TGs	1	1	7.58	0.0059
TGs	1	1	4.44E-7	0.9995
Video Event Cycles 1, 2, and 3	2	2	3.50	0.1741
Video Event Cycles * Video Event Category	2	2	1.96E-8	1.0
Video Event Cycles * TGs	2	2	4.64	0.0983
Video Event Cycles * Video Event Category * TGs	2	2	6.02E-9	1.0

We found that the mean percent of teachers with growth averaged across the Initial/Intermediate Event Groups was 44.4% in contrast to an average of 85.2% of the teacher's exhibiting growth in the Terminal Event Groups a statistically significant difference at a significance level of less than 0.0001. This data is summarized in Table 10.

Table 10. Logistic regression means estimates and test of equality of mean teacher growth of initial/intermediate video events in comparison to terminal video events

Video Event Categories			
Initial/Intermediate Video Event Group	**Terminal Video Event Group**	**Test Statistic**	**Statistical Significance**
44.4	85.2	LR Effect (2): LR Chi-Square = 25.13	<0.0001

Table 11 is a two-way classification table of the percent of teachers with growth for each of the two teacher groups and each of the two Video Event Categories (Initial/Intermediate and Terminal Events). LR Effect (3) of Table 8 reports this is a statistically significant interaction effect. Figure 10 is a corresponding graph of this interaction effect from which it is evident that the Teacher Group 1 and Teacher Group 2 lines are not parallel. Specifically, we note that while Teacher Group 2 has a low mean percent of teachers with growth on the Initial/Intermediate Events compared to Teacher Group 1, the two teacher groups have comparable mean growth estimates on the Terminal Events.

Table 11. Logistic regression means percent teacher growth estimates of interaction effects of teacher groups and video event categories of initial/intermediate versus terminal egs

Teacher Groups	**Initial/Intermediate Event Group**	**Terminal Event Group**	**Interaction Effect**	**Statistical Significance**
Teacher Group 1	58.3	83.3	LR Effect (3): LR Chi-Square = 7.58	0.0059
Teacher Group 2	16.7	88.9		

Figure 11 is a graph of the 2-way effect: Video Event Category by Video Event Cycle ID. This 2-way effect is found not to be statistically significant (LR Chi-square = 1.96E-8, Prob > Chi-Square = 1.0). A non-statistically significant 2-way effect is evident by noting in Figure 11 that teacher mean growth differences of the Initial/Intermediate and Terminal Event levels of the Video Event Category do not significantly differ across Video Cycles 1, 2, and 3.

Figure 12 is a graph of the three-way effect: Teacher Group by Video Event Category by Video-Cycle Identification. This 3-way effect is found not to be statistically significant (LR Chi-Square = 6.02E-9, Prob > Chi-Square = 1.0). A non-statistically significant 3-way effect is evident by noting in Figure 12 that for both Teacher Groups 1 and 2, the 2-way Video Category by Video-Cycle Interactions are similar, that is, the mean percent of teachers with growth is relatively low for Initial/Intermediate Events and high for Terminal Events.

Table 12. Logistic regression mean estimate of initial/intermediate terminal video event teacher means growth effects for each of three video cycles

Video-Cycle ID	Initial/Intermediate Video Events: %Teachers With Growth	Terminal Video Events: %Teachers with Growth	Interaction of Video Event Category (Initial/Intermediate - Terminal) and Video-Cycle IDs 1, 2, and 3	Statistical Significance
1	37.8	88.9	LR Effect (6): Chi-Square = 1.96E-8	ns
2	44.4	77.8		
3	51.1	88.9		

Summary

The logistics regression analysis provides evidence of three video cycles of the video events with the Initial/Intermediate Events averaging a teacher growth rate of 44.4% and the associated Terminal Events averaging 85.2%, statistically significant at a significance level of < 0.0001 (See Table 9). The finding provides evidence of teachers' growth in noticing the children's argumentation. The data also provide evidence that the significant mean teacher growth of a Terminal Event relative to its associated Initial/Intermediate Events is consistent across the three observed video cycles as observed in Figure 9 and based upon a non-statistically significant video category by Video-Cycle Identification interaction as displayed in Table 11 and described in Figure 11.

Two different groups of teachers were identified, one group with a mean growth level on the Initial/Intermediate Events of 58.7% and one group with a mean growth level on the Initial/Intermediate Events of 16.7%. However, the mean growth levels on the associated Terminal Events were comparable at 83.3% and 88.9% respectively. The statistical significance of this conclusion is based upon a statistically significant video event category by teacher group interaction at a statistically significant level < 0.0059 (see Table 10 and Figure 10). The data also provide evidence that the video event category by Video-Cycle Identification interaction is consistent across the two teacher groups. This conclusion is based upon a non-statistically significant video category by Video-Cycle Identification by teacher interaction as displayed in Figure 11 and shown in Table 8 with LR Effect = 6.02E-9. These results indicate the nature of teacher group differences in examining teacher growth across the series of 15 classroom videos.

It is evident from these data that teachers recognized more of the children's argumentation after the intervention. Teachers noticed, for example, more claims, data, backing, counterclaims, and counterarguments and their connectedness after the intervention. The natural growth in the children's argumentation noticed by the teachers occurred repeatedly over 3 video event cycles after the presentations of arguments and counterarguments in the respective video cycle rather than after the observation of a single event that does not portray the full argument.

Conclusions and Implications

After the initial analysis of teachers' growth from the pre-assessment descriptions to the post-assessment descriptions, the unit of analysis shifted from individual events to clusters of events. The results indicate that teachers are more able to recognize argumentation when they see a complete story, rather than a particular episode. This research supports what other researchers have found, that studying classroom videos of children's argumentation supported teachers' noticing of argumentation in student discourse (Sherin & Van Es, 2005; Van Es & Sherin, 2002; 2008). However, this study adds to the literature in the finding that teachers are supported by observing video stories, which are evidenced in the different cycles in the video narrative. Each cycle is comparable to a developing story and the evidence shows that teachers' growth in noticing increases as the story unfolds. When a new story begins (i.e., in an Initial Event), the growth in teacher noticing is relatively low. The height of the growth in teacher noticing is at the end of the story, or in the Terminal Event, right before the discourse shifts to a new argument. This indicates that teachers noticed more argumentation after all of the students shared their arguments about the solution than after a single or pair of students posed an argument to support the solution. This study, then, shows strong evidence that the video story that is told is an important factor in teachers' growth in noticing children's argumentation. The fact that this pattern of growth was replicated three times throughout the 15 video clips makes this a compelling result. Thus, the implication of the research is, that rather than having teachers analyze short instances of children engaging in argumentation, teachers benefit from studying full cycles of argumentation, or classroom interactions in which there are opportunities to view the contributions of all of the children who engaged in the argument.

The findings in this study also suggest that: (1) The functionality in the RUanalytic tool that allowed teachers to write descriptions and titles associated with video clip events was useful, since with it, teachers generated the descriptions that became the data that were analyzed in this study, (2) Recognizing argumentation can be learned, (3) As suggested by Brunvand and Fishman (2006), the analysis of VMCAnalytics designed to illustrate argumentation can be an effective tool that supports this learning, and (4) With video narratives of students engaging in authentic argumentation, teachers can better attend to student interaction and discourse.

In this study, teachers, after studying story-line narratives, described more details in the children's argumentation in terms of the number of elements and structure of the argumentation. Thus, the results of this research suggest that the recognition of argumentation can be supported by studying video narratives developed to show what student argumentation looks like in problem-solving settings that foster argumentation and provide strong evidence that effective narratives tell a complete story of children's authentic classroom experience in which all voices were heard. The findings suggest that the presentation of a single child's argument which may or may not be correct has less of an impact on teachers than when several students present their reasoning, especially when they point out invalid reasoning that led to faulty solutions

Another finding of this study is that teachers can learn to notice argumentation, an important outcome, considering the importance that current National Teachers of Mathematics Standards (NCTM 1989; 2000) and extant mathematics education literature (Bieda & Lepak, 2014; Krummheuer, 1995; Schwarz, 2009; Whitenack & Yackel, 2002) place on argumentation and also in light of Wagner and colleagues assertion that it is "critically important" to "foster and support student argumentation," and Jacobs et al (2010), and Whitenack and Yackel's (2002) emphasis that professional noticing (i.e., of mathematical practices such as argumentation) needs to be specifically supported. Supporting teachers' growth in noticing student argumentation is a logical first step toward helping teachers promote argumentation in their teaching.

With a growing emphasis on online learning and teaching, the readily accessible database of the Video Mosaic Collaborative (www.videomosaic.org) with its resources such as the RUAnalytic tool to create more video narratives, accessible worldwide, can provide more resources for research and practice, such as in-person or online courses as part of teacher preparation or teacher professional development programs, as well as for onsite or remote workshop series designed to support participants understanding and recognition of student argumentation. We recommend the use of video narratives as a valuable resource for preservice and in-service teacher education, especially when it is not feasible to observe in-person classroom instruction and student learning.

Additionally, when more complete episodes of student reasoning are offered, including invalid as well as valid arguments, teachers have the opportunity to notice obstacles in reasoning that might not otherwise be anticipated. The finding that after observing a succession of arguments about a model (a cycle) as displayed in the video narratives, in contrast to observing a single argument about a model, preservice-teacher noticing improved, suggests that in offering more complete stories of children's reasoning about a particular problem, more argumentation is likely to be noticed.

Limitations and Areas of Further Study

It is widely accepted that videos of children's learning are effective tools in teacher education. The assessment video narrative created for this study enabled the tracing of the evolution of growth in children's understanding of fraction comparisons as they engaged in argumentation. The use of classroom video narratives of children engaged in this argumentation with preservice teachers was shown to be an effective tool in this study. The mathematical content of this study was limited to fraction topics. We recommend the creation of video narratives that tell a complete story of the engagement of children in mathematics learning in other mathematical content areas.

Teachers viewing cycles of children's argumentation showed improved success as compared with viewing isolated events, suggesting the benefit of viewing cycles of video narratives. The participants in this study were limited to 9 pre-service teachers in a secondary teacher preparation program. A study with in-service teachers, both at the elementary and secondary levels, followed by actual classroom implementations of the same or similar tasks, might provide insight into whether teachers who learn to recognize children's argumentation from video narratives are successful in promoting argumentation in their classrooms.

A conservative approach to coding for growth was taken in this study to not overstate the growth in teachers' noticing of children's argumentation from pre-assessment to post-assessment, where any growth, was coded as 1, and no growth was coded as 0. A deeper analysis of teacher growth could provide further insights into how video narratives can support teachers' noticing of student argumentation.

In recent times, the need for readily available online intervention tools, such as the VMCAnalytics created for this project, has never been more evident. The contributions of this study include the finding that it is the video story, showing children's co-construction of cycles of arguments, rather than single episodes of children's reasoning, that provides added value in supporting teachers' learning about children's argumentation. The Video Mosaic Collaborative and its available tools provide for the creation of new video narratives that can be widely accessible for in-person or online use. Expanding the development and use of these video stories can support teachers' noticing of children's arguments which can provide a foundation for creating classroom environments that promote engagement in student argumentation.

References

Agnew, G., Mills, C. M., & Maher, C. A. (2010). VMC analytic: Developing a collaborative video analysis tool for education faculty and practicing educators. In *Proceedings of the Annual Hawaii International Conference on System Sciences*. https://doi.org/10.1109/HICSS.2010.438

Akkus, M. (2016). The common core state standards for mathematics. *International Journal of Research in Education and Science*, *2*(1), 49–54. https://doi.org/10.21890/ijres.61754

Ball, D. L., Hoyles, C., Jahnke, H. N., & Movshovitz-Hadar, N. (2002). The teaching of proof. In L. I. Tatsien (Ed.), In *Proceedings of the International Congress of Mathematicians*. Retrieved from https://doi.org/10.1007/978-3-031-04313-0_8

Berenson, S. B. (2012). Improving Teacher Practice with Action Learning. *Teachers' Life-cycle from Initial Teacher Education to Experienced Professional*, 112.

Bieda, K. N. (2010). Enacting proof-related tasks in middle school mathematics: Challenges and opportunities. *Journal for Research in Mathematics Education*, *41*(4), 351–382. https://doi.org/10.5951/jresematheduc.41.4.0351

Bieda, K. N., & Lepak, J. (2014). Are You Convinced? Middle-Grade Students' Evaluations of Mathematical Arguments. *School Science and Mathematics*, *114*(4), 166-177. https://doi.org/10.5951/mathteacmiddscho.20.4.0212

Brunvand, S., & Fishman, B. (2006). Investigating the impact of the availability of scaffolds on preservice teacher noticing and learning from video. *Journal of Educational Technology Systems*, 35(2), 151-174. https://doi.org/10.2190/l353-x356-72w7-42l9

Conner, A., Singletary, L. M., Smith, R. C., Wagner, P. A., & Francisco, R. T. (2014a). Identifying Kinds of Reasoning in Collective Argumentation. *Mathematical Thinking and Learning*, 16(3), 181-200. https://doi.org/10.1080/10986065.2014.921131

Conner, A., Singletary, L. M., Smith, R. C., Wagner, P. A., & Francisco, R. T. (2014b). Teacher support for collective argumentation: A framework for examining how teachers support students' engagement in mathematical activities. *Educational Studies in Mathematics*, 86(3), 401-429. https://doi.org/10.1007/s10649-014-9532-8

Derry, S. J. (2007). *Guidelines for video research in education: Recommendations from an expert panel*. Chicago: Data Research and Development Center, University of Chicago. Retrieved from http://drdc.uchicago.edu/what/video-research-guidelines.pdf

Gomoll, A. S., Sigley, R., Winter, E., Hmelo-Silver, C., & Maher, C. (2015). Constructing multimedia artifacts with pre-service and in-service teachers: Problem solving in a heterogeneous technology learning environment. In *CEUR Workshop Proceedings*. Retrieved from https://ceur-ws.org/Vol-1411/paper-08.pdf

Hmelo-Silver, C. E., Maher, C. A., Agnew, G., Palius, M., & Derry, S. J. (2010). The video mosaic: design and preliminary research. In *Proceedings of the 9th International Conference of the Learning Sciences, 2*, 425-426. Retrieved from https://repository.isls.org/bitstream/1/2888/1/425-426.pdf

Hmelo-Silver, C. E., Maher, C. A., Alston, A., Palius, M. F., Agnew, G., Sigley, R., & Mills, C. (2013). Building multimedia artifacts using a cyber-enabled video repository: The VMCAnalytic. In *2013 46th Hawaii International Conference on System Sciences*, 3078-3087. IEEE. https://doi.org/10.1109/hicss.2013.122

Hmelo-Silver, C. E., Maher, C. A., Palius, M. F., Sigley, R., & Alston, A. (2014). Showing what they know: Multimedia artifacts to assess learner understanding. In *Proceedings of International Conference of the Learning Sciences,1*, 410–417. Retrieved from https://repository.isls.org/bitstream/1/1143/1/410-417.pdf

Jacobs, V. R., Lamb, L. L., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, 169-202. https://doi.org/10.5951/jresematheduc.41.2.0169

Krummheuer, G. (1995). The ethnography of argumentation. In Cobb, Paul, Bauersfeld, Heinrich (Eds). (1995). *The emergence of mathematical meaning: Interaction in classroom cultures. Studies in mathematical thinking and learning series*., (pp. 229-269). Hillsdale, NJ, England: Lawrence Erlbaum Associates, Inc, xi. (https://doi.org/10.4324/9780203053140-11)

Krummheuer, G. (2007). Argumentation and participation in the primary mathematics classroom: Two episodes and related theoretical abductions. *The Journal of Mathematical Behavior*, 26(1), 60-82. https://doi.org/10.1016/j.jmathb.2007.02.001

Maher, C. A. (2010). Children's reasoning: Discovering the idea of mathematical proof. In *Teaching and learning proof across the grades* (pp. 120-132). Routledge. https://doi.org/10.4324/9780203882009-7

Maher, C. A. (2011). Supporting the development of mathematical thinking through problem solving and reasoning. In *Proceedings of 35th Conference of the International Group for the Psychology of Mathematics Education*, *1*, 85-90.

Maher, C. A., & Sigley, R. (2014). Task-based interviews in mathematics education. In *Encyclopedia of Mathematics Education* (pp. 579-582). Springer Netherlands. https://doi.org/10.1007/978-94-007-4978-8_147

Maher, C. A., & Yankelewitz, D. (2017). *Children's reasoning while building fraction ideas*. Springer. https://doi.org/10.1007/978-94-6351-008-0

Maher, C. A., Landis, J., & Palius, M. F. (2010). Teachers attending to student reasoning: Using videos as tools. *Journal of Mathematics Education*, *3*(2), 1–24.

Maher, C. A., Palius, M. F., & Mueller, M. (2010). Challenging beliefs about children's mathematics learning through video study. In *Proceedings of the 32nd annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, *4*, 885-992.

Maher, C. A., Palius, M. F., Maher, J. A., Hmelo-Silver, C. E., & Sigley, R. (2014). Teachers Can Learn to Attend to Students' Reasoning Using Videos as a Tool. *Issues in Teacher Education*, 23(1), 31-47. Retrivied from https://www.itejournal.org/wp-content/pdfs-issues/spring-2014/07maheretal.pdf

Maher, C., & Martino, A. (1996). The development of the idea of mathematical proof: A 5-year case study. *Journal of Research in Mathematics Education*, 27 (2), 194-214. https://doi.org/10.5951/jresematheduc.27.2.0194

Martinez, M. V., Castro Superfine, A., Carlton, T., & Dasgupta, C. (2015). Examining the Impact of a Videocase-based Mathematics Methods Course on Secondary Preservice Teachers' Skills at Analyzing Students' Strategies. *Journal of Research in Mathematics Education*, *4*(1), 52–79. https://doi.org/10.4471/redimat.2015.59

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics (NCTM). (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NTCM.

Otto, J. J., & Ralston, S. L. (2012). Disseminating equine research and teaching videos through the institutional repository: A collaboration. *Journal of Agricultural & Food Information*, 13(1), 64-77. https://doi.org/10.1080/10496505.2012.638247

Palius, M. F., & Maher, C. A. (2011). Teacher education models for promoting mathematical thinking. In *Proceedings of 35th Conference of the International Group for the Psychology of Mathematics Education*, *1*, 321-328.

Palius, M. F., & Maher, C. A. (2013). Teachers learning about student reasoning through video study. *Mediterranean Journal for Research in Mathematics Education*, *12*(1-2), 39-55. Retrieved from https://www.cymsjournal.com/

Pedemonte, B. (2007). How can the relationship between argumentation and proof be analysed. *Educational Studies in Mathematics*, 66(1), 23-41. https://doi.org/10.1007/s10649-006-9057-x

Powell, A. B., Francisco, J. M., & Maher, C. A. (2003). An analytical model for studying the development of learners' mathematical ideas and reasoning using videotape data. *The Journal of Mathematical Behavior*, 22(4), 405-435. https://doi.org/10.1016/j.jmathb.2003.09.002

Schwarz, B. B. (2009). Argumentation and learning. *Argumentation and education: Theoretical foundations and practices*, 91-126. https://doi.org/10.1007/978-0-387-98125-3_4

Sherin, M. G., & Van Es, E.A. (2005). Using video to support teachers' ability to notice classroom interactions. *Journal of Technology and Teacher Education*, *13*(3), 475–491. Retrieved from file:///Users/maryloutardif/Dropbox/Articles_Textes/Mendeley/2005/Journal

Sigley, R., & Wilkinson, L. C. (2015). Ariel's cycles of problem solving: An adolescent acquires the mathematics register. *The Journal of Mathematical Behavior*, 40, 75- 87. https://doi.org/10.1016/j.jmathb.2015.03.001

Sriraman, B., & Umland, K. (2020). Argumentation in mathematics education. *Encyclopedia of Mathematics Education*, 63-66. https://doi.org/10.1007/978-3-030-15789-0_11

Star, J. R., & Strickland, S. K. (2008). Learning to observe: Using video to improve preservice mathematics teachers' ability to notice. *Journal of Mathematics Teacher Education*, 11(2), 107-125. https://doi.org/10.1007/s10857-007-9063-7

Tall, D., Yevdokimov, O., Koichu, B., Whiteley, W., Kondratieva, M., & Cheng, Y. H. (2012). Cognitive development of proof. In Proof and proving in mathematics education (pp. 13-49). Springer, Dordrecht.Thompson, D. R. (1996). Learning and Teaching Indirect Proof. *Mathematics Teacher*, *89*(6), 474-82. https://doi.org/10.1007/978-94-007-2129-6_2

Toulmin, S. (1958). *The uses of argument*. United Kingdom: Cambridge University Press. https://doi.org/10.1017/s0008197300003937

Toulmin, S. E. (2003). *The uses of argument*. Cambridge University Press. https://doi.org/10.1017/cbo9780511840005 Pedemonte, B. (2007). How can the relationship between argumentation and proof be analysed. *Educational Studies in Mathematics*, 66(1), 23-41. https://doi.org/10.1007/s10649-006-9057-x

Towers, J. (2007). Using video in teacher education. *Canadian Journal of Learning and Technology/La revue canadienne de l'apprentissage et de la technologie*, 33(2). https://doi.org/10.21432/t2dg6t

Van Es, E. A., & Sherin, M. G. (2002). Learning to Notice : Scaffolding New Teachers ' Interpretations of Classroom Interactions. *Journal of Technology and Teacher Education*, *10*, 571–596.

Van Es, E. A., & Sherin, M. G. (2008). Mathematics teachers' "learning to notice" in the context of a video club. *Teaching and teacher education*, 24(2), 244-276. https://doi.org/10.1016/j.tate.2006.11.005

Van Ness, C. K. (2015a) *Eighth Grader Stephanie's Argumentation about Meaning for the Square of a Binomial using Algebraic Reasoning*. [video]. Retrieved from http://dx.doi.org/doi:10.7282/T3FN180C

Van Ness, C. K. (2015b) *Eighth Grader Stephanie's Argumentation about Meaning for the Square of a Binomial using Geometric Reasoning*. [video]. Retrieved from http://dx.doi:10.7282/T3QZ2CRF

Van Ness, C. K. (2015c). *Fourth graders' argumentation about the density of fractions between 0 and 1*. [video]. Retrieved from http://dx.doi.org/doi:10.7282/T39K4CZC

Van Ness, C. K. (2016). Creating and Using Video Narratives for Secondary Preservice Teachers' Studying of Argumentation. In R. Huang and M. Strutchens (Co-chairs), *Preservice mathematics education of secondary teachers*. Topic Study Group 48 conducted at the 13th International Congress on Mathematical Education, Hamburg, Germany. https://doi.org/10.1007/978-3-319-62597-3_75

Van Ness, C. K. (2017). *Creating and using VMCAnalytics for Preservice Teachers' Studying of Argumentation*. [Unpublished doctoral dissertation]. Rutgers The State University of New Jersey, School of Graduate Studies. Retrieved from https://rucore.libraries.rutgers.edu/rutgers-lib/52283/PDF/1/

Van Ness, C. K., & Alston, A. S. (2017). Justifying the Choice of the Unit. In C. A. Maher & D. Yankelewitz (Eds.). *Children's Reasoning While Building Fraction Ideas* (pp. 83-94). Rotterdam, The Netherlands: Sense Publishers. https://doi.org/10.1007/978-94-6351-008-0_9

Van Ness, C. K., & Maher, C. A. (2019). Analysis of the argumentation of nine-year-olds engaged in discourse about comparing fraction models. *The Journal of Mathematical Behavior*, *53*, 13-41. https://doi.org/10.1016/j.jmathb.2018.04.004

Wagner, P. A., Smith, R. C., Conner, A., Singletary, L. M., & Francisco, R. T. (2014). Using Toulmin's Model to Develop Prospective Secondary Mathematics Teachers' Conceptions of Collective Argumentation. *Mathematics Teacher Educator*, 3(1), 8-26. https://doi.org/10.5951/mathteaceduc.3.1.0008

Weber, K., Maher, C., Powell, A., & Lee, H. S. (2008). Learning opportunities from group discussions: Warrants become the objects of debate. *Educational Studies in Mathematics*, 68(3), 247-261. https://doi.org/10.1007/s10649-008-9114-8

Whitenack, J., & Yackel, E. (2002). Making mathematical arguments in the primary grades: The importance of explaining and justifying ideas. *Teaching Children Mathematics*, 8(9), 524. https://doi.org/10.5951/tcm.8.9.0524

Wilson, M. C., & Jantz, R. C. (2011). Building value-added services for institutional repositories (IRs): Modeling the Rutgers experience. *Social Science Libraries: A Bridge to Knowledge for Sustainable Development*, 0–18. Retrieved from http://hdl.handle.net/2142/25875

Yackel, E. (2002). What we can learn from analyzing the teacher's role in collective argumentation. *The Journal of Mathematical Behavior*, 21(4), 423-440. https://doi.org/10.1016/s0732-3123(02)00143-8

Yankelewitz, D. (2009). *The Development of Mathematical Reasoning in Elementary Schools' Exploration of Fraction Ideas*. [Unpublished doctoral dissertation]. Rutgers The State University of New Jersey, School of Graduate Studies. Retrieved from https://rucore.libraries.rutgers.edu/rutgers-lib/28410/PDF/1/play/

Zack, V. (1997). "You have to prove us wrong": proof at the elementary school level. *21st Conference of the International Group for the Psychology of Mathematics Education*, *4*(1982), 291–298.